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Calculated heat capacity and magnetization of two-dimensional electron systems

Qiang Li, X. C. Xie, and S. Das Sarma
Center for Theoretical Physics, Department of Physics, University of Maryland, College Park, Maryland 20742
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We calculate the heat capacity and the magnetization of a weakly disordered two-dimensional electron gas as a function of temperature, electron density, and external magnetic field. The electronic density of states entering our theory includes the effects of Landau-level broadening through a self-consistent theory that treats impurity scattering and dielectric screening self-consistently. Our calculated results, in good qualitative agreement with existing measurements, show that the actual density of states is much broader than what would be inferred on the basis of the standard short-range scattering model. We find that, depending on relative magnitudes of temperature, level broadening, cyclotron energy, and chemical potential, the temperature dependence of the specific heat could show an interesting experimentally observable nonmonotonic behavior.

In the last few years it has become possible[1,2] to directly measure thermodynamics properties of a two-dimensional electron gas (2D EG) in the presence of a strong external quantizing magnetic field. These measurements (e.g., specific heat, magnetic susceptibility), which complement transport studies, are important because they provide *direct* information about the density of states (DOS) at the chemical potential (μ) of a 2D EG. Both extended and localized states contribute equally in equilibrium to a thermodynamic quantity such as the specific heat (C_v) or the magnetization (M). Transport, on the other hand, depends crucially on whether the chemical potential lies in the extended or the localized states in the DOS. In this paper, we will provide a theoretical calculation of the electronic specific heat[1] and the orbital magnetization[2] of a 2D EG in the presence of a strong, external magnetic field oriented perpendicular to the system. Our results are in very good qualitative agreement with the measurements,[1,2] and in agreement with the experimental results, we find that the strong-field DOS of a 2D EG is, in general, much broader than that implies by the zero-field mobility and the short-range[3] impurity scattering model. We also predict a novel temperature dependence of the electronic specific heat in these systems. Depending on the temperature, level broadening, and the cyclotron energy, the behavior of specific heat as a function of temperature could be very non-Fermi-liquid-like.

There have been earlier calculations[4] of the magnetic field dependence of electronic thermodynamic properties in a 2D EG, but these calculations usually neglect the subtle interplay between impurity scattering ("disorder") and dielectric screening that is invariably present in a 2D EG. In a pure (*no* disorder) 2D EG the DOS, $D(E)$, has δ-function-like singularities in the presence of a perpendicular magnetic field B:

$$D(E)=\frac{1}{2\pi l^2}\sum_n \delta(E-E_n)\,, \tag{1}$$

where $l=(c\hbar/eB)^{1/2}$ is the Landau radius and $E_n=(n+\frac{1}{2})\hbar\omega_c$ are the Landau levels (we neglect *any* spin splitting throughout this paper). This singular DOS is, of course, not seen in experimental measurements of thermodynamic properties. The standard[1-4] method of handling this divergence has been to use a Landau-level broadening (Γ_n) to suppress the singularity in Eq. (1). Theoretical results then become functions of the broadening parameter Γ_n which is taken[3] to be fixed by the mobility of the system at zero magnetic field:

$$\Gamma_n\equiv\Gamma=\hbar(2\omega_c/\pi\tau)^{1/2}\,, \tag{2}$$

where τ is the relaxation time corresponding to the zero-field mobility μ_0:

$$\tau=m\mu_0/e\,. \tag{3}$$

Equations (2) and (3) are derived[3] on the basis of the assumption that the randomly distributed impurity scattering centers interact with the electrons via a δ-function zero-range potential (the so-called "short-range" potential). In reality, the bare unscreened potential is Coulombic and the actual screened potential could be long ranged or short ranged depending on the parameters B and the Fermi level E_F. Specifically, when E_F lies at the middle (edge) of a Landau level screening is strong (weak) and the screened potential is short ranged (long ranged). Thus, scattering and screening determine[5] each other self-consistently and Γ reflects this self-consistent screening effect. In contrast to Eqs. (2) and (3), the broadening then becomes a complicated oscillatory function of B and E_F. The DOS, therefore, looks very different from the one implied by Eqs. (1)–(3). The actual situation[5] is somewhat more complicated by Landau-level coupling effects which introduce some asymmetry in the DOS.

We have recently developed[5] a theory for the DOS of a 2D EG which takes into account this self-consistent level broadening effect. In this paper we apply this theory to calculate thermodynamic properties of a 2D EG. Details for the calculation of the DOS can be found in Ref. 5, and in the following we use this self-consistent DOS for all our calculations. We neglect any temperature dependence of the DOS which is a reasonable assumption because we are

interested only in rather low temperature (0–10 K) properties where the temperature dependence of the DOS is exponentially small.

