Diffusion and fluctuations in a nonequilibrium electron gas with electron-electron collisions

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We investigate the equivalence between the Boltzmann (or field) approach and the particle approach for the definition of the diffusion coefficient in the presence of electron-electron collisions. Accordingly, we introduce a "model" collision operator as well as a "model" electron distribution function which can be used in a one-particle numerical procedure. In this way one can overcome the difficulties connected with the calculation of the diffusion coefficient in the presence of electron-electron collisions within the framework of the particle approach.

I. INTRODUCTION

As is customary in the study of the motion of an ensemble of particles, two equivalent approaches can be used.[1] The first one (Boltzmann or field approach), by introducing the single-particle distribution function, considers the ensemble as a continuum. The second one (particle approach), by following the motion of each particle (pointlike), considers the discretized nature of the ensemble.

In the theory based on the Boltzmann equation there are no difficulties in defining the diffusion coefficient in the presence of *e-e* collisions.[2–4] However, the situation is not so simple in the particle approach, where the appropriate estimator for the diffusion coefficient in the presence of *e-e* collisions is to the best of our knowledge still missing. In this paper, a proper definition of this estimator will be found by establishing a close parallel between the Boltzmann equation and the particle approach. Such a parallel will help us in laying down a standard procedure to include the *e-e* collisions in the response problem. Finally, we shall provide a prescription for a one-particle simulation of the diffusion coefficient, associated with Fick's law, in the presence of *e-e* collisions.

The paper is organized as follows. Section II establishes the parallel between the particle approach and the Boltzmann equation in the absence of an electric field and *e-e* collisions. Then, the influence of an external field as well as of *e-e* collisions is considered in Secs. III and IV, respectively. Section V is devoted to the determination of the appropriate estimator for evaluating the diffusion coefficient in the presence of *e-e* collisions. Some conclusions are finally drawn in Sec. VI.

II. INDIVIDUAL AND AVERAGE MOTION OF AN ENSEMBLE OF PARTICLES IN THE ABSENCE OF AN ELECTRIC FIELD

Let us consider a gas of particles (electrons) colliding with scatterers in the absence of both an external electric field and *e-e* collisions. As usual we assume that the collision duration Δt is much shorter than the mean-free-flight duration τ. An electron in the gas performs a random walk in space changing its velocity $\mathbf{v}$ (momentum $\mathbf{p}$) after each collision. It is well known that this process can be simulated numerically with desired accuracy, for example through the Monte Carlo technique.[5] Let us imagine an electron with momentum $\mathbf{p}_0$ at time $t=0$ which, after a number of collisions, acquires momentum $\mathbf{p}$ at the time t. The intermediate moments $\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n$ and collision times $t_1,t_2,\ldots,t_n$ (or flight durations t_1-0, t_2-t_1, etc.) are stochastic variables, so that the random process may be characterized by a probability to find the electron at time t with momentum $\mathbf{p}$, provided that it had the momentum $\mathbf{p}_0$ at $t=0$ and suffered n collisions. Let us denote this (conditional) probability as $P_n(\mathbf{p},t|\mathbf{p}_0)$. By definition, the total probability for an electron to have a momentum $\mathbf{p}$ at a time t is given by the sum over all possible numbers of collisions:

$$P(\mathbf{p},t|\mathbf{p}_0)=\sum_{n=0}^{\infty}P_n(\mathbf{p},t|\mathbf{p}_0)\ . \tag{1}$$

Figure 1 provides a diagrammatic representation of Eq. (1). Here the first term in this series corresponds to the probability for an electron to travel without collisions during the time interval t. This probability is given by

