# dc conductivity of arsenic-doped silicon near the metal-insulator transition

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dc conductivity measurements have been made on uncompensated Si:As in the concentration range $6.85\times10^{18}<N<32.8\times10^{18}$ cm$^{-3}$ for $0.5<T<77$ K, although three samples were studied in a dilution refrigerator. Insulating samples exhibited variable-range hopping (VRH) behavior of the form $\sigma(T)=\sigma_0(1/T)^s\exp[-(T_0/T)^p]$ for $0.86N_c<N<0.99N_c$ for $T<8$ K. The most satisfactory overall fit to the data is $p\sim\frac{1}{4}$ and $s\sim0$, namely Mott VRH. A new criterion is given to decide the temperature regimes where Mott or Efros-Shklovskii VRH ($p=\frac{1}{2}$) should be observed. The characteristic Mott temperature $T_0\propto(1-N/N_c)^{3\nu}$ yielded $0.77<\nu<0.97$. Strong deviations from VRH behavior were observed for $N<0.84N_c$. Metallic samples showed a $\sigma(n,T)=\sigma(n,0)+m(n)T^{1/2}$ dependence at sufficiently low temperatures. The results yield $\sigma(n,0)=\sigma_0(n/n_c-1)^\mu$ with $\sigma_0=376$ S/cm, $8.55<n_c<8.60\times10^{18}$ cm$^{-3}$, and $\mu=0.60\pm0.05$. The coefficient $m(n)$ shows behavior similar to that for Si:P, but shows a second sign crossing for $n/n_c\sim2.4$. The $m(n)$ results seem to confirm the dominance of the Hartree interaction for the many-valley case.

## I. INTRODUCTION

Extensive experimental results from dc conductivity measurements on different barely metallic metal-insulator transition (MIT) systems as a function of dopant density (or metal atom composition for semiconductor-metal alloys) have confirmed the scaling behavior[1] of $\sigma_{dc}(n,T=0$ K) and have apparently ruled out Mott's minimum metallic conductivity.[2] These experimental results for $3d$ MIT systems all yield $\sigma_{dc}(T\to0)\propto(n/n_c-1)^\mu$ where $n_c$ is the critical density and we note two different values of the conductivity exponent $\mu$ have been obtained dependent on the type of MIT system. For uncompensated $n$-type crystalline semiconductors such as Si:P,[3] Si:As,[4] Si:P+As,[5] Si:Sb,[6] and Ge:As,[7] $\mu$ is close to $\frac{1}{2}$. Si:P has been particularly carefully studied by Paalanen *et al.*[3] in the millikelvin temperature regime utilizing uniaxial stress to tune $n-n_c$ to within 0.1% of $n_c$, thus obtaining $\mu=0.51\pm0.05$. On the other hand, the amorphous alloys *a*-Si:Au,[8] *a*-Si:Al,[9] *a*-Ge:Au,[10] *a*-Si:Nb,[11] and *a*-Ge:Mo (Ref. 12) all show $\mu\approx1$ in agreement with the prediction[1] of weak localization scaling theory. Different results[13,14] for $\mu$ have been obtained for crystalline Ge:Sb,[13,14] where the compensation may not be as small as in the $n$-type Si, with the most recent results of Ootuka *et al.*[14] showing $\mu\sim0.9$ in zero magnetic field. There is no generally accepted theory of $\mu=\frac{1}{2}$ for uncompensated $n$-type semiconductors although it is thought to relate to the dominance of electron-electron interactions near $n_c$ for these MIT systems. This exponent puzzle situation has been reviewed by Thomas.[15]

On the insulating side of the MIT the $T\to0$ behavior of the low-frequency dielectric response $\epsilon'(N)$ exhibits a polarization catastrophe[16] [$\epsilon'(N)\to\infty$ as $N\to N_c-$]. It has been experimentally demonstrated that $\epsilon'(N)=\epsilon_h+4\pi\chi'(N)$ exhibits scaling behavior [$\chi'(N)\propto(N_c-N)^{-\zeta}$] with $\zeta=1.0$ for Si:P (Ref. 17) and $\zeta=1.18\pm0.08$ for Si:As.[18] These results are consistent with theoretical predictions of weak localization scaling theory[1] that $\zeta=2\mu$, neglecting *e-e* interactions although a scaling theory incorporating *e-e* interactions, such as that formulated by McMillan,[19] could also lead to a similar result. The only semiconductor-metal alloy MIT system for which there is a dielectric constant enhancement is the *a*-Si:Au system.[20] There has been no effort to verify the $\zeta=2\mu$ prediction for the semiconductor-metal alloy MIT systems.

Despite the fact the MIT is considered a $T=0$ phase transition critical behavior can also be observed in finite temperature corrections to the $T=0$ behavior. For metallic samples ($n>n_c$) the dc conductivity exhibits a correction

$$\delta\sigma_{dc}(n,T)=\sigma_{dc}(n,T)-\sigma_{dc}(n,0)=m(n)T^{1/2}$$

at sufficiently low temperatures, as has been reported in many systems.[3–6,11,12,21–24] This $T^{1/2}$ correction, resulting from *e-e* interactions in the presence of strong impurity scattering, was first calculated by Altshuler and Aronov.[25] Of particular interest very close to $n_c$ as $n\to n_c+$ is the sign change in $m(n)$. $m(n)$ changes sign to become positive with $m(n)\propto D(n)^{-1/2}$. Since $D\propto(n/n_c-1)^p$ ($p=\mu$ from weak localization theory) $m(n)$ increases rapidly for $n_c/n_c-1<0.02$. The behavior of $m(n)$ has been accurately determined for Si:P by Thomas *et al.*[3] Susceptibility studies[26–28] near $n_c$ have shown $\chi(N,T)\propto T^{-\alpha(N)}$ with $\alpha(N)$ approaching 0.6 as $N\to N_c-$.