The specific heat can be written as

$$C_v = \int_{-\infty}^{+\infty} \frac{df(E)}{dT}(E-\mu)D(E)dE\,, \tag{4}$$

where $f(E)=(e^{\beta(E-\mu)}+1)^{-1}$ is the Fermi factor and $\beta=(k_BT)^{-1}$. Using the fact that the experimental systems employ constant electron densities, one can rewrite Eq. (4) as

$$C_v = \frac{1}{T}\int_{-\infty}^{+\infty} A(E,\beta)(E-\mu)^2 D(E)dE - \frac{1}{T}\frac{\left[\int_{-\infty}^{+\infty} A(E,\beta)(E-\mu)D(E)dE\right]^2}{\int_{-\infty}^{+\infty} A(E,\beta)D(E)dE}\,, \tag{5}$$

where

$$A(E,\beta)=\beta e^{\beta(E-\mu)}/(e^{\beta(E-\mu)}+1)^2\,. \tag{6}$$

The chemical potential μ is obtained from the total electron density N_s in the system:

$$N_s=\int_{-\infty}^{+\infty} f(E)D(E)dE\,. \tag{7}$$

The function $A(E,\beta)$ defined by Eq. (6) peaks around $E\approx\mu$ with a width of about k_BT. In zero magnetic field and for $k_BT\ll\mu$, Eq. (5) can be expanded in a Sommerfeld expansion to give the simple result

$$C_v=\frac{\pi^2}{3}k_B^2TD(E_F)\,, \tag{8}$$

where $E_F=\mu(T=0)$ is the Fermi energy and we retain only the linear order term in temperature. We find that even at the low temperatures explored experimentally (1–10 K) a result like Eq. (8) is not valid for a 2D EG in a strong magnetic field. In fact, C_v is nonlinear even in the temperature range 1–10 K (cf. Fig. 1) at a fixed value of B. We find theoretically that for $\hbar\omega_c\gg k_BT\gg\Gamma$, specific heat varies as Γ^2/T^2 instead of behaving like Eq. (8). We find that the actual behavior of $C_v(T)$ depends sensitively on relative magnitudes of k_BT, E_F, $\hbar\omega_c$, and Γ. In particular, $C_v(T)$ can show very pronounced nonmonotonic behavior [cf. Fig. 1(a)]. At fixed T, $C_v(B)$ looks similar to $D(E_F)$ apart from small additional structures (cf. Fig. 2) in between Landau levels. These structures can be identified as contributions from inter-Landau-level contributions.

In Figs. 1 and 2 we show our calculated specific heat for a 2D EG in GaAs heterojunction as functions of temperature (Fig. 1) for fixed B and of magnetic field (Fig. 2) for fixed T, respectively. For the purpose of comparison, we show our calculated DOS, $D(E_F)$, as insets in Fig. 2. One can see that $D(E_F)$ and C_v are similar in shape except for the inter-Landau-level contributions. In Fig. 1, C_v is linear in T only up to about 3 K, beyond which nonlinearity is pronounced. It turns out that the actual regime of linear behavior depends sensitively on whether $k_BT>\Gamma$ and $\hbar\omega_c>k_BT$. The temperature dependence of C_v is easily understood on the basis of the detailed shape of the DOS and the position of the chemical potential μ at some particular B and N_s values. The linear T dependence at small T in Fig. 1(a) corresponds to the usual Fermi-liquid-like behavior arising from intra-Landau-level contribution to C_v when μ lies well within a broadened Landau level. As T increases, C_v eventually decreases since there can be no excitation in the gap region between Lan-

