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Violation of the integral quantum Hall effect: Influence of backscattering and the role of voltage contacts

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Heterostructure ($GaAs$-$Al_xGa_{1-x}As$) Hall-bar devices with short cross gates are studied in the quantum Hall regime. A potential barrier introduced by the gate causes back scattering of edge currents, making steplike structures in the curves of the Hall and the diagonal resistances versus the gate bias voltage. The Hall resistance on either side of the gate deviates largely from expected quantized values, indicating a nonequilibrium occupation of edge states and its stability over a macroscopic distance (50 μm). An essential role of imperfect voltage contacts in probing the nonequilibrium edge currents is noted.

The most fundamental characteristic of the integral quantum Hall effect (QHE) is that, when current I is transmitted through a channel of two-dimensional electron gas (2D EG) where bulk Landau levels are occupied, a quantized voltage of $(h/\nu e^2)I$ appears across the channel, where ν is the number of the filled Landau levels, h is the Planck constant, and e the unit charge.[1] Significant deviation of the quantized Hall voltage has not been reported even in the studies on small devices of micron and submicron scales[2-5] or on samples with an electron-density discontinuity.[6] We report here a significant violation of the QHE, which occurs when a potential barrier is introduced in a 2D EG channel by a short cross gate to cause a back scattering of electrons. According to our interpretation, the deviation of the Hall voltage is a consequence of a nonequilibrium occupation of edge states and imperfect voltage contacts that selectively probe different edge states. The experimental results further indicate that edge currents travel ballistically over a surprisingly long distance (50 μm).

The experiments are made on $Al_{0.3}Ga_{0.7}As$-GaAs single interface heterostructure devices with a 2D EG density, n_S, of $3.4\times10^{11}/cm^2$ and a 4.2-K mobility of about 1.1×10^6 cm^2/Vs.[7] The heterostructure consists of a 1-μm-thick undoped GaAs layer, a 200-Å-thick undoped $Al_{0.3}Ga_{0.7}As$ spacer layer, a 900 Å-thick n-type Si-doped $Al_{0.3}Ga_{0.7}As$ layer, and a top cap layer of 100-Å-thick n-type GaAs. The area of the 2D EG is photolithographically pattered into standard Hall bridges of the total length of 400 μm as shown in Fig. 1. The electrical contacts are prepared by evaporation of a 300-Å-thick Au-Ge layer and a 3000-Å-thick Au layer on top of the GaAs cap layer and subsequent alloying at 450 °C in nitrogen atmosphere. A short Al front gate spanning the channel is deposited on top of a GaAs cap layer (Fig. 1). We have studied two devices with a gate length of $L=1$ μm (H1-1 and H1-2) and one device with $L=3$ μm (H3). In each sample, gate leakage current is negligible (<25 pA) when the gate bias with respect to the 2D EG, V_G, is between $+0.6$ and -1.0 V. The dependence of the 2D EG density n_{SG} in the gated region on V_G is determined from the studies of capacitance-voltage characteristics of the gate as described in Ref. 7. When $n_{SG}=n_S$, usual Schubnikov–de Haas oscillation is observed. Most of the measurements are carried out by transmitting ac currents (10 Hz) through contacts 1 and 2, and by detecting a potential difference, $U_{ij}=U_i-U_j$, between contacts i and j (Fig. 1) by using a lock-in amplifier. The input impedance is about 50 $M\Omega$ in the ac measurements, and is higher than 1000 $M\Omega$ in additional dc-current measurements.

Figure 2 shows the resistance between contacts 4 and 7, $R_{47}=U_{47}/eI$, as a function of V_G at $T=1.65$ K for different even-filling factors ν in the ungated region. The amplitude of the current I is 10 nA and the magnetic field is "positive" according to the specification in Fig. 1. As V_G decreases, the resistance R_{47} is quantized to, or approaches $(h/e^2)(1/\nu_G)$ at the V_G positions where the filling factor ν_G in the gated region takes on even integers. Precursor structures, probably due to spin splitting, are also noted at the positions of odd-integer fillings ν_G in the curves for $\nu=4$ and 6. These data are accountable within a simple picture regarding the QHE to be a bulk phenomenon: By assuming the gated and the ungated regions