On the insulating side of the MIT one expects $\sigma_{dc}(N<N_c,T=0)=0$. For finite but sufficiently low temperatures one observes variable-range hopping (VRH) conduction of the form

$$\sigma_{dc}(N<N_c,T)=\sigma_0(T)e^{-(T_0/T)^p} \quad (1)$$

where $p=\frac{1}{4}$ for $3d$, [$p(d)=1/(d+1)$] for the Mott type[29] of VRH conduction for which *e-e* interactions are

 

neglected, and $\sigma_0(T) \propto (1/T)^s$. For the Efros-Shklovskii case,[30] including long-range $e$-$e$ interactions, $p = \frac{1}{2}$ (in all $d$). In both cases the characteristic temperature $T_0$ (different expressions for the two cases) is expected to scale to zero as $N \to N_c-$. The literature includes innumerable observations of VRH conduction in amorphous semiconductors, polymers, doped crystalline semiconductors, and even in high-$T_c$ superconducting materials. Here we shall limit the discussion to group-IV and III-V crystalline semiconductors such as $n$-type Ge,[31–34] $n$-type Si,[35–39] $n$-type GaAs,[40–43] $n$-type InSb,[44,45] and $n$-type InP.[46–50] VRH conduction in the critical regime as $N \to N_c-$ has not received sufficient attention. In this regime $T_0 \to 0$ as $N \to N_c-$ and the mean hopping distance $R_{\rm hop}(T)$ becomes comparable to or less than the localization length $\xi$ as $T_0$ becomes less than the measuring temperature $T$. This is an unfamiliar regime for classical phonon-assisted hopping theory.[51] In this regime $e$-$e$ interactions are expected to be important and correlated many-electron hopping[52,53] may be very significant. After our preliminary results for Si:As suggested VRH, probably of the Mott type, the primary objective of the present study was to document the critical behavior of the VRH conduction and $T_0(N)$ as $N \to N_c-$ and to compare with the Si:P results of Hess *et al.*[54]

A secondary objective of this work was to extend the critical behavior of barely metallic Si:As first reported by Newman and Holcomb[4] closer to $n_c$ and also to more metallic samples with $n > 2n_c$. It is of interest to ascertain how closely the temperature-dependent $m(n)T^{1/2}$ behavior and the $T=0$ conductivity exponent $\mu$ of Si:As resemble those of Si:P in order to determine the universality of the critical behavior.

## II. BACKGROUND

### A. Variable-range hopping conduction

The theory of VRH conduction [Eq. (1)] has ultimately been based on the Miller-Abrahams (MA) expression[51] for the phonon-assisted electron hopping rate $\nu(R,E)$ between localized states separated by a distance $R$ and differing in energy by $E$. The MA expression for phonon absorption is of the form

$$\nu(R,E) = bER^k e^{-[2R/a(N)+E/kT]} , \tag{2}$$

where $a(N)$ is a localization length, $b$ is a constant including deformation potential and phonon velocity, while $k=2$ for isotropic donor wave functions. Mott[29] obtained Eq. (1) with $s=\frac{1}{2}$ by minimizing the exponent in Eq. (2) with the hopping distance $R(T)$, utilizing a mean hopping energy given by $E = 3/4\pi N(E_F)R^3$. This led to a mean hopping distance $\langle R(T)\rangle$, a mean hopping energy $\langle E_{\rm hop}(T)\rangle$, and the Mott characteristic temperature $T_0$ given by

$$\bar{R}_{\rm hop} \equiv \langle R(T)\rangle \simeq 0.4a(N)(T_0/T)^{1/4} , \tag{3a}$$

$$\bar{E}_{\rm hop} \equiv \langle E(T)\rangle \simeq 0.24kT^{3/4}T_0^{1/4} , \tag{3b}$$

$$T_0 = \beta/kN(E_F)[a(N)]^3, \quad \beta \approx 18 . \tag{3c}$$

Mott's approach assumes a slowly varying density of states (DOS) in the vicinity of the Fermi level and specifically neglects $e$-$e$ interactions. The Mott result was also obtained by Ambegaokar, Halperin, and Langer[55] by a careful application of percolation theory. Efros and Shklovskii[30] (ES), taking account of $e$-$e$ interactions, obtained $N(E) \propto (E-E_F)^2$ and a Coulomb gap width $\Delta_{\rm Cg} = e^3N_0(E_F)^{1/2}/\epsilon'^{3/2}$ where $N_0(E_F)$ is the DOS without $e$-$e$ interactions and $\epsilon'$ is the static dielectric constant. Their analysis led to $p=\frac{1}{2}$ in all dimensions and $T_0' = 2.8e^2/\epsilon' a(N)$. A detailed discussion of VRH theory, both with and without $e$-$e$ interactions, and of the Coulomb gap problem is given in the book by Shklovskii and Efros.[56]

In considering VRH conduction in the critical regime as $n \to n_c-$ it is essential to consider the magnitudes of the characteristic temperatures $T_0$, $T_0'$, and $T_{\rm Cg}$ ($T_{\rm Cg} \equiv \Delta_{\rm Cg}/k$) relative to each other and to the thermodynamic temperature $T$ of the sample. One also needs to compare the relative magnitudes of $\bar{R}_{\rm hop}$ and $a(N)$. The MA derivation of Eq. (2) was in the dilute limit where $\bar{R}_{\rm hop} \gg a(N)$. In the dilute regime $T_0$ is large and the conventional view of Mott VRH conduction is that $\bar{R}_{\rm hop} \gg a(N)$ and $T_0 \ggg T$ and the temperature dependence in Eq. (1) is dominated by the exponential term. However, in the critical regime ($N \to N_c-$) $a(N)$ is identified as the localization length $\xi(N) = \xi_0(1-N/N_c)^{-\nu}$, which diverges as $N \to N_c-$. The dielectric constant is given by $\epsilon'(N) = \epsilon_h + 4\pi\chi'(N)$ where $\chi'(N) = e^2N(E_F)\xi^2$ and $\epsilon_h$ is the host dielectric constant. In the critical regime as $N \to N_c-$, $4\pi\chi'(N) \gg \epsilon_h$ and all three temperatures $T_0$, $T_0'$, and $T_{\rm Cg}$ scale to zero as $[N(E_F)\xi^3]^{-1}$. The ratios of these quantities are, respectively,

$$T_0/T_0' \simeq 80; \quad T_0/T_{\rm Cg} \simeq 800; \quad T_0'/T_{\rm Cg} \simeq 10 . \tag{4}$$

One expects Mott VRH when $\bar{E}_{\rm hop}(T) > \Delta_{\rm Cg}$ because the DOS looks smooth in this case. Similarly, to observe ES VRH one requires $\bar{E}'_{\rm hop} < \Delta_{\rm Cg}$. Using the ratios in Eq. (4) and also Eq. (3b) this leads one to expect Mott VRH when $T > T_0/1170$ and ES VRH when $T$ is enough less that $T_0/2000$. The Si:As data discussed in Sec. IV seem to be in qualitative agreement with these inequalities. These inequalities illustrate the practical difficulty of seeing ES VRH when $T_0$ is less than 1 K as $N \to N_c-$. However, the crossover from Mott to ES VRH as the temperature is lowered has recently been reported for $a$-Ge:Sn alloys by Glukhov *et al.*[57] The above arguments show there is a large range of temperature $T > T_0$ where from Eq. (3a) one obtains $\bar{R}_{\rm hop} < \xi(N)$. This is an unfamiliar range and is the opposite limit of the MA calculation.

