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Methods to measure the charge of the quasiparticles in the fractional quantum Hall effect

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We propose various experimental circumstances in which the longitudinal resistance of a two-dimensional electron gas in a high transverse magnetic field depends, in a simple and characteristic way, on the charge of the quasiparticle excitations. We propose that experiments of this sort could be used to directly measure the charge of the quasiparticle excitations which carry the dissipative part of the current. While it has been persuasively argued by Laughlin that the Hall conductance itself measures the quasiparticle charge, the connection is indirect, since the Hall current is carried by the condensate, *not* by the quasiparticles.

INTRODUCTION

One of the most remarkable features of the theory[1,2] of the fractional quantum-Hall effect is that it predicts the existence of quasiparticles with fractional charge. In particular, for the incompressible state corresponding to a value of the Hall conductance $\sigma_H=(e^2/h)(n/m)$, for which the density of electrons per flux quantum $\nu=n/m$, the quasiparticles have charge $e^*=e/m$ where e is the elementary quantum of charge and m is an odd integer. To date, there has been no direct experimental verification of this prediction of the theory. Laughlin constructed a remarkable argument[3] in the context of the integer quantum-Hall effect which is based on the notion that if the center-of-mass motion of the electron gas can equivalently be described *as if* an integer number of quasiparticles were transported from one side of the sample to the other, then the Hall conductance can be shown to be e^*e/h times an integer. Thus, it seems clear that the observation of a Hall plateau at a fractional multiple of e^2/h implies the existence of fractionally charged quasiparticles. While this argument is compelling, the inference is indirect. In the Hall plateau region the entire current is carried by the collective motion of the electron gas; it is important to the entire scheme that there be no quasiparticle component to the current. Thus, the Hall conductance measures the properties of the condensate, not the properties of the quasiparticles. Recently there was also an interesting experimental[4] observation that in the middle of the Hall plateau corresponding to filling factor $\nu=(n/m)$ flux quanta per electron, the longitudinal conductance obeys an Arrhenius law with temperature and has a prefactor, inferred by extrapolating to infinite temperature, which is equal to $(e/m)^2/h$. This was taken as evidence that the quasiparticle charge is e/m. However, this inference is again indirect and there is presently no theoretical understanding of why this experiment should measure the charge of the quasiparticle.

In this paper we propose several experiments which would directly probe the charge of the quasiparticle excitations. The experiments all probe one-particle properties, so they should not depend on the statistics of the quasiparticles. Thus, we imagine that on length scales that are large compared with the quasiparticle radius (basically, the magnetic length), the quasiparticles can be treated as point particles which satisfy an effective Schrödinger equation with charge e^*, so long as we only consider one quasiparticle in a simply connected region of the two-dimensional electron gas. (If the region is not simply connected, the quasiparticle wave function may not be single valued.[5])

In the high magnetic field limit we can use semiclassical methods to determine the spectrum of single quasiparticle states.[6-9] So long as the characteristic length scale of the potential is large compared to the magnetic length, this method should be very accurate. The classical equations of motion for the guiding center coordinates are vortex dynamics,[9] and hence, the particles move along equipotential contours with a speed proportional to the magnitude of the gradient of the potential. The eigenenergies of bound states can be determined from the Bohr-Sommerfeld quantization condition which implies that successive orbits enclose one more effective flux quantum, $\phi_0^*=hc/e^*$, than the previous orbit

$$B(A_{p+1}-A_p)=\phi_0^* , \tag{1}$$

where A_p is the area enclosed by the pth classical orbit (i.e., the area enclosed by the corresponding equipotential contour). Among other things, this implies that there are semiclassical bound states associated with orbits which enclose local maxima in the potential as well as those which enclose local minima. Tunneling amplitudes between different semiclassical eigenstates can be computed easily and accurately by studying trajectories for imaginary time dynamics in complex space.[6] The result is always of the form

$$t(R)=t_0\exp\left(-f\frac{R^2}{l^2}+i\theta\right), \tag{2}$$

where R is the shortest distance through the classically forbidden region between the two classical trajectories, l is the magnetic length,

$$2\pi l^2 B = \phi_0^* , \tag{3}$$

t_0 is a dimensionless prefactor which depends weakly on the form of the potential, f is a number of order one which depends on the shape, but not the magnitude of the potential in the classically forbidden region, and θ is a Aharonov-Bohm phase associated with the tunneling path which we expect to be zero in all the cases that we consider in this paper.