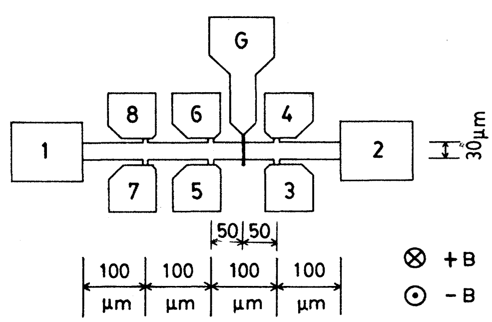

FIG. 1. A schematic of the Hall-bar devices with a short cross gate. The gate length is either 1 or 3 μm.

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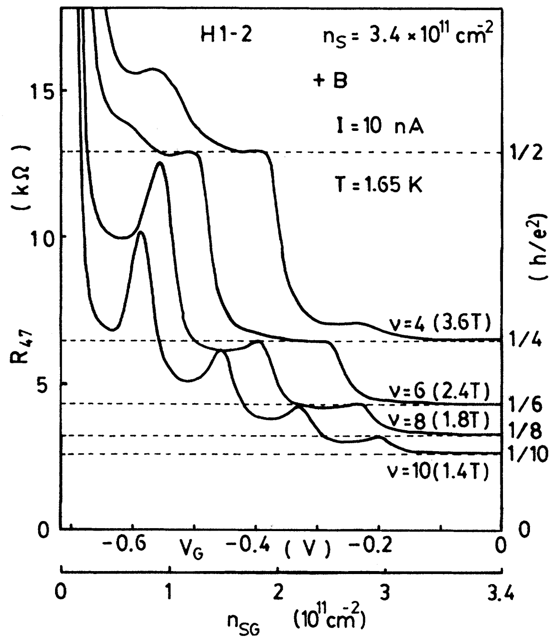

FIG. 2. R_{47} as a function of V_G at different magnetic fields B corresponding to even filling factors.

as respectively quantized-independent regions characterized by the ideally quantized Hall "resistivities" and the vanishing diagonal "resistivities," as discussed earlier by Syphers and Stiles,[6] we can easily derive the relations

$$R_{47}(+B)=R_{45}(+B)=R_{38}(-B)$$
$$=R_{36}(-B)=(h/e^2)(1/\nu_G)\,, \qquad (1)$$

where $+B$ or $-B$ specifies the direction of magnetic field according to Fig. 1. The considerable deviations of R_{47} from the expected values, appreciable in the lower-magnetic-field curves of $\nu=8$ and 10, are supposed to be due to nonvanishing diagonal resistivity in the gated region. For the later discussion, we note here that the relations

$$R_{xy}=(h/e^2)(1/\nu) \qquad (2)$$

and

$$R_{xx}=(h/e^2)(1/\nu_G-1/\nu) \qquad (3)$$

can be also derived from the simple picture, respectively, for the Hall resistance R_{xy} measured on either side of the gate and for the diagonal resistance R_{xx} measured across the gate.

The failure of the simple bulk picture of the QHE in describing the real situation is revealed when the resistances, $R_{ij}=U_{ij}/eI$, between other pairs of contacts i and j are measured. Figure 3(a) displays the resistances measured at all the side-arm contacts at 4.2 K as a function of V_G in the case of $\nu=4$. In the following we will refer the magnitude of resistance in units of h/e^2. The Hall resistances, R_{67} and R_{87}, deviate largely from $\frac{1}{4}$ when ν_G approaches and becomes smaller than 2. This implies that the gate has an unexpected but definite influence on a 2D