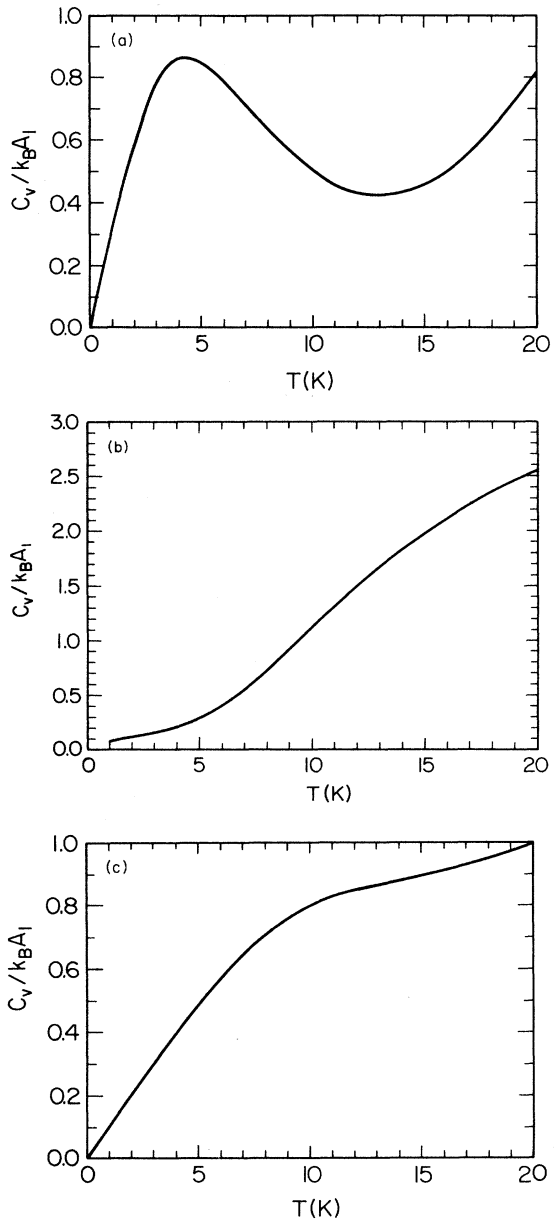

FIG. 1. Shows the electronic specific heat C_v as a function of temperature at fixed magnetic fields (B) for three different situations: (a) $N_s=2\times10^{11}$ cm^{-2}, $\mu_0=295000$ cm^2/V sec, $B=5.8$ T, $E_F=1.4749\hbar\omega_c$; (b) same as in (a) except $B=4.2$ T; $E_F=1.8577\hbar\omega_c$ is at the edge of Landau level; (c) $N_s=3\times10^{11}$ cm^{-2}, $\mu_0=80000$ cm^2/V sec, $B=8$ T, $E_F=1.5305\hbar\omega_c$. Here k_B is Boltzmann's constant, $A_1=eB/ch$.

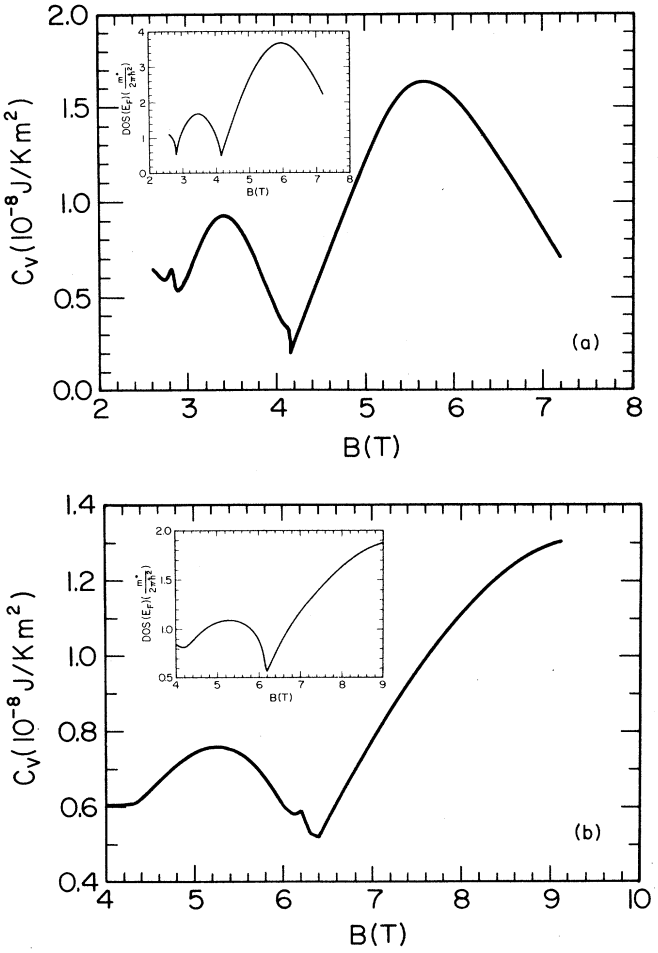

FIG. 2. Shows C_v as a function of B at fixed T: (a) sample corresponding to Figs. 1(a) and 1(b) with $T=4.2$ K; (b) sample corresponding to Fig. 1(c) with $T=4.2$ K. Insets show the density of states, $D(E_F)$, at the Fermi level calculated self-consistently (Ref. 5) corresponding to (a) and (b), respectively. m^* is the effective mass of 2D electron, for GaAs, $m^*=0.0665m_e$.

dau levels. For even higher temperatures, inter-Landau-level transitions become possible and C_v increases with T again. Thus, C_v shows pronounced nonmonotonic behavior for the high-mobility (gaps between Landau levels) and high-field situation [Fig. 1(a)]. The situations for high mobility and low field [Fig. 1(b)], and low mobility and high field [Fig. 1(c)], can also be explained in the same way. One finds monotonic behavior in both of these cases. Another noteworthy feature of our calculated C_v is the occurrence of the structure associated with an extra peak [compared with $D(E_F)$ which does not have any peak at the same B value] around $B\approx 2.5$ T in Fig. 2(a). A shoulderlike structure arising from such an extra peak can also be seen near the valley around $B\approx 4$ T. These structures in C_v are quite general features of Eq. (5) and in our system arise from inter-Landau-level excitations.