By now we have considerable experience in calculating tunneling amplitudes in model potentials (see discussions in Refs. 6–8) and, given an analytic form for the potential, we can easily calculate f and estimate t_0. In simple cases, f is proportional to $(U_{xx}/U_{yy})^{1/2}$ where U_{xx} and U_{yy} are, respectively, the average curvature of the potential in the directions parallel and perpendicular to the tunneling path. [Note, for comparison, that the direct overlap of two localized wave functions a distance R apart are of the form of Eq. (2) with $f=\frac{1}{4}$.] Unfortunately, for cases of interest, we often do not know the precise form of the potential so we cannot compute f explicitly. However, unless the potential is very eccentrically shaped, we generally expect that $f\sim 1$, and in particular, if the potential is approximately constant in the classically forbidden regime, we expect that $f\approx\frac{1}{4}$. In Ref. 6, the prefactor t_0 was computed for a variety of model problems. While, in general, t_0 can depend somewhat on the nature of the potential (for example, in one model considered in Ref. 6 $t_0 \propto l/R$), in practice, over a very wide range of model parameters, the prefactor varies very little, certainly when compared to the variation of the exponent; it is usually sufficient to take t_0 to be a number of order one.

Thus, we find that the areas of bound-state orbits and the magnitudes of tunneling amplitudes in problems of known geometry are independent of the details of the external potential, the electron-electron interactions, and all properties of the quasiparticles other than their charge e^*. We propose to use this feature of the dynamics of particles in a high field to design experiments to test the validity of the quasiparticle description of the excited states of the two-dimensional electron gas in the extreme quantum limit, and to verify that the quasiparticles actually have the fractional charge predicted by theory.

We now show that by studying the behavior of the longitudinal conductance as a function of magnetic field in narrow Hall channels, one can measure the quasiparticle charge. We consider two geometries shown in Fig. 1; in both the conducting channel is sufficiently narrow that edge currents determine the Hall conductance, so the total current is the difference between the edge currents on the two sides of the channel.[10–12] The longitudinal resistance is determined by the scattering rate from the upper edge to the lower edge according to a Landauer-type formula.[11–13] The dashed lines in the figure represent the equipotential contours corresponding to the energy of the highest occupied states, i.e., the chemical potential. We have considered these two particular geometries because they are simple to analyze, but similar effects could be observed in other experimental setups.