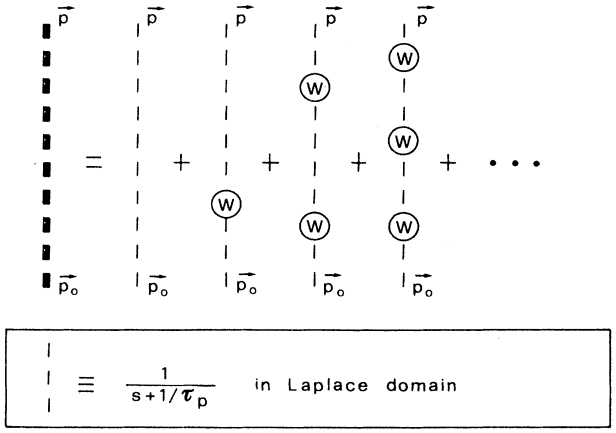

FIG. 1. Diagrammatic representation of Eq. (1). In the rhs of the figure the open circles represent the scattering operator and the dashed lines joining two consecutive circles (propagators) correspond to the probability a carrier has of not making a scattering with the given momentum.

$\exp(-t/\tau_p)$, where the scattering rate $1/\tau_p$ is

$$\frac{1}{\tau_p}=\sum_{p'} W_{p'p} \tag{2}$$

and $W_{p'p}$ is the transition probability per unit time for the electron having the momentum $\mathbf{p}$ before a collision to acquire a momentum $\mathbf{p}'$ after the collision. Let us write the first term in Fig. 1 explicitly. It is given by

$$P_0(\mathbf{p},t|\mathbf{p}_0)=\exp(-t/\tau_p)\delta_{pp_0} . \tag{2a}$$

The second term in Fig. 1 represents the process with one collision at some time moment t_1 $(0<t_1<t)$. The corresponding probability is given by

$$\begin{aligned} P_1(\mathbf{p},t|\mathbf{p}_0)&=\int_0^t dt_1 \exp\left[-\frac{t-t_1}{\tau_p}\right] \\ &\quad\times\sum_{p_1} W_{pp_1}\exp\left[-\frac{t_1}{\tau_{p_1}}\right]\delta_{p_1p_0} \\ &=\int_0^t dt_1 \exp\left[-\frac{t-t_1}{\tau_p}\right] W_{pp_0}\exp\left[-\frac{t_1}{\tau_{p_0}}\right] . \end{aligned} \tag{3}$$

By writing Eq. (3) we implicitly have assumed that the collision duration Δt_{coll} is much shorter than the mean free flight τ_p:

$$\Delta t_{\text{coll}} \ll \tau_p . \tag{4}$$

This is a usual condition of applicability of the Boltzmann equation.

The next lines in Fig. 1 with two or more points (vertices) represent the contributions from processes with two, three, and more collisions. The expression for $P_n(\mathbf{p},t|\mathbf{p}_0)$ is organized quite like Eq. (3) but is more cumbersome because of many subsequent integrations over collision times $t_1,t_2,\ldots,t_n$ and the summation over intermediate momenta. To simplify the formula we write its Laplace transform, $\tilde{P}_n(\mathbf{p},s|\mathbf{p}_0)$:

$$\tilde{P}_n(\mathbf{p},s|\mathbf{p}_0)=\frac{1}{s+1/\tau_p}W\frac{1}{s+1/\tau_p}W\cdots\frac{1}{s+1/\tau_p}\delta_{pp_0} , \tag{5}$$

where s is the Laplace variable and the scattering operator W is defined as

$$Wx_p=\sum_{p'} W_{pp'}x_{p'} . \tag{6}$$

Now it is easy to sum the series and obtain the expression for the Laplace transform of the total probability $\tilde{P}(\mathbf{p},s|\mathbf{p}_0)$. We have

$$\begin{aligned} \tilde{P}(\mathbf{p},s|\mathbf{p}_0)&=\sum_{n=0}^{\infty}\tilde{P}_n(\mathbf{p},s|\mathbf{p}_0) \\ &=\frac{1}{s+1/\tau_p}\sum_{n=0}^{\infty}\left[W\frac{1}{s+1/\tau_p}\right]^n\delta_{pp_0} \\ &=\frac{1}{s+1/\tau_p-W}\delta_{pp_0}=\frac{1}{s+I_p^{\text{th}}}\delta_{pp_0} . \end{aligned} \tag{7a}$$

