Generalized Drude approach to the conductivity relaxation time due to electron-hole collisions in optically excited semiconductors

Bo E. Sernelius
Department of Physics and Measurement Technology, University of Linköping, S-581 83 Linköping, Sweden
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We apply the generalized Drude approach to obtain the zero-frequency relaxation time from carrier scattering in electron-hole plasmas in semiconductors. To our knowledge this is the first time this approach has been tested on these kinds of scattering processes. It is found to work quite well and lacks the disadvantage of the Boltzmann approach for giving results in only the quantum and classical limits. Furthermore, the calculations are much simpler than those in the Boltzmann approach.

Interest in the effects from carrier collisions in electron-hole (e-h) plasmas has increased recently. This increase has been triggered by experiments[1] on extremely dense e-h plasmas created by femtosecond laser irradiation of silicon samples. The experiments have shown a free-carrier absorption much larger than what is usually obtained from carrier-phonon scattering. The experiments indicate a relaxation time of the order of 10^{-15} s, which is believed to be due to e-h scattering. This inspired Combescot and Combescot[2] to investigate if e-h collisions could occur at this high rate. They used the Boltzmann approach and found an "exact" solution in the quantum limit and an approximate one in the classical limit. The Boltzmann approach is, apart from being very complicated, unable to produce results in the intermediate regime, where the maximum scattering rate is actually found.

We use an alternative approach, the generalized Drude approach, which, if it works, is not just simpler and more physically transparent but can furthermore be used for all densities and temperatures. This approach has been shown to work[3–5] with impurity scattering as well as with acoustical phonon scattering, but to our knowledge it has never been applied to e-h scattering. It consists of three steps. In the first step the high-frequency limit of the dynamical conductivity is derived within the Kubo formalism and diagrammatic perturbation theory; in the second step, this result is compared to the high-frequency expansion of the generalized Drude expression for the dynamical conductivity and the relaxation time τ is hereby identified (the generalization of the Drude expression consists of allowing the relaxation time to be frequency dependent); in the third, and last, step the obtained expression for τ is assumed to be valid at zero frequency. This is a bold step since τ was obtained form a high-frequency treatment.

For simplicity we study a model semiconductor where the effective masses of the electrons and holes are taken to be equal and have the value $m^*=0.3m_e$. The background dielectric constant is chosen to be 10.0. These are typical values for semiconductors. We have refrained from treating the more complicated situation in silicon with its many-valley conduction band. However, the treatment here can easily be generalized to handle that more complicated situation.

The Drude expression for the real part of the dynamical conductivity and its high-frequency expansion is given by

$$\sigma_1=\frac{2ne^2}{m^*}\frac{1/\tau}{(1/\tau)^2+\omega^2}\underset{\omega\to\infty}{\approx}\frac{2ne^2}{m^*\omega^2}\frac{1}{\tau(\omega)} ,$$

where n is the density of each of the components in the plasma. Comparison with the derived high-frequency conductivity and identification of τ gives

$$\frac{1}{\tau(0)}=\lim_{\omega\to 0}\frac{m^*\omega^2}{2ne^2}\sigma_1^{\text{high}}(\omega) .$$

Substituting σ_1 by the first term of Eq. (2.30) in Ref. 6, properly simplified from taking into account the fact that we are considering a system with two components of equal mass, we obtain

$$\frac{1}{\tau(0)}=\frac{\hbar^2\beta}{6nm^*\pi^3}\int_0^\infty d\omega'\frac{1}{\sinh^2(\beta\omega'/2)}\times\int_0^\infty dq\frac{[q^2\text{Im}\alpha(q,\omega')]^2}{|1+2\alpha(q,\omega')|^2} ,$$

where the function $\alpha(q,\omega')$ is the polarizability from one of the components. It is the retarded, finite temperature function and its denominator contains the background dielectric constant. The parameter β is the ordinary temperature parameter $1/k_BT$, where k_B is the Boltzmann constant and T the temperature. The expression is very simple and to obtain numerical results is straightforward. However, the calculation is rather time consuming. The reason for this is that only the imaginary parts of the polarizabilities can be obtained analytically.[7] The real parts have to be found numerically from the imaginary parts with the use of the Kramers-Kronig dispersion relations.