The MA calculation was also carried out in the limit $qR > 1$ where $q$ is the phonon wave number. For a characteristic phonon velocity $c_s$ and using $\bar{q} = \bar{\omega}/c_s = \bar{E}_{\rm hop}/\hbar c_s \propto T^{3/4}$ and $\bar{R}_{\rm hop} = 0.4\xi(N)(T_0/T)^{1/4}$ one finds it possible to have $\bar{q}\bar{R}_{\rm hop} \ll 1$ for $T < 1$ K and the terms neglected by MA need to be considered in the critical regime. In the Appendix these extra terms are considered. The primary effect of these terms is to change

the value of $k$ in Eq. (2) which correspondingly changes the value of $s$ in Eq. (1).

Whether pairwise hopping of the Miller-Abrahams type is valid as $N \rightarrow N_{c-}$ or whether highly correlated multielectron hopping, as considered by Knotek and Pollak,[52,53] is dominant as $N \rightarrow N_{c-}$ is a question that is not easy to resolve. However, the experimental observation of the Mott VRH conduction law in Eq. (1) should not be used to rule out correlated multielectron hopping in the critical regime.

### B. Metallic behavior

The temperature dependence of barely metallic samples has the form

$$\sigma(n,T)=\sigma(n,0)+m(n)T^{1/2}+B(n)T^{p} \tag{5}$$

where the first term is the $T=0$ weak localization result $\sigma(n,0)\approx\sigma_B[1-3/(k_F l)^2(1-l/L)]$ where $\sigma_B$ is the Boltzmann conductivity, $l$ is the elastic mean free path, and $L$ is the size of the sample, although for finite temperatures $L$ can be the inelastic diffusion length $L_{\rm in}$ which then leads to a term like the third term in Eq. (5) with $p\approx 1$ or $p\approx\frac{3}{4}$. As $T\rightarrow 0$ the leading temperature-dependent correction to $\sigma(n,T)$ is the $m(n)T^{1/2}$ term which results from $e$-$e$ interactions in a disordered metal. Altshuler and Aronov[25] found a $\sqrt{E}$ correction to the electron DOS near $E_F$ and this in turn led to the $m(n)T^{1/2}$ term in Eq. (5) with $n(m)$ given by

$$m(n)=c(e^2/\hbar)[k/\hbar D(n)]^{1/2}(\tfrac{4}{3}-2F) \tag{6}$$

where $c=0.915/4\pi^2$ and $D(n)$ is the diffusion coefficient. The $\frac{4}{3}$ factor results from the exchange term and $F$ is the Hartree factor. Additional corrections were later found[58,59] and the $2F$ in Eq. (6) is replaced by the quantity $(3\tilde{F}/2)$ where $\tilde{F}$ for the $d=3$ case is given by

$$\tilde{F}=\tfrac{32}{3}[(1+F/2)^{3/2}-(1+3F/4)]/F\ . \tag{7}$$

For small $F$ one finds $\tilde{F}\rightarrow F$. $F$, resulting from the Hartree interaction, has been given as $F=[\ln(1+x)]/x$ with $x=(2k_F/K)^2$, where $k_F$ is the Fermi wave number and $K$ is the Thomas-Fermi screening wave vector. The results in Eqs. (6) and (7) are for the single-valley case. Additional corrections are required for the multiple-valley case where there is a mass anisotropy to be considered and these corrections have been considered by several authors.[60,61] The situation for the many-valley case is considerably more complex and one needs to also consider the possible importance of intervalley scattering. A detailed analysis of $m(n)$ for the multiple-valley cases of Si and Ge has been made comparing with Si:P[3] and Ge:Sb[22] data.

Using $D(n)=D_0(n/n_c-1)^{\mu}$ with $D_0=v_F^2\tau/3$, we express Eq. (6) as

$$m(n)=c(e^2/\hbar)S_0(k/\hbar D_0)^{1/2}(n/n_c-1)^{-\mu/2}A(F) \tag{8}$$

for the purposes of data analysis. $S_0$ results from the many-valley feature and mass anisotropy and for Si, $S_0=1.26$. All of $m(n)$ in Eq. (8) except $A(F)$ will be regarded as known experimentally from $\sigma_0(n)$ and the experimental values of $m(n)$ will be utilized to determine $A(F(n))$ which can then be compared with theoretical predictions for the different many-body corrections.

## III. EXPERIMENTAL DETAILS

### A. Samples and sample preparation

Almost all the data reported below are on nominally uncompensated Si:As samples cut from a single ingot originally 5.0 cm in diameter and 35 cm in length obtained from MA-COM. The ingot was Czochralski grown and featured an approximate linear concentration gradient along the growth axis with the As concentration spanning the range $5.0\times10^{18}/\text{cm}^3$ to $11\times10^{18}/\text{cm}^3$. The acceptor (boron) concentration was stated to be less than $10^{14}/\text{cm}^3$ by MA-COM. The critical density $n_c$ for the onset of metallic behavior was near the center of the ingot and a fine concentration grid of both insulating and metallic samples in the critical regime was readily obtained. The first reported results of scaling behavior of $\sigma_{\rm dc}(n\rightarrow n_{c+},T\rightarrow 0)$ for Si:As were obtained by Newman and Holcomb[4] on samples cut from this ingot.

Since the room-temperature resistivity $\rho$(RT) on any individual wafer from the above ingot can vary as much as 10–15% it was necessary to carefully profile each wafer to find the desired As concentration and regions of high doping homogeneity. All $\rho$(RT) measurements were made with a Signatone four-point probe with osmium tips (probe spacing is 0.050 in.). $\rho$(RT) was determined using standard formulas[62] to within $\pm1.5\%$ accuracy, which led to $\pm3\%$ errors in the donor density $N_D$ in this density regime. For the critical regime this accuracy was insufficient and the final donor densities were obtained from the resistivity ratio $r_0(N_D)=\rho(4.2\text{ K})/\rho(\text{RT})$ which varies far more rapidly near $n_c$ than $\rho$(RT) and can be measured on the final samples after etching and lead attachment. $r_0(N_D)$ can be measured to $\pm0.2\%$. The sample dimensions are not required and thermal contraction corrections are negligible ($\sim2\times10^{-4}$). Figure 1 shows $r_0(N_D)$ versus $N_D$ for Si:As in the critical regime. Our $r_0(N_D)$ and $\rho$(RT) values were compared with corresponding values obtained by Newman and Holcomb,[4] which in turn were tied to the absolute scale of $\rho$(RT) versus $N_D$ for Si:As determined by Newman, Hirsch, and Holcomb[63] (NHH) using the neutron activation analysis technique. With the use of Fig. 1 the relative concentrations, the important quantity for studying critical behavior near $n_c$, could be obtained to better that 0.5%. Any systematic errors in the absolute $\rho_{\rm RT}$ versus $N_D$ scale are not important to the results reported below.