The diagrammatic representation of Eq. (7a) is the same as in Fig. 1 where, being in the Laplace domain, the propagator is now represented by the factor $(s+1/\tau_p)^{-1}$, as recalled in the inset of the figure. By inverse Laplace transformation we obtain

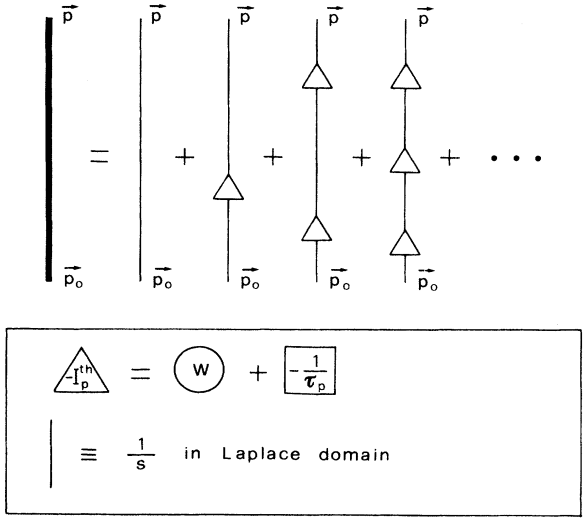

FIG. 2. Diagrammatic representation of Eq. (9). In the rhs of the figure the triangles represent the negative of the collision operator and the continuous lines joining two consecutive triangles (propagators) correspond to the carrier free flight.

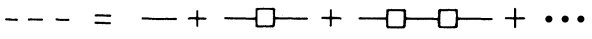

FIG. 3. Diagrammatic representation of the resummation procedure which connects the propagators of Fig. 1 with those of Fig. 2.

$$P(\mathbf{p},t|\mathbf{p}_0)=\exp(-I_p^{\rm th}t)\delta_{pp_0}\ , \tag{7b}$$

where $I_p^{\rm th}$ is the collision operator describing the collisions of electrons with phonons and/or impurities given by

$$I_p^{\rm th}x_p=(1/\tau_p-W)x_p=\sum_{p'}(W_{p'p}x_p-W_{pp'}x_{p'})\ . \tag{8}$$

The expression (7b) proves that $P(\mathbf{p},t|\mathbf{p}_0)$ is nothing but the solution of the Boltzmann equation:

$$\left[\frac{\partial}{\partial t}+I_p^{\rm th}\right]f_p(t)=0 \tag{9}$$

with the initial condition $f_p(0)=\delta_{pp_0}$. The identity $f_p(t)=P(\mathbf{p},t|\mathbf{p}_0)$ confirms the correspondence between the theory based upon the Boltzmann equation and the particle approach as noticed in Refs. 6–8.

The solution of the Boltzmann equation (9) can be also represented as a series of collisions and free flights in a different way, as is depicted in Fig. 2. In the Laplace domain the propagators are represented by the factors $1/s$ according to the series expansion

$$\frac{1}{s+I_p^{\rm th}}=\frac{1}{s}\sum_{n=0}^{\infty}\left[\frac{-I_p^{\rm th}}{s}\right]^n\ . \tag{10}$$

The Boltzmann equation and the particle approaches differ only in their methods of summation of the series. Indeed, making use of the identity (see inset in Fig. 2)

$$-I_p^{\rm th}=W-\frac{1}{\tau_p}\ , \tag{11}$$

we can expand each term $(W-1/\tau_p)^n$ into series containing different powers of W (represented, as above, by circles) and $-1/\tau_p$ (represented by squares). Then, each graph in Fig. 2 would contain a different sequence of circles and squares connected by straight lines. The resummation can be done as is depicted in Fig. 3 and so the correspondence between the two types of expansion shown in Figs. 1 and 2, respectively, is established.

III. INFLUENCE OF AN EXTERNAL ELECTRIC FIELD

The electron gas in an electric field $\mathbf{E}$ has the stationary distribution function F_p which satisfies the Boltzmann equation:

$$\left[e\mathbf{E}\frac{\partial}{\partial\mathbf{p}}+I_p^{\rm th}\right]F_p=I_pF_p=0\ , \tag{12}$$

e being the electron charge.