The numerical room-temperature results are displayed in Fig. 1. For comparison, we have also included the classical and quantum limits as obtained from the Boltzmann approach. The result for the quantum limit given in Ref. 2 has been slightly corrected. The quantum

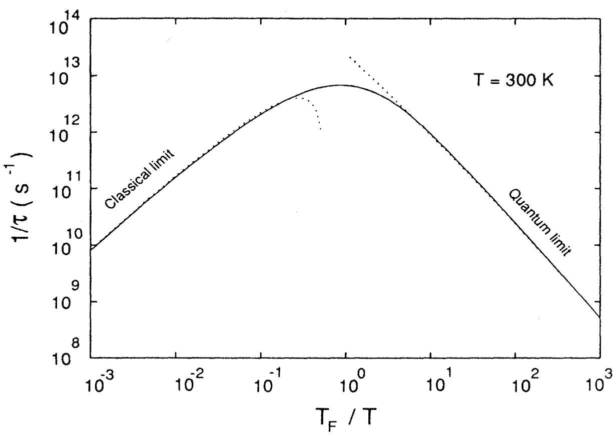

FIG. 1. The scattering rate from electron-hole collisions as a function of T_F/T. The results are valid for an electron-hole plasma at $T=300$ K. The solid curve is the result from the present approach. The dotted curves are the results from the Boltzmann approach, as obtained in Ref. 2, in the classical and quantum limits. The quantum-limit result has been modified (see the main text).

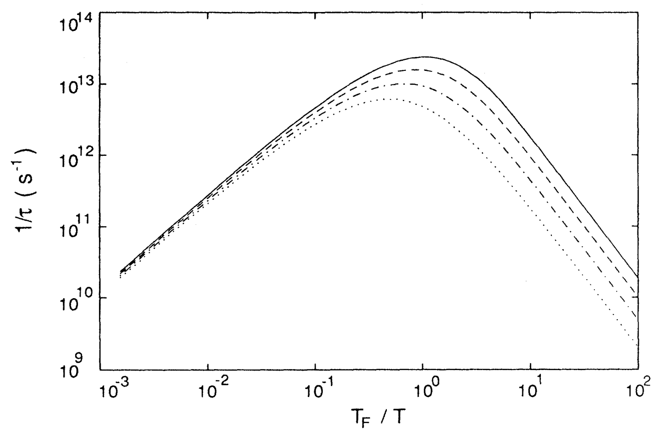

FIG. 2. The scattering rate from electron-hole collisions as a function of T_F/T. Each curve is valid for an electron-hole plasma at fixed density. The dotted, dash-dotted, dashed, and solid curves are for the plasma densities 1×10^{18}, 1×10^{19}, 1×10^{20}, and 1×10^{21} cm^{-3}, respectively.

limit is the limit when T/T_F, where T_F is the Fermi temperature, goes to zero. This can be achieved in two ways; either T goes to zero or T_F goes to infinity. The result for the quantum limit given in Ref. 2 is valid if the limit is approached according to the second alternative, i.e., if T_F or equivalently the density goes to infinity. However, one has to reach very high densities before the limiting expression is a good approximation. In Ref. 2 the authors have assumed that the integrands in their Eq. (21) vanish before the upper integration limits are reached and have replaced those limits with infinity. We have redone their derivation of the quantum-limit result without this replacement. The so-obtained quantum-limit result is what we have included in the figure. In the notation of Ref. 2 the corrected expression reads

$$\frac{\hbar}{\tau_{e\text{-}h}} = k_B T_{\rm Ryd}\left[\frac{T}{T_n}\right]^2\left[x\tan^{-1}x - \frac{x^2}{1+x^2}\right] \times \frac{\pi}{6}\nu^{4/3}\left[\frac{m_e^2 m_h^2}{m^4}\right],$$

where

$$x = \frac{\nu^{-1/6}\pi^{1/2}m^{1/2}}{(\nu m_e+\nu^{1/3}m_h)^{1/2}}\left[\frac{T_n}{T_{\rm Ryd}}\right]^{1/4}.$$