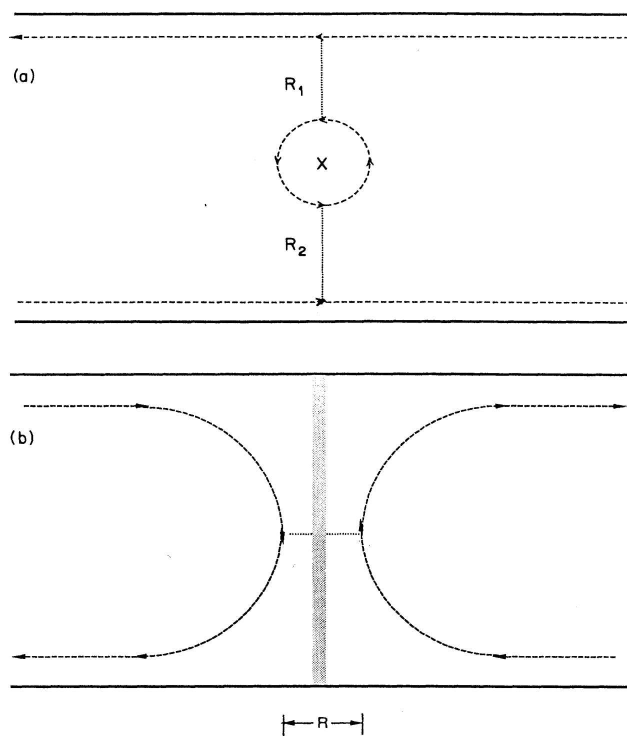

FIG. 1. (a) Schematic view of a narrow Hall device suitable for studying the spectrum of quasiparticle bound states by measuring the resonances in the longitudinal resistivity as a function of the impurity voltage. The solid lines represent the edges of the conducting channel. The dashed lines represent equipotential contours at the energy of the highest occupied quasiparticle states, and the arrows signify the direction of the drift of the particle guiding centers. The x marks the position of the impurity potential which we imagine is produced by a voltage probe placed slightly above the two-dimensional electron gas. R_1 and R_2 are, respectively, the distances from the edges to the impurity. The magnetic field is directed out of the page. (b) Schematic view of a narrow Hall device suitable for studying the tunneling amplitude as a function of the transverse magnetic field. The conventions are the same as in (a). The dotted line represents the shortest tunneling path from one side of the barrier to the other.

TUNNELING SPECTROSCOPY OF THE BOUND-STATE SPECTRUM

In Fig. 1(a), we picture a narrow-channel Hall device, formally equivalent to one considered in Ref. 14, in which an "impurity" potential lies roughly half way between the two edges of the sample. In the present case, we consider the case in which the "impurity" potential is externally applied and can be controlled experimentally, for instance, by placing a voltage probe slightly above the plane of the electron gas and varying the applied voltage. Since the presence of the impurity caused only a weak perturbation on the current-carrying edge states, there is very little interedge scattering; the longitudinal resistance is thus small and proportional to the interedge scattering amplitude, T.

It was shown in Ref. 14 that, as is usual with resonant tunneling, the tunneling amplitude is much smaller when none of the bound states of the impurity are at the same energy as the edge states,

$$T_{nr} \sim \exp\left[-f\frac{R_1^2+R_2^2}{l^2}\right], \quad (4)$$

than when they are resonant with a bound-state energy

$$T_{\text{res}} \sim T_{nr} \exp\left[+f\frac{R_{\min}^2}{l^2}\right], \quad (5)$$

where R_1 and R_2 are the distances from the two edges to the impurity, as shown in Fig. 1(a), $R_{\min}$ is the lesser of R_1 and R_2, and $f \approx \frac{1}{4}$. The width of each resonance is proportional to its decay rate, hence to $\exp[-f(R_{\min}^2/l^2)]$. We see, therefore, that the longitudinal resistance can be used as a spectroscopic probe of the bound-state energies; a peak in the resistance corresponds to a resonance with a bound state.

We have as parameters that we can readily vary the strength of the impurity potential V and the transverse magnetic field B. (In general, it is not easy to change the concentration of electrons keeping other parameters fixed.) In practice, we cannot expect to predict the spectrum of bound states, since the "impurity" potential is likely to be quite complicated. However, we can predict how the bound-state spectrum will change as we change from one Hall plateau to another. For a given impurity potential, let $A_p(e^*/e, B_1/B)$ be the area enclosed by the pth quantized orbit of a charge e^* quasiparticle in a magnetic field B, where B_1 is a reference magnetic field, which we choose to be the magnetic field corresponding to exactly one full Landau level $B_1 = \rho\phi_0$ where ρ is the density of electrons per unit area. From Eq. (1), it follows that A_p obeys the following scaling equation:

$$A_p\left[\frac{e^*}{e}, \frac{B_1}{B}\right] = \left[\frac{e}{e^*}\right]\left[\frac{B_1}{B}\right] A_p(1,1), \quad (6)$$

or, for $B_1/B = (n/m) = \nu$, and the expected value of $e^*/e = (1/m)$,

$$A_p\left[\frac{1}{m}, \frac{n}{m}\right] = A_{np}(1,1). \quad (7)$$

Conversely, if we hold the magnetic field fixed and vary the impurity potential, we can define $V_p(B_1/B)$ to be the magnitude of the applied potential at which the pth resonance in the scattering amplitude occurs. Since all potentials are electrostatic in origin, all energies are scaled by a factor of e^*. However, the resonant tunneling measures the energy of the bound state relative to the chemical potential for quasiparticles at the edge (i.e., the energy to remove one quasiparticle from the edge), so an additional factor of e^* does not occur in the resonance condition. It therefore follows from Eq. (7) that if the quasiparticles have the expected charge, then

$$V_p\left[\frac{n}{m}\right] = V_{np}(1). \quad (8)$$

Observation of this scaling relation would be a direct confirmation of the fractional charge of the quasiparticles.