The operator in the left-hand side (lhs) of this equation, which we denote as I_p, enters the formulas for the nonequilibrium state created by the field $\mathbf{E}$ instead of the operator $I_p^{\rm th}$. For instance, the evolution of the state with a given initial momentum $\mathbf{p}_0$ now is described by

$$f_p(t)=\exp(-I_pt)\delta_{pp_0}\ . \tag{13}$$

In the particle approach the influence of the external field is taken into account via the changes in momentum during the free flight due to the acceleration of the electron by the field according to the Newton law $\mathbf{p}(t)=\mathbf{p}+e\mathbf{E}t$. The operator $e\mathbf{E}\partial/\partial\mathbf{p}$ is the displacement operator representing this acceleration: $x_p(t)=\exp(-te\mathbf{E}\partial/\partial\mathbf{p})x_p(0)$. Since the motion of an electron is not entirely free but rather a motion with collisions, the acceleration due to the electric field can be taken into account by inclusion of the new vertices $e\mathbf{E}\partial/\partial\mathbf{p}$ into propagator lines in a manner a similar to that shown in Fig. 2. Such an inclusion can be done either before the summation over other vertices or afterwards with the same final result.

IV. INFLUENCE OF ELECTRON-ELECTRON COLLISIONS

In this case the Boltzmann equation for the stationary distribution function in an electric field is

$$I_pF_p+S_p(F|F)=0\ , \tag{14}$$

where $S_p(F|F)$ is the nonlinear e-e collision term which is given by[3]

$$S_p(F|F)=\sum_{p'p_1p_1'}(W_{p'p_1'}^{pp_1}F_pF_{p_1}-W_{pp_1}^{p'p_1'}F_{p'}F_{p_1'})\ . \tag{15}$$

A small fluctuation of the distribution function evolves in time according to the same formula as before:

$$\Delta F_p(t)=\exp(-I_p^{ee}t)\Delta F_p(0) \tag{16}$$

with the linearized response operator I_p^{ee} given by

$$I_p^{ee}=I_p+S_p(F)\ , \tag{17}$$

where $S_p(F)$ is the "linearized" e-e collision operator acting on a function x_p as

$$\begin{aligned}S_p(F)x_p&=S_p(F|x)+S_p(x|F)\\&=\sum_{p'p_1p_1'}(W_{p'p_1'}^{pp_1}F_px_{p_1}-W_{pp_1}^{p'p_1'}F_{p'}x_{p_1'}\\&\quad+W_{p'p_1'}^{pp_1}F_{p_1}x_p-W_{pp_1}^{p'p_1'}F_{p_1'}x_{p'})\ .\end{aligned} \tag{18}$$

The stationary distribution function F_p does not vanish under the action of the operator I_p^{ee}. Indeed, from the definition (17) and (18) it follows that

$$I_p^{ee}F_p=S_p(F|F|)\neq0\ . \tag{19}$$

On the other hand, by differentiating the Boltzmann equation (14) with respect to the number of electrons N,

we have

$$I_p^{ee}\frac{\partial F_p}{\partial N}=0 \ . \tag{20}$$

Due to the nonlinearity of the Boltzmann equation (14), the distribution function F_p is not necessarily proportional to $\partial F_p/\partial N$ and, in general, it is

$$F_p \neq N\frac{\partial F_p}{\partial N} \ . \tag{21}$$

Then, an arbitrary (small) deviation of the distribution function at $t\rightarrow\infty$ becomes proportional not to F_p but to $\partial_p F/\partial N$ (if, of course, it does not vanish at all). In other words, the expression

$$f_p(t)=\exp(-I_p^{ee}t)\delta_{pp_0} \tag{22}$$

at $t\rightarrow\infty$ does not go to F_p/N and therefore cannot be considered as the proper distribution function in this case.