From the comparison with the Boltzmann results we can safely conclude that we calculate the same thing. In other words our approach is valid. We further notice that the modified "exact" quantum-limit result from the Boltzmann approach is correct. The small deviations between the results are entirely due to the fact that in the Boltzmann approach Thomas-Fermi screening was used while we use the full RPA (random-phase approximation) screening. One can also conclude that the approximations used to arrive at the "approximate" classical limit within the Boltzmann approach were sound, since those limiting results agree quite well with our results towards the classical limit.

In Fig. 2 we give the results for a series of fixed plasma densities, i.e., T_F is constant and T varies for each curve. The dotted, dash-dotted, dashed, and solid curves are for the plasma densities 1×10^{18}, 1×10^{19}, 1×10^{20}, and 1×10^{21} cm^{-3}, respectively. We find from Fig. 2 that the maximum scattering rate increases with density. This density dependence is displayed in more detail in Fig. 3.

The e-h plasma in the simple model semiconductor we have considered here has two components. An e-h plasma in silicon has eight components; six electron components and two hole components. Since the conduction-band minima are anisotropic, electrons from two valleys can scatter against each other and contribute to the type of scattering mechanism considered here.

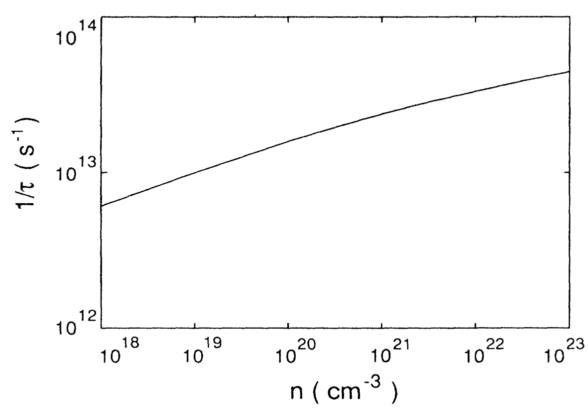

FIG. 3. The maximum scattering rate from electron-hole collisions as a function of density. The results are valid for an electron-hole plasma in the model semiconductor described in the text.

Twenty-five combinations of pairs of plasma components can take part in the processes. This leads to an increased scattering rate in silicon as compared to the model semiconductor. A factor of 10 is probably a reasonable estimate of this increase. From these considerations we can conclude that the scattering rate is found to be high enough to make credible the explanation, that electron-hole collisions are responsible for the extremely high free-carrier absorption in the experiments of Ref. 1. This paper will be followed by a more detailed paper in which the present treatment is extended in such a way that the real situation in silicon can be handled.

Possibly, the approach presented here can be of use in the interpretation of other experiments in the field of laser excitation of high-density electron-hole plasmas. Experimentally, this is a burgeoning field, particularly in III-V materials.[8-13] Also on the theoretical side there is important activity. Extensive Monte Carlo work[14-16] has been done on electron and electron-hole plasmas as well as simpler Boltzmann equation work.[17]

In summary, we have shown that the generalized Drude approach works well in the case of carrier-carrier scattering in an electron-hole plasma. It has advantages over the Boltzmann approach in that it is simpler and can be used for all densities and temperatures. The Boltzmann approach can be used only in the quantum and classical limits. The maximum scattering rate occurs in the intermediate regime where the Boltzmann approach fails. We found that the rate of electron-hole scattering was large enough to explain the very small relaxation times found in the experiments of Ref. 1. It should be mentioned that we kept only the lowest-order contribution in the perturbation expansion of the high-frequency conductivity. This corresponds to the Born approximation. This approximation was also used in the Boltzmann approach of Ref. 2. The Born approximation is asymptotically exact in the quantum and classical limits. It is difficult to predict how good this approximation is in the region between the limits. One should, if possible, include higher-order perturbation terms to test this.

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