Several disk van der Pauw samples ($n/n_c>2$) that were more metallic were studied and concentrations were determined from a combination of $\rho$(RT), $r_0(N_D)$, and Hall coefficient measurements taking account of the Hall correction factor $A(N_D)$ [$R_H=A(N_D)/ne$]. For these samples the relative values of $n$ are less accurate, but they are much further from $n_c$ and the errors are less important.

The bar-shaped samples were cut with a Micromech

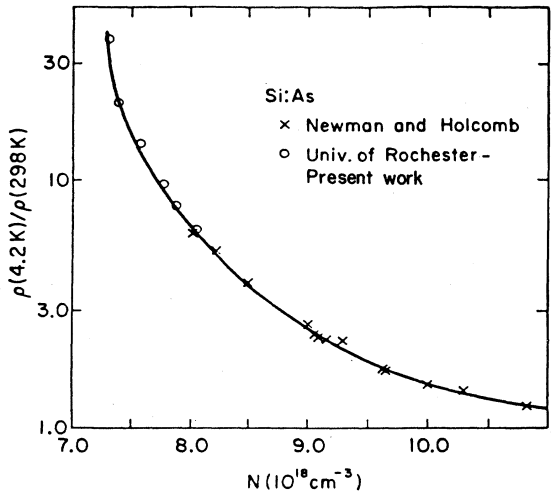


FIG. 1. $r_0=\rho(4.2\text{ K})/\rho(298\text{ K})$ vs $N_D$ for $7.3<N<11\times10^{18}$ cm$^{-3}$. Our Si:As $\rho$(RT) values were compared with those of Newman and Holcomb (Ref. 4) which were based on absolute scale of $\rho$(RT) vs $N_D$ determined by Newman *et al.* (Ref. 63) employing neutron activation analysis.

wafering machine to approximately $0.4\times0.75\times6$ mm$^3$. After hand lapping with 600 grit emery paper they were cleaned with isopropanol, and finally were etched with a CP-4 solution (3 parts HF, 3 parts $HNO_3$, 5 parts acetic acid) for about 90 sec. They were stored in distilled water until the leads (0.002 in. $Au_{0.98}Sn_{0.02}$ wire) were attached using the wire-welding method of Capik.[64] Cleanliness during the "welding" process was essential. The contact area of the weld is equivalent to a circle of diameter $\sim0.1$ mm and the four leads were placed 1 to 1.5 mm apart on the bar samples. For the van der Pauw disk-shaped samples (6 mm and 3 mm diameter) the four leads were placed 90° apart on the perimeter. With the leads in place the samples were again cleansed in trichloroethylene and isopropanol and stored in distilled water until mounted for measurements. The four lead wires are soldered to BeCu pins with indium solder on one of several low-temperature inserts. Sample can be successfully recycled for several different low-temperature experiments, although occasionally a lead will pull off. Usually the break is not right at the Si surface, but just above the contact. In such cases the lead can be reattached with the same welding procedure.

### B. Low-temperature measurement details

Measurements at Rochester were made between 1.4 and 77 K employing an insert featuring a Cu block and a heater coil positioned inside a vacuum tight can containing $^4$He exchange gas. The temperature of the Cu block was monitored above 4.2 K with a Lakeshore Cryotronics Ge resistor calibrated between 1.4 and 100 K. Temperatures $T$ below 4.2 K were determined from $^4$He vapor pressure-temperature curves. The BeCu pins were thermally heat sunk to the Cu block. For all measurements between 4.2 and 77 K $T$ was accurate to $\pm0.2\%$.

Measurements at the Francis Bitter National Magnet Laboratory (FBNML), as a part of a larger series of magnetoresistance studies to 15 T and higher, were made in either a $^3$He cryostat or in a dilution refrigerator (DR) (with Brooks).[65] Most of the zero-field results discussed below resulted from temperature sweeps between 4.2 and 0.5 K in a $^3$He cryostat because of the precise temperature control (employing $^4$He and $^3$He vapor pressures). In several experiments we reached $T\approx0.37$ K with a booster diffusion pump. Excellent thermal contact between the samples and $^3$He liquid was obtained, although for $T>2.2$ K the sample contact with the $^3$He exchange gas was such that it was necessary to wait as long as ten minutes between temperatures to achieve sample-$^3$He exchange gas-$^4$He bath thermal equilibrium. In the DR experiments the sample leads were directly soldered to feed-through leads inside the epoxy mixing chamber and the samples were in direct contact with the $^3$He-$^4$He mixture. The temperature was monitored with calibrated carbon resistors and temperatures reached were about 50 mK. The precision of the temperature control was not as high as with the $^3$He refrigerator.

The conductivity $\sigma(T)$ of the bar samples is determined from the relation

$$\sigma(T)=\frac{1}{R(T)}\frac{l}{A}\simeq\frac{1}{R(T)}\frac{R(\text{RT})}{\rho(\text{RT})}\ , \tag{9}$$

where $l$ is the spacing between the voltage leads and $A$ is the cross-sectional area. The assumption in the second part of Eq. (9) is that $l/A$ changes negligibly between RT and $T\sim1$ K. $R$(RT) can be determined to within 0.1% while $\rho$(RT) is determined to $\pm1.5\%$ as discussed above. A direct geometric measurement of $l/A$ cannot be better than $\pm10$–$20\%$ because the $Au_{0.98}Sn_{0.02}$ contact areas are slightly irregular and the effective cross section between the voltage leads is not readily measured. The absolute magnitudes of $\sigma(T)$ are limited by the $\rho$(RT) determination, but the relative temperature dependence is much more accurately determined and is independent of errors in $\rho$(RT). The details of the van der Pauw geometry measurements are discussed elsewhere.[66]

Because of the accuracy of the temperature control in the $^3$He refrigerator we attempted to experimentally determine, as proposed by Hill,[67] the logarithmic derivative $\delta=-d\ln\sigma/d\ln(1/T)$ by choosing temperatures in groups of three with equal intervals in $1/T$ between the three. The experimental $\delta(T)$ is determined from

$$\delta(T_2)=-\frac{1}{\sigma(T_2)T_2}\left[\frac{\sigma(T_3)-\sigma(T_1)}{(1/T_3)-(1/T_1)}\right], \tag{10}$$

where $T_3>T_2>T_1$. In obtaining $R(T)$ the current (from a Keithley 225 or 261 current source) was measured with a Keithley 195 multimeter and the voltage with either a Keithley 181 nanovoltmeter or a HP 3456A digital volt meter (DVM). The current and voltage were measured with both current polarities to eliminate thermal emf's.