A proper iterative procedure to find the stationary electron distribution function in the presence of *e-e* collisions can be formulated by using a formula similar to Eq. (22) but with the evolution operator of another form:

$$\tilde{f}_p(t)=\exp(-\tilde{I}_p^{ee}t)\delta_{pp_0} \ . \tag{23}$$

This new operator, which is a functional of the distribution function we are looking for, is defined as follows:

$$\tilde{I}_p^{ee}=I_p+\tilde{S}_p(F) \ , \tag{24}$$

where

$$\tilde{S}_p(F)x_p=S_p(x|F) = \sum_{p'p_1p_1'}(W_{p'p_1'}^{pp_1}F_{p_1}x_p-W_{pp_1}^{p'p_1'}F_{p_1'}x_{p'}) \ . \tag{25}$$

As long as $\tilde{I}_p^{ee}$ can be considered a linear operator it has the property

$$\tilde{I}_p^{ee}F_p=0 \ . \tag{26}$$

Therefore, the expression (23) can serve to represent the average motion of an electron. Indeed, at $t\rightarrow\infty$ the function $\tilde{f}_p(t)$, by satisfying Eq. (14), tends to F_p/N and the expression

$$\frac{F_p}{N}=\lim_{t\rightarrow\infty}\exp(-\tilde{I}_p^{ee}t)\delta_{pp_0} \tag{27}$$

may be used to obtain the stationary distribution F_p through an iterative procedure. In this way the distribution function can be obtained by iterating a single-particle simulation. On the other hand, the expression (22) should be used, as we have seen above, to describe properly the response problem in the presence of *e-e* collisions. As we have seen in the previous sections, expressions of the type of (22) and (23) can be put into correspondence to a one-particle simulation in which one should use the diagonal of the momentum part of the collision operator $(1/\tau_p)\delta_{pp'}$ for the determination of the free flight or the acceleration by the field and the nondiagonal part $W_{pp'}$ to describe collisions. The simulation of the process described by Eq. (23) presents no special problems as compared to the case where *e-e* collisions are absent. The diagonal part of the operator $\tilde{S}_p(F)$, $1/\tau_p^{ee}$, given by

$$\frac{1}{\tau_p^{ee}}=\sum_{p'p_1'p_1}W_{p'p_1'}^{pp_1}F_{p_1} \ , \tag{28}$$

changes (to the due extent) the free flight duration, and thus gives the probability per unit time for an electron in state $\mathbf{p}$ to make an *e-e* collision. On the other hand, the off-diagonal part of $\tilde{S}_p(F)$, $\tilde{W}_{pp'}^{ee}$, given by

$$\tilde{W}_{pp'}^{ee}=\sum_{p_1p_1'}W_{pp_1}^{p'p_1'}F_{p_1'} \ , \tag{29}$$

serves to determine the final state $\mathbf{p}'$ after an *e-e* collision has occurred. Thus, for the particle simulation of the stationary homogeneous problem we should use the stochastic law of motion described by the operator $\tilde{I}_p^{ee}$ with the transition rate $\tilde{W}_{pp'}=W_{pp'}^{\text{th}}+\tilde{W}_{pp'}^{ee}$.

These formula show that, for an electron, the other electrons are simply additional scatterers (the dependence of scattering rate on the distribution function being quite analogous, for example, to the dependence of electron scattering on phonons upon the distribution function of phonons). Using the expression (23) in this way, we can obtain (by iteration) the stationary distribution function F_p as we have already mentioned and find the drift velocity of electrons, $\mathbf{V}$:

$$\mathbf{V}=\frac{1}{N}\sum_p \mathbf{v}F_p \ . \tag{30}$$

To find the diffusion coefficient entering Fick's law or, more generally, to solve a response problem in the case of *e-e* scattering, we should set up a one-particle simulation of Eq. (22). In such a simulation, the flight duration remains the same as in the simulation of Eq. (23) since the diagonal terms in $\tilde{S}_p(F)$ and $S_p(F)$ coincide. On the contrary, the off-diagonal terms in these operators differ significantly. In the case of the linearized *e-e* collision operator $S_p(F)$ we have