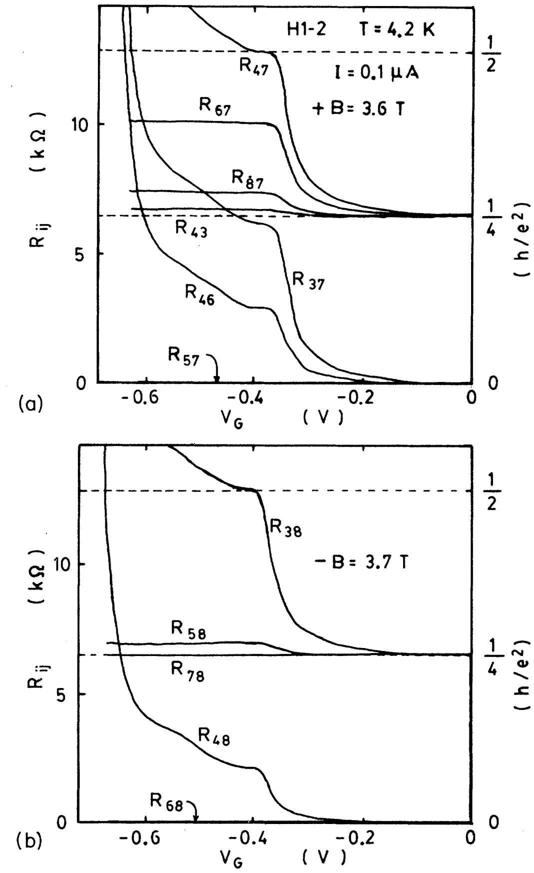

FIG. 3. (a) R_{ij} of all the voltage contacts vs V_G at $B=+3.6$ T ($\nu=4$). The magnetic field is "positive" according to the specification given in Fig. 1. (b) R_{ij} vs V_G for the "negative" magnetic field.

EG region at least 50 μm away from it. The Hall resistance on the other side of the gate, R_{43}, also deviates appreciably from $\frac{1}{4}$ in the range of $\nu_G \lesssim 2$. Since R_{57} remains exactly zero in the entire range of ν_G, and R_{47} is correctly quantized to $\frac{1}{2}$ at $\nu_G=2$, the large deviations of R_{67} and R_{87} are ascribed to upward shifts of the potentials U_6 and U_8. Similarly, the deviation of R_{43} is to be ascribed to a downward shift of the potential U_3. We have confirmed that additional measurements of any other pairs of the contacts gave consistent results. When magnetic field is reversed, the contacts exhibiting anomalous potential deviations and those exhibiting normal potentials change their positions: The data in Fig. 3(b) indicate that U_5 and U_4, instead of U_6 and U_3, deviate largely from the expected quantized values when $\nu_G \lesssim 2$. Similar singularities also appear in the case of $\nu=6$.[7]

We have obtained identical results in dc-current measurements, and found that the reversal of dc current simply reverses the sign of the potential measured at each contact. Similar results have been obtained in all the three samples studied. No evidence of a systematic dependence of the magnitude of the potential deviation on the gate length (1 or 3 μm) was noted; in any case, its dependence on an individual contact is evident, as readily seen from Fig. 3. Temperature dependence studied in the same device shows that the deviation of the potentials gradually increases with decreasing T from 4.2 to 1.7 K, but it rapidly decreases with increasing T above 5–7 K, recovering an approximate quantization everywhere in the channel.

These anomalous phenomena are extremely sensitive to the magnitude of current, as we pointed out in Ref. 7: It practically vanishes and the usual quantization nearly recovers on each side of the gate when I reaches a few microamperes. This is shown in Fig. 4(a), where the anomaly of U_6 is represented for different levels of dc-current I by the curves of R_{65} vs V_G. We have studied the derivatives of U_{68} and U_{34} with respect to I as a function of dc-bias current by modulating the current with an amplitude of 10 or 20 nA at 10 Hz. The results shown in Fig. 4(b) further indicate that the deviations of dU_{68}/dI and dU_{34}/dI practically vanish when I exceeds a critical current of $I_c = 320$–420 nA. This implies that the potential deviation does not completely vanish but is saturated to a finite constant magnitude when I exceeds I_c, although not clearly discernible in Fig. 4(a). The potential difference across the gate $(h/\nu_G e^2)I$ at $I = I_c$ is 4.1–5.4 meV, which is comparable to the Landau-level spacing $\hbar\omega_c = 6.0$ meV at $B = 3.6$ T.