In order to avoid non-Ohmic conduction effects[68] care was taken to keep the sample power dissipation sufficiently small. Periodically $R(T)$ was measured with

current levels over 2 orders of magnitude. The current level was set a factor of 10 below the onset for non-Ohmic conduction. For the DR experiments this required $I^2R(T) \ll 10^{-9}$ W for $T < 0.1$ K. As observed for Si:P,[68] it was necessary to reduce the current as the temperature was lowered and this meant keeping $E/T < 3$ mV/cm K.

## IV. EXPERIMENTAL RESULTS

### A. Insulating samples

The insulating bar samples studied are listed in Table I along with the respective values of $r_0$, $N_D$, $\rho$(RT), and the temperature range in which they were measured. One observes from Table I that for a 20% change in $N_D$, $\rho$(RT) changes only 12% while $r_0 = \rho(4.2\ \text{K})/\rho(\text{RT})$ changes by a factor of 77, thus demonstrating (as in Fig. 1) the usefulness of $r_0$ in determining the relative position of these insulating samples as $N_D \rightarrow n_c -$. While one cannot achieve the fine tuning of $|n - n_c|$ obtained by Paalanen *et al.*[3] with stress tuning, it is nevertheless readily possible to distinguish samples 0.5% different in $N_D$.

$\ln\sigma(N_D, T)$ versus $T^{-1/4}$ is shown for nine Si:As samples in Fig. 2. The six most concentrated samples apparently fit the Mott VRH law for $T^{-1/4} > 0.6$ ($T < 8$ K) and, as will be discussed more below, fit activated hopping [$\ln\sigma \propto -(E_{\text{act}}/kT)$] for $T > 40$ K. The three more dilute samples (7.39, 7.30, and 6.98) may exhibit the Mott law in a narrow temperature range, but these three clearly show a stronger temperature dependence at much lower temperatures, which might be associated with the opening of a Coulomb gap. The deviations from the Mott-like behavior will be deferred for later discussion. It is generally expected that at sufficiently low temperatures when $kT \ll \Delta_{Cg}$ ($\Delta_{Cg}$ the Coulomb gap width) the VRH behavior should change to ES behavior [$p = \frac{1}{2}$ in Eq. (1)]. Three samples in Table I ($\tilde{N}_D = 7.66$, 8.07, and 8.23) were run in Brooks's DR at FBNML. The $\ln\sigma$ versus $T^{-1/4}$ plot for these samples in Fig. 3 shows a good fit to the Mott law for the 8.23 and 8.07 samples down to $T \sim 60$ mK, however the 7.66 sample shows upward curvature below $T \sim 160$ mK. We carefully checked for non-Ohmic behavior for this sample but found none. The deviation for this sample is in the opposite direction to that for the 7.39, 7.30, and 6.98 samples in Fig. 2. We have no explanation for this behavior although we note that more complex $\sigma(T)$ behavior for Si:Sb has been observed by Long and Pepper.[6]

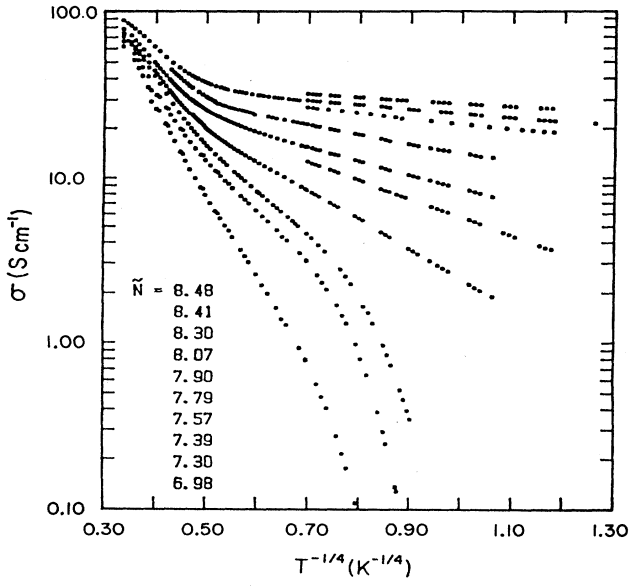


FIG. 2. $\log\sigma(N,T)$ vs $T^{-1/4}$ for ten insulating Si:A bar samples ($N = \tilde{N} \times 10^{18}$ cm$^{-3}$).

It has frequently been argued that one cannot reliably distinguish between the Mott VRH law ($p = \frac{1}{4}$) and the

TABLE I. Insulating samples.

| Sample | $r_0$ | $N$ ($10^{18}$ cm$^{-3}$) | $\rho_{RT}$ (mΩ cm) | Measurements |
|---|---|---|---|---|
| 11*c*-2 | 302.9 | 6.85 | 8.78 | $^3$He |
| 11*a*-1 | 143.0 | 6.98 | 8.69 | $^4$He |
| 16*c*-3 | 37.71 | 7.30 | 8.45 | $^4$He |
| 18*a*-1 | 25.85 | 7.39 | 8.39 | $^4$He |
| 18*x*-1 | 24.10 | 7.42 | 8.37 | $^4$He |
| 30-3 | 14.14 | 7.57 | 8.27 | $^4$He, $^3$He |
| 30-2 | 12.0 | 7.66 | 8.21 | DR |
| A17-42 | 9.65 | 7.79 | 8.12 | $^3$He |
| A4-1 | 7.93 | 7.90 | 8.04 | $^4$He, $^3$He |
| B4-2 | 6.27 | 8.07 | 7.93 | $^4$He, $^3$He |
| B4-4 | a | ~8.07 | ~7.93 | DR |
| A'2-3 | 5.09 | 8.23 | 7.83 | DR |
| B16-1 | 4.71 | 8.30 | 7.79 | $^3$He, DR |
| B'3-33 | 4.26 | 8.41 | 7.72 | $^4$He, $^3$He |
| B1-1 | 3.94 | 8.48 | 7.68 | $^3$He |

[a] $r_0$ was not determined, but the data indicate that $N$ is very close to sample B4-2.