$$W_{pp'}^{ee}=\tilde{W}_{pp'}^{ee}+\sum_{p_1p_1'}(W_{pp_1}^{p'_1p'}F_{p_1'}-W_{p_1p_1'}^{pp'}F_p) \tag{31}$$

or, due to the indistinguishability of the particles,

$$W_{pp_1}^{p'p'_1}=W_{pp_1}^{p'_1p'}=W_{p_1p}^{p'p'_1}=W_{p_1p}^{p'_1p'} \ , \tag{32}$$

$$W_{pp'}^{ee}=2\tilde{W}_{pp'}^{ee}-F_p\sum_{p_1p_1'}W_{p_1p_1'}^{pp'} \ . \tag{33}$$

We expect that, at least for the majority of cases, $W_{pp'}^{ee}$ in Eq. (33) is a positive quantity; in any case we shall leave this problem to a future investigation. Let us note that the expression

$$\sum_{p'p_1'}W_{p'p_1'}^{pp_1}=\frac{1}{\tau_{pp_1}} \tag{34}$$

entered as a diagonal element in the collision integral for collisions of a "pair" of particles,

$$S_{pp_1}(F|F)=\sum_{p'p'_1}\left(W^{pp_1}_{p'p'_1}F_pF_{p_1}-W^{p'p'_1}_{pp_1}F_{p'}F_{p'_1}\right), \tag{35}$$

which play a significant role in the theory of fluctuations.[3,9] Thus, for the particle simulation of the response problem we should use the "stochastic law of motion" described by the operator I_p^{ee} with the transition rate $W_{pp'}=W^{\rm th}_{pp'}+W^{ee}_{pp'}$, i.e., to solve some specific "model problem" described by Eq. (22).

V. DIFFUSION COEFFICIENT

If we agree that for our "model" problem the law of stochastic motion of a particle is prescribed to its full extent by the form of the evolution operator $\exp(-I_p^{ee}t)$ and for the related problem of finding the stationary distribution by the operator $\exp(-\tilde{I}_p^{ee}t)$, we can find, by the corresponding simulations, the quantities

$$P_M(\mathbf{p},t|\mathbf{p}_0)=\exp(-I_p^{ee}t)\delta_{pp_0}, \tag{36}$$

$$\tilde{P}(\mathbf{p},t|\mathbf{p}_0)=\exp(-\tilde{I}_p^{ee}t)\delta_{pp_0}. \tag{37}$$

The stationary distribution function F_p is determined from the limit of $\tilde{P}(\mathbf{p},t|\mathbf{p}_0)$ at large time,

$$\lim_{t\to\infty}\tilde{P}(\mathbf{p},t|\mathbf{p}_0)=\frac{F_p}{N}, \tag{38}$$

whereas

$$\lim_{t\to\infty}P_M(\mathbf{p},t|\mathbf{p}_0)=\frac{\partial F_p}{\partial N}. \tag{39}$$

Of course, the quantity in Eq. (39) can be evaluated by differentiating with respect to N the results obtained from Eq. (38). From the knowledge of $\partial F_p/\partial N$ we can find the differential (with respect to the number of carriers) drift velocity $\mathbf{V}_N$ given by[3]

$$\mathbf{V}_N=\sum_p \mathbf{v}\frac{\partial F_p}{\partial N} \tag{40}$$

as the drift velocity of the model problem.

The diffusion tensor D_{ik} entering Fick's law for the relaxation of a smooth space-inhomogeneous fluctuation of the electron density δn,

$$\frac{\partial}{\partial t}\delta n=V_{Ni}\frac{\partial}{\partial x_i}\delta n+D_{ik}\frac{\partial^2}{\partial x_i\partial x_k}\delta n, \tag{41}$$

is microscopically defined by[3,4]

$$D_{ik}=\sum_p v_i(I_p^{ee})^{-1}(v_k-V_{Nk})\frac{\partial F_p}{\partial N}. \tag{42}$$

The spectral density of current fluctuations at low frequency, $(\delta j_i\delta j_k)$, includes as a part the contribution proportional to the tensor D_{ik} (Ref. 3)

$$(\delta j_i\delta j_k)=\frac{e^2n_0}{V_0}(D_{ik}+D_{ki}-\Delta_{ik}). \tag{43}$$