Our specific-heat results agree, qualitatively, rather well with the recent experimental data of Wang *et al.*[1] Our results are, however, in qualitative disagreement with

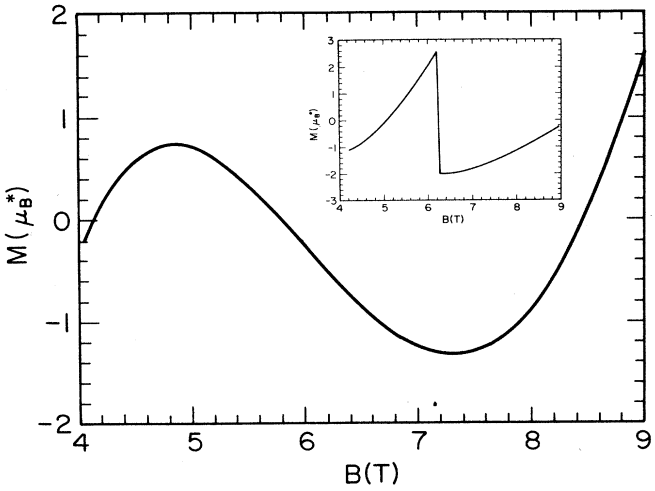

FIG. 3. Shows the orbital magnetization M as a function of the magnetic field B for the sample of Fig. 1(c) using the fully self-consistent theory. The inset shows the result assuming a short-range broadening. μ_B^* is the effective Bohr magneton of the system.

the earlier experimental results[6] of Gornik *et al.* (which are also in disagreement with the results of Ref. 1). Our model clearly supports the B-dependent oscillation of Γ_n that Wang *et al.* infer on the basis of their specific-heat data —oscillatory level broadening is a feature[5] of our self-consistent DOS. Totally consistent with our theoretical results, the level broadening needed to explain the experimental data is larger than that inferred from the zero-field mobility [cf. Eqs. (2) and (3)]. Even though the experimental resolution is not good enough to resolve the additional structure found (and discussed above) in our theoretical results, the data of Ref. 1 is quite suggestive and we predict that further improvement in experimental accuracy would enable one to see the structure predicted here. The overall qualitative agreement between experiment and theory is the best one can do at the present time because of large inaccuracies in the absolute values of the experimental data. Our theoretical DOS depends crucially on the actual impurity distribution in the experimental samples which is, in general, not known. Also, the typical experimental samples of Ref. 1 have rather large electron densities with many occupied Landau levels. Our theoretical calculation of the self-consistent DOS becomes intractably complex when many Landau levels are occupied. For these reasons, a detailed quantitative comparison between theory and experiment is not possible at the present time.

The orbital (or the Landau) magnetization of the system is given by (at $T=0$)

$$M=\int_{-\infty}^{E_F}(E_F-E)\frac{\partial D(E)}{\partial B}dE\,. \tag{9}$$

Numerical calculation of M based on Eq. (9) is tricky because of the derivative term—instead we use a polynomial fit to the free energy as a function of B and evaluate M directly from the free energy. Thus our calculated M is not as accurate as our calculated C_v. Our calculated mag-

netization (at $T=0$) is shown as a function of the magnetic field for a GaAs-based 2D EG in Fig. 3. We also show as an inset in Fig. 3 the calculated magnetization on the basis of the short-range theory which is much spikier in structure. Such sharp spikes in M are not seen experimentally. Our results are in good qualitative agreement with experimental results[2] but a detailed quantitative comparison is difficult for reasons already discussed. The level broadening needed to account for the experimental data is much broader than the short-range scattering result as emphasized before, and this is in accord with our theory.

To summarize, we present theoretical calculations of electronic specific heat and magnetization in a two-dimensional electron system as a function of the applied magnetic field. The DOS entering our calculations is obtained from a self-consistent theory[5] where the impurity scattering-induced level broadening and screening determine each other. Our results are in qualitative agreement with experiment, indicating that short-range scattering approximation is inadequate and that the level broadening could be significantly broader than that inferred from the zero-field mobility value. The paucity of experimental results and, particularly, lack of information about how the impurity scattering centers are distributed in experimental samples rule out a detailed quantitative comparison between theory and experiment at the present time. However, the good qualitative agreement between theory and experiment shows that electronic screening plays an important role in determining the strong-field scattering potential in two-dimensional systems and any theory which employs a model potential (without accounting for self-consistency) is to some extent suspect with respect to its quantitative conclusions. Our predicted nonmonotonic dependence of C_v on temperature should be experimentally verifiable.

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