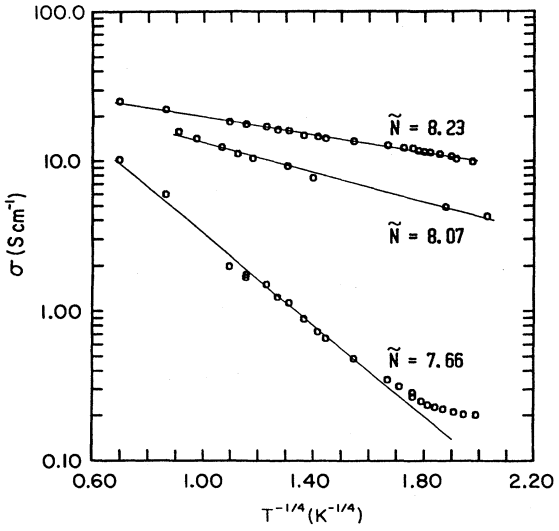


FIG. 3. Dilution refrigerator results for $\log\sigma(N,T)$ vs $T^{-1/4}$ for three Si:As bar samples ($\tilde{N}=N/10^{18}$). The upward curvature for $T<0.16$ K for the $\tilde{N}=7.66$ sample is not a result of Ohmic heating or non-Ohmic effects.

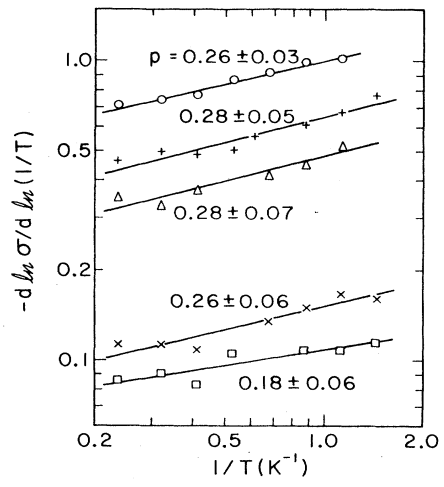


FIG. 4. $\delta(T)=-d\ln\sigma(T)/d\ln(1/T)$ vs $1/T$ for five Si:As samples. □, $\tilde{N}=8.48$; ×, $\tilde{N}=8.41$; △, $\tilde{N}=7.90$; +, $\tilde{N}=7.79$; ○, $\tilde{N}=7.57$. Note that Eq. (1) yields $\delta(T)=p(T_0/T)^p-s$.

ES VRH law ($p=\frac{1}{2}$) unless one has a sufficiently large range in both $\sigma(T)$ and in $T$. The problem becomes particularly severe as $N\to N_c-$ since $T_0\to 0$ and the overall variation in $\sigma(T)$ becomes very small. As the variation in the exponent $e^{-(T_0/T)^p}$ becomes small it becomes important to consider the temperature dependence of the prefactor $\sigma_0(T)$ which on theoretical grounds from Eq. (2) is proportional to $(1/T)^s$ ($s=\frac{1}{2}$ from Mott[29]). Many experimentalists have ignored the prefactor temperature dependence in their data analysis. In Fig. 4 we plot the logarithmic derivation $\delta(T,s,p,T_0)$ versus $1/T$ on a log-log plot for the 7.57, 7.79, 7.90, 8.41, and 8.48 samples from Fig. 2. From Eq. (1) $\delta=p(T_0/T)^p-s$ we see that within experimental error $\delta(T)$ is consistent with $p\sim\frac{1}{4}$ and $s\sim 0$, however it is not possible to reliably determine the intercept in Fig. 4 because of the slow variation of $p(T_0/T)^p$

TABLE II. Least-squares fits to VRH data. $N$ is in $10^{18}$ cm$^{-3}$, $T_0$ in K, and $A$ in S K$^s$. The error $\chi$ is the percent error given by $\chi=(100/n)\{\sum_{i=1}^{n}[\sigma(T,p,s,A)-\sigma_i(T_i)]^2\}^{1/2}$, with $\sigma(T,p,s,A)$ given by Eq. (4.1), $\sigma_i(T_i)$ the measured conductivity at temperature $T_i$, and $n$ the number of measurements on a given sample.

| | Fit 1 $p=\frac{1}{4}$ or $\frac{1}{2}$, $s=0$ | | | | Fit 2 $p=\frac{1}{4}$ or $\frac{1}{2}$, $s$ variable | | | | | Fit 3 $p$ variable, $s=0$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | $p$ | $T_0$ | $A$ | $\chi$ | $p$ | $T_0$ | $s$ | $A$ | $\chi$ | $p$ | $T_0$ | $A$ | $\chi$ |
| 7.57 | $\frac{1}{4}$ | 278 | 145 | 0.135 | $\frac{1}{4}$ | 474 | 0.11 | 258 | 0.040 | 0.285 | 100 | 99 | 0.042 |
| | $\frac{1}{2}$ | 7.3 | 32 | 0.691 | $\frac{1}{2}$ | 2.3 | −0.34 | 11 | 0.053 | | | | |
| 7.79 | $\frac{1}{4}$ | 45.6 | 77 | 0.053 | $\frac{1}{4}$ | 41.6 | −0.01 | 73 | 0.056 | 0.245 | 53.0 | 81 | 0.056 |
| | $\frac{1}{2}$ | 2.14 | 25 | 0.333 | $\frac{1}{2}$ | 0.51 | −0.29 | 12 | 0.057 | | | | |
| 7.90 | $\frac{1}{4}$ | 14.1 | 60 | 0.083 | $\frac{1}{4}$ | 10.5 | −0.03 | 52 | 0.098 | 0.235 | 21.0 | 66 | 0.099 |
| | $\frac{1}{2}$ | 1/35 | 27 | 0.313 | $\frac{1}{2}$ | 0.30 | −0.21 | 15 | 0.105 | | | | |
| 8.07 | $\frac{1}{4}$ | 3.69 | 54 | 0.137 | $\frac{1}{4}$ | 0.41 | −0.11 | 26 | 0.085 | 0.204 | 9.0 | 66 | 0.089 |
| | $\frac{1}{2}$ | 0.79 | 31 | 0.356 | $\frac{1}{2}$ | 0.076 | −0.19 | 17 | 0.086 | | | | |
| 8.30 | $\frac{1}{4}$ | 0.358 | 49 | 0.076 | $\frac{1}{4}$ | 0.125 | −0.04 | 41 | 0.063 | 0.193 | 0.79 | 57 | 0.065 |
| | $\frac{1}{2}$ | 0.193 | 35 | 0.205 | $\frac{1}{2}$ | 0.028 | −0.10 | 27 | 0.065 | | | | |
| 8.41 | $\frac{1}{4}$ | 0.136 | 45 | 0.089 | $\frac{1}{4}$ | 0.030 | −0.05 | 37 | 0.067 | 0.239 | 0.15 | 66 | 0.084 |
| | $\frac{1}{2}$ | 0.107 | 35 | 0.165 | $\frac{1}{2}$ | 0.011 | −0.10 | 27 | 0.071 | | | | |
| 8.48 | $\frac{1}{4}$ | 0.036 | 44 | 0.041 | $\frac{1}{4}$ | 0.0037 | −0.04 | 36 | 0.020 | 0.153 | 0.09 | 51 | 0.021 |
| | $\frac{1}{2}$ | 0.055 | 36 | 0.122 | $\frac{1}{2}$ | 0.0045 | −0.07 | 30 | .021 | | | | |