Here $n_0=N/V_0$ is the electron concentration, V_0 is the volume of the system, and the symmetric tensor Δ_{ik} describes the contribution due to the e-e collision correlation. This last is given by[3]

$$\Delta_{ik}=\frac{1}{N}\sum_{pp_1}v_iv_k(I_p^{ee})^{-1}(I_{p_1}^{ee})^{-1}\times\left[S_{pp_1}(F|F)-(I_p^{ee}+I_{p_1}^{ee})\left[F_p-N\frac{\partial F_p}{\partial N}\right]\delta_{pp_1}\right]. \tag{44}$$

Let us introduce the "chaotic" electron velocity of the model problem:

$$\mathbf{u}=\mathbf{v}-\mathbf{V}_N. \tag{45}$$

Then the diffusion tensor D_{ik} of Eq. (42) can be also defined as

$$D_{ik}=\sum_p u_i\int_0^\infty dt\,\exp(-I_p^{ee}t)u_k\frac{\partial F_p}{\partial N}. \tag{46}$$

[We use the property $\sum_p(I_p^{ee})^{-1}\cdots=0$ (Ref. 3) to replace v_i with u_i.] Equation (46) can be rewritten as [see Eq. (36)]

$$D_{ik}=\sum_{pp_0}u_i\int_0^\infty dt\,P_M(\mathbf{p},t|\mathbf{p}_0)u_{0k}\frac{\partial F_{p_0}}{\partial N}. \tag{47}$$

The integrand in the rhs of Eq. (47) is nothing but the autocorrelation function of the chaotic velocity of the model problem:

$$\sum_{pp_0}u_iP_M(\mathbf{p},t|\mathbf{p}_0)u_{0k}\frac{\partial F_{p_0}}{\partial N}=\langle u_i(0)u_k(t)\rangle. \tag{48}$$

Thus, in analogy with the case when e-e collisions are neglected,[5,10] we have

$$D_{ik}=\int_0^\infty dt\langle u_i(0)u_k(t)\rangle, \tag{49}$$

where angular brackets denote the ensemble average for a particle with the law of stochastic motion prescribed by our model, i.e., by the linearized collision operator I_p^{ee}.

Let $\mathbf{v}(t)$ be the velocity of a given electron in the simulation of such a model motion. Then, $\mathbf{v}(\infty)$ [or $\mathbf{v}(t)$ for the large t] averaged over many realizations becomes $\mathbf{V}_N$. If we observe the wandering of an electron with the "law of motion" given by the evolution operator $\exp(-I_p^{ee}t)$ we could find the diffusion tensor D_{ik} and the differential drift velocity $\mathbf{V}_N$ according to the usual formula:

$$\frac{d}{dt}\langle\mathbf{r}(t)\rangle=\mathbf{V}_N,\quad\langle\mathbf{r}(0)\rangle=0 \tag{50}$$

$$\frac{1}{2}\frac{d}{dt}\langle[x_i(t)-V_{Ni}t][x_k(t)-V_{Nk}t]\rangle=D_{ik}. \tag{51}$$

On the other hand, the substitution in the evolution operator $\tilde{I}_p^{ee}\to I_p^{ee}$ [i.e., the substitution $\tilde{S}_p(F)\to S_p(F)$] gives us the drift velocity $\mathbf{V}$ by the same procedure.

VI. CONCLUSIONS

In this paper the equivalence between the Boltzmann equation and the particle approach in the presence of electron-electron collision has been investigated and given on a rigorous basis. In particular, we have proven that the diffusion coefficient can be still expressed through a time integral of the single-particle autocorrelation function of velocity fluctuations. However, the law of the stochastic motion of the electron should be determined by the linearized electron-electron collision operator. By introducing a model collision operator and distribution function, we propose an iterative scheme to evaluate the quantities of interest through a standard one-particle simulation. Further research will be devoted to a quantitative calculation for practical cases of interest.

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