To interpret these phenomena we regard the gated region as a potential barrier which causes elastic back scattering of electrons in the 2D EG channel. For our consideration, a Landauer-type picture of the QHE is instructive.[8–11] When ν and ν_G are even integers, the barrier height is $(\nu - \nu_G)\hbar\omega_c/2$, where the factor of 2 represents the spin degeneracy. The edge currents arising from the first ν_G Landau levels perfectly transmit the barrier, while the other edge currents from the rest of the $\nu - \nu_G$ Landau levels are totally reflected as schematically shown by Figs. 5(a) and 5(b) for the case of $(\nu, \nu_G) = (4,2)$. The Landauer-type picture[8–11] predicts that, when a small current I is forced to flow, a difference in chemical potentials, given by $(\mu_1 - \mu_2)/e = (h/\nu_G e^2)I$, arises between the upper-right edge (μ_1) and the lower-left edge (μ_2) of the channel shown in Fig. 5(a) for the specified magnetic field. This is in accord with Eq. (1) and gives an explanation to the data in Fig. 2 analogous to that for the ballistic one-dimensional quantum transport.[12] It is trivial that the edges of μ_1 and μ_2 appear on the lower-right and upper-left sides when the magnetic field is reversed.

In this situation, the outer- and inner-edge states on the opposite sides of the channel are occupied up to the different energies μ_1 and μ_2 as shown in Figs. 5(a)–5(c).[9,11] If voltage contacts perfectly absorb the edge currents to read the mean potential of the differently occupied edge states, the former results (2) and (3) would be reproduced as pointed out by Büttiker.[11] However, our data shown in Figs. 3(a) and 3(b) demonstrate that the side-arm contacts on the upper-left edge in Fig. 5 (except contact 7 in the case of $-B$) do not indicate $(\mu_1 + \mu_2)/2$ but deviate toward μ_1 and the contacts on the lower-right edge deviate from $(\mu_1 + \mu_2)/2$ toward μ_2, while any contacts on the other edges correctly exhibit the expected potentials. Hence, we hypothesize that (i) the nonequilibrium edge currents ballistically travel over the distance

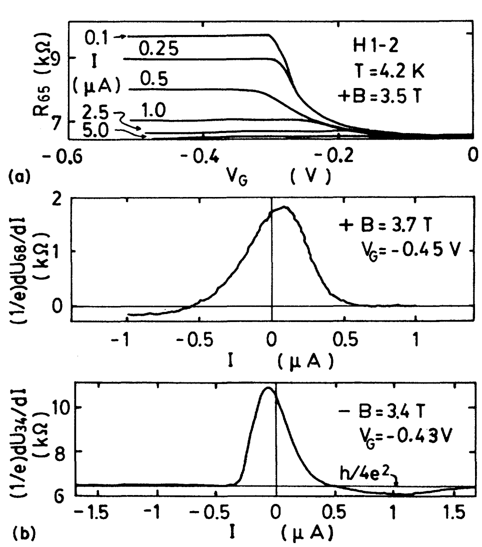

FIG. 4. (a) R_{65} as a function of V_G at different channel currents I for $B = +3.5$ T. (b) Derivative resistances $(1/e)dU_{68}/dI$ and $(1/e)dU_{34}/dI$ vs I at $\nu_G = 2$, respectively, for the positive and the negative magnetic fields. The current I is defined as positive when contact 1 serves as the source of electrons.