with $1/T$. One could increase $p$ and add a negative $s$ and still fit the $\delta(T)$ data in Fig. 4. Thus, although the data in Fig. 4 are consistent with $p \approx \frac{1}{4}$ and $s \sim 0$, the data cannot rule out a more complex two-parameter fit.

In Table II we give the results of least-squared fits for the seven most concentrated insulating samples for three types of fits, namely fit 1, $p = \frac{1}{4}$ or $\frac{1}{2}$, $s = 0$; fit 2, $p = \frac{1}{4}$ or $\frac{1}{2}$, $s$ is a variable; and fit 3, $p$ is variable and $s = 0$. Fit 1 shows that the percent error $\chi$ is clearly much smaller for $p = \frac{1}{4}$ than for $p = \frac{1}{2}$, even for the three samples closest to $n_c$. Fit 2 with $s$ now a variable still shows a very slightly better fit to $p = \frac{1}{4}$ but the $s(n)$ dependence is decidedly different for $p = \frac{1}{4}$ and $p = \frac{1}{2}$. For $p = \frac{1}{4}$ $s$ is small and there is no systematic trend with density while for $p = \frac{1}{2}$ $s(n)$ starts small near $n_c$ and becomes increasingly negative as the donor density is decreased to 7.57. None of the theoretical calculations of VRH conduction yield a negative $s$. Fit 3 essentially agrees with the data shown in Fig. 4 and yields almost as good a fit as fit 2 with $p = \frac{1}{4}$. The Mott-like fit with $p \sim \frac{1}{4}$ and $s \sim 0$ is the simplest two-parameter fit, however the $p = \frac{1}{2}$ fit with $-0.07 > s(n) > -0.34$ is almost as good a fit. A fit with $p = \frac{1}{2}$ and $s$ = const for all samples is not tenable. One should also note the different density dependences in fit 2 for the prefactor $A(n)$ for the $p = \frac{1}{2}$ and $p = \frac{1}{4}$ cases. As we shall see in the discussion there are physical reasons why the Mott fit with $p = \frac{1}{4}$ and $s \simeq 0$ is the most probable explanation for the data in Fig. 2.

Figure 5 shows the prefactor $\sigma_0(N_D)$ [$\sigma(T) = \sigma_0 e^{-(T_0/T)^p}$] obtained from fit 1 with $p = \frac{1}{4}$ and $s = 0$. $\sigma_0(N_D)$ approaches a constant $\sigma_0(n_c) \approx 43$ S/cm as $N_D \to N_c -$ and increases rapidly for $N_D < 8 \times 10^{18}/\text{cm}^3$.

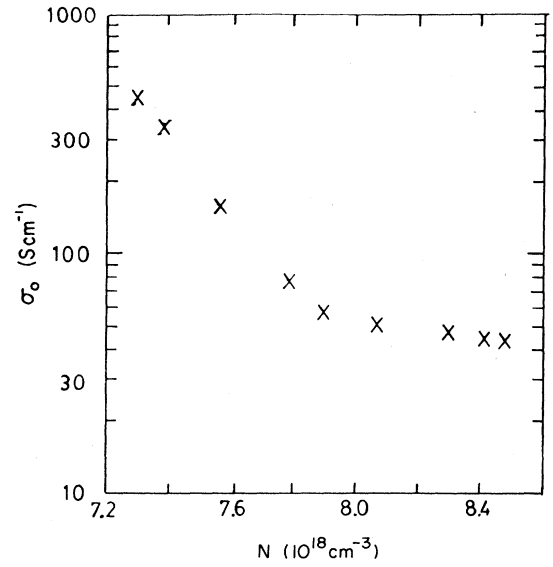


FIG. 5. $\sigma_0$ vs $N$ based on the Mott fit 1 in Table II: $p = \frac{1}{4}$ and $s = 0$. For $\tilde{N} < 7.5$ the Mott fit does not adequately describe the data. As $N \to N_c -$ ($\sim 8.6 \times 10^{19}$ cm$^{-3}$) $\sigma_0 \to 43 \pm 2$ s/cm.

The Mott prediction would predict $\sigma_0(N_D) \propto \bar{R}_{\text{hop}}^{\,2} \propto \xi^{1/2}$ which would predict a divergence of $\sigma_0$ as $N_D \to N_c -$. This is one more indication of the need for a revised theory of VRH conduction as $N_D \to N_c -$. Shklovskii and Efros have reviewed other calculations of $\sigma_0(N_D, T)$, however none of the theoretical results adequately describe the temperature and donor concentration dependences of $\sigma_0(N_D, T)$. We note that $\sigma_0(N_D \to N_c)$ is slightly larger than Mott's minimum metallic conductivity for Si:As.