As an aside, it should be noted that it is possible, in principle, to imagine another type of spectroscopy in which for fixed impurity potential and magnetic field one looked directly at photoinduced transitions between different bound states. Let $E_p(e^*/e, B_1/B)$ be the energy of the pth bound state. Then,

$$E_p\left[\frac{e^*}{e}, \frac{B_1}{B}\right] = e^* V_p\left[\frac{B_1}{B}\right], \quad (9)$$

from which it follows that the energy absorbed in a transition between two bound states satisfies the relation

$$\left[E_p\left[\frac{1}{m}, \frac{n}{m}\right] - E_{p-1}\left[\frac{1}{m}, \frac{n}{m}\right]\right] = \frac{1}{m}[E_{np}(1,1) - E_{np-n}(1,1)]. \quad (10)$$

While the observation of such a scaling law would be quite spectacular evidence of fractional charge, we generally expect the matrix elements for such a transition to be rather small and so the effect may be hard to observe. One could also, in principle, deduce e^* from the magnitude of the scattering amplitude off resonance since l depends on the quasiparticle charge according to Eq. (3). However, this dependence is more simply explored in the experimental geometry in Fig. 1(b).

THE MAGNITUDE OF THE TUNNELING MATRIX

In Fig. 1(b) we show a second experimental geometry in which a variable potential barrier is placed across the conducting channel. Samples with this geometry have already been studied in Ref. 15. We consider the case in which there are no classical trajectories at energies less than or equal to the energy of the edge states which cross the barrier, so almost all quasiparticles are reflected at the barrier. The contours corresponding to the highest occupied quasiparticle states are shown as dashed lines in Fig 1(b). The resistance is therefore large and the conductance is proportional to the square of the transmission coefficient through the barrier. Because the tunneling amplitude falls so rapidly with distance, the transmission coefficient will be dominated by tunneling through the point in the barrier where the two classical trajectories have their closest approach, as shown by the dotted line in Fig. 1(b). Thus, according to Eq. (2), the tunneling amplitude will be a Gaussian function of the distance across the barrier at the point of closest approach, R, divided by l, where l depends on the quasiparticle charge according to Eq. (3). Notice that the transmission does not depend on the particle effective mass or on the *magnitude* of the barrier, just on the geometrical factor R and the charge e^*. (In particular, f does not depend on either e^*, which merely effects the magnitude of the potential, or on B.)

Again, by comparing the magnitude of the tunneling current at different values of the magnetic field with the barrier held fixed, we can determine the quasiparticle charge. For instance, if $T(B_1/B, e^*/e)$ is the tunneling amplitude for fixed barrier voltage as a function of B and e^*, then

$$\ln\left[T\left(\frac{B_1}{B},\frac{e^*}{e}\right)\right]=\left(\frac{e}{e^*}\right)\left(\frac{B_1}{B}\right)\ln[T(1,1)]\,, \quad (11)$$

or for $B_1/B = n/m$ and $e^*/e = 1/m$,

$$\ln\left[T\left(\frac{n}{m},\frac{1}{m}\right)\right]=\frac{1}{n}\ln[T(1,1)]\,. \quad (12)$$

Thus, by measuring the *conductance* as a function of B in the parameter region where the transmission coefficient is small, one can obtain a measure of the effective magnetic length, and hence of the quasiparticle charge. This process is similar to the process studied in Ref. 16 which can determine the bulk critical current.

DISCUSSION

All of our conclusions are based on a very simple picture of the excitation spectrum of the electron gas in a high magnetic field. We have simply assumed that the low-energy excitations can be well represented as few quasiparticle states and that the quasiparticles satisfy the same sort of Schrödinger equation as the bare particles, but with fractional charge. We have ignored possible complications that could arise if there is more than one species of quasiparticle (as there would be for noninteracting electrons in a situation in which excitations from more than one Landau level are relevant in the center of the channel). More importantly, we have also ignored possible complications that could arise due to the multivaluedness of the many quasiparticle wave functions which occur due to their presumed fractional statistics. It is our feeling that this is justified for the single quasiparticle states relevant to the present discussion.

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