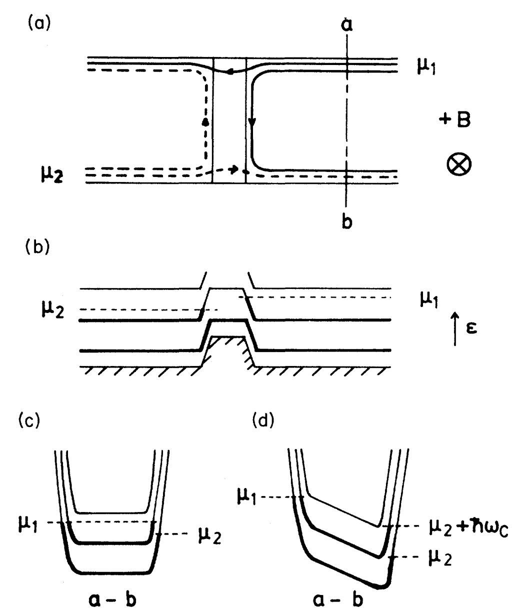

FIG. 5. (a) A schematic representation of edger currents for the case of $\nu = 4$ and $\nu_G = 2$. Magnetic field is "positive." (b) A sketch of the energy diagram of bulk Landau levels along the sample for $\nu = 4$ and $\nu_G = 2$. The thicker portions of the lines indicate the occupation of the states. (c) A sketch of the energy spectrum of Landau levels across the channel along the line a-b in (a) and the occupation of $\mu_1 - \mu_2 < \hbar\omega_c$. (d) Similar sketch of our expectation for the case of $\mu_1 - \mu_2 > \hbar\omega_c$.

reaching the contacts, and (ii) the contacts are not perfect and preferentially probe the outermost edge states. When I increases so that $\mu_1 - \mu_2 > \hbar\omega_c$, the fraction of edge currents having the energies $\varepsilon > \mu_2 + \hbar\omega_c$ will cause a partial transmission, leading us to expect that the energy difference between the edge currents is limited to $\hbar\omega_c$ as illustrated in Fig. 5(d). This is in harmony with the saturation of the potential deviation observed in the range $I \gtrsim I_c$.

The spatial separation between neighboring edge currents arising from low-lying Landau levels is 200–300 Å, according to a calculation based on the assumption of an infinite confining potential.[13] Although the actual spacing, probably somewhat larger than the above values, may be larger than the magnetic length $(h/eB)^{1/2} = 135$ Å at $B = 3.6$ T, and although the different edge states are strictly orthogonal in an ideal condition, finite relaxation may occur between the edge states due to elastic and/or inelastic scatterings. We can estimate the drift velocity $v_{\rm edg}$ of electrons in edge states through $ev_{\rm edg} = (d\varepsilon/dy)/B$, where y is the coordinate in the direction normal to the edge and ε is the energy of edge states.[14] Assuming $d\varepsilon/dy = \hbar\omega_c/300$ Å, we have $v_{\rm edg} = 5.6 \times 10^6$ cm/s. Our hypothesis (i) thus requires the relaxation time to be comparable to or longer than 50 μm$/v_{\rm edg} = 9 \times 10^{-10}$ s. As for hypothesis (ii), it is important that our voltage contacts exhibit relatively large series resistances R_i such that R_1, $R_2, \ldots,$ and R_8 are 2.7, 0.7, 2.3, 8.7, 2.1, 6.2, 5.7, and 1.4 $k\Omega$, respectively, in sample $H1$-2 at $B = 3.6$ T and $T = 4.2$ K. These resistances were determined from $R_i = R_{ij,ik}$ or $R_i = R_{ij,ik} - (h/4e^2)$ depending on the configuration of contacts i, j, and k, where current was passed through contacts i and k and voltage was measured between contacts i and j. We have found that R_i were practically independent of the choice of contacts j and k. We have also found that R_i were nearly independent of the magnitude of current in a range $I \lesssim 2$ μA. Generally, selective probing of different edge currents is possible when the contacts have finite resistance as argued by Büttiker.[11] Particularly, the property (ii) can be expected if an imperfect contact is characterized by a tunneling of electrons between 2D EG and an electron reservoir through a potential barrier.[15]

Recently, Washburn *et al.*[16] and Haug *et al.*[17] made similar measurements but reported data which agrees with Eqs. (1)–(3). Probably, the edge currents adequately equilibrate before reaching voltage contacts and/or the contacts are nearly perfect in their experiments.

We believe that this work is the first experimental demonstration of the existence and the physical significance of edge currents, and is also the first observation of an explicit influence of voltage contacts on the QHE. Further, this work definitely indicates that macroscopic samples with typical dimensions of order 50 μm are small enough to observe essential size effects in the regime of QHE.

Note added in proof. After the present work was completed, we developed a general treatment of contacts in the presence of nonequilibrium population[18] and quantitatively analyzed the data presented here.[19,20] Also, we became aware of the experiments by van Wees and co-workers where related effects were observed.[21,22]

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