The logarithmic derivative $\delta(T)$ for four samples is shown in Fig. 6 for $1 < T < 77$ K. While it is not obvious from Fig. 2, it is clear from Fig. 6 that there is an intermediate region between activated hopping ($p = 1$, $T > 40$ K) and VRH ($p = \frac{1}{4}$, $T < 8$ K) with a very different temperature dependence. It is straightforward to show that one cannot obtain the negative slope region in Fig. 6 solely with the sum of an activated hopping term (with temperature-dependent prefactor) and a VRH term [Eq. (1)]. The negative slope of $\delta(T)$ suggests a third contribution of the form $\delta\sigma(T) \propto \exp(aT^q)$. Hurd[69] has proposed a quantum-mechanical (QM) tunneling mechanism resulting from an oscillating barrier width that results from thermal vibrations that yields $q = 1$. Adding such a term definitely improves the fit for the 8.41 and 7.90 samples, but this term is much less important for the 7.57 and 7.39 samples. If one employs the Hurd mechanism one does not obtain a very reasonable estimate of the tunneling distance $R_{\text{tun}}$ for appropriate phonon vibration frequen-

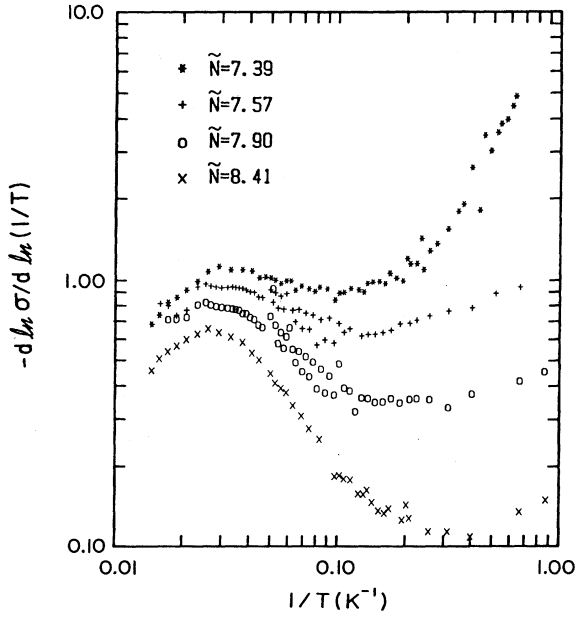


FIG. 6. $\delta(T)$ vs $1/T$ for four insulating Si:As samples for $1 < T < 77$ K. For $T > 30$ K the results suggest activated behavior ($p \sim 1$) while for $T < 5$ K the results imply Mott VRH for $\tilde{N} = 8.41$, $\tilde{N} = 7.90$, and $\tilde{N} = 7.57$, but close to activated behavior for $\tilde{N} = 7.39$.

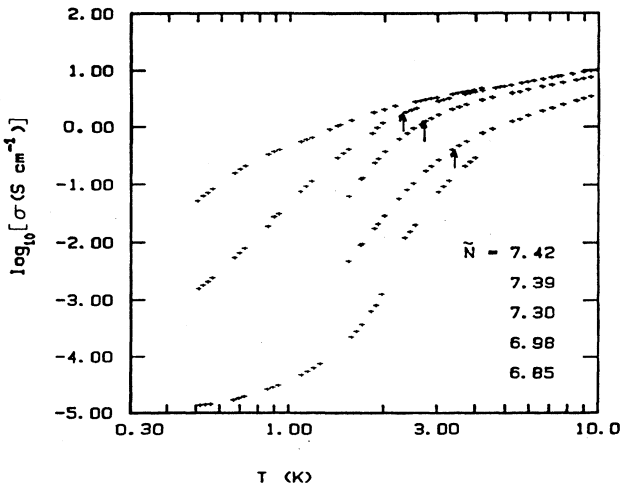


FIG. 7. $\log_{10}\sigma(T)$ vs $T$ for five more insulating Si:As samples with $1-N/N_c>0.135$. The three arrows show the crossover temperatures from Mott VRH to a new stronger temperature dependence for the 7.39, 7.30, and 6.98 samples. The approximate crossover temperatures are 2.2, 2.6, and 3.3 K, respectively.

cies. At present the negative slope region in Fig. 6 is not understood.

Figure 6 also indicates a slope $p\sim\frac{3}{2}$ for the 7.39 sample for $T<4$ K indicating a very strong deviation from Mott VRH. The other samples with $N\leq 7.42$ also show strong deviations from Mott's law for $T$ less than a characteristic temperature with $1<p<2$. The data for these more insulating samples have been replotted in Fig. 7 on a log-log $\sigma(T)$ versus $T$ plot. The arrows on three samples indicate the crossover between the Mott VRH dependence and the much steeper unexplained $T$ dependence at lower temperatures. The crossover temperatures are $T_g\sim 2.2$, 2.6, and 3.3 K for the 7.39, 7.30, and 6.98 samples, respectively. Although a greater temperature range is needed to accurately document this lower-$T$ dependence, one can make some qualitative observations. A simple power law ($\sigma\propto T^s$), which yields a constant $\delta(T)$, with $s\sim 5$, can explain some of the data over a limited temperature range for the 7.39, 7.30, and 6.98 samples, but is clearly inadequate for the 6.85 sample below 2 K. This latter sample shows a flattening for $\sigma<2\times10^{-4}$ ($\rho>5\times10^3$ Ω cm) which is unexplained, but might result from parallel path currents[70] that are not bulk current or from nonthermal equilibrium effects. However, based on the results in Figs. 6 and 7 below $T_g$ the data are not consistent with ES VRH conduction with $p=\frac{1}{2}$. This may be because the temperature is not yet low enough. Measurements to very much lower temperatures will be required to accurately document the temperature dependence well below the Mott VRH regime.

### B. Results for metallic samples

The metallic samples studied, including both bar and van der Pauw disc samples, are listed in Table III. All of the samples except the last three came from the original 5-cm-diam 35-cm-long ingot. The last three samples were obtained from individual wafers purchased separately from MA-COM. These last three exhibited very small temperature dependences between 1.4 and 4.2 K and were only studied in the $^4$He range and above.

Figure 8 shows $\sigma(T)$ versus $T^{1/2}$ results for the five bar samples listed in Table III. With the exception of the 8.67 sample all of the data are a very good fit to Eq. (5) without the third term (i.e., $B\sim 0$). However, the 8.67 sample shows a deviation (with $B$ negative) for $T>2$ K. The deviation from the third term for this sample is less than 7% of $\sigma(0,n)$ and only about 10% of the $m(n)T^{1/2}$ term at 4.2 K. The data are an excellent fit to the $m(n)T^{1/2}$ law with $m(n)$ changing sign very close to the 8.91 sample from positive to negative with increasing $n$ in agreement with the Si:P results. Figure 9 shows $\sigma(T)$ versus $T^{1/2}$ data for six of the van der Pauw samples listed in Table III. The lower-density samples (8.78, 9.14, 9.30, and 10.2) agree with the results obtained on the bar samples shown in Fig. 8. The three largest density samples (17.9, 20.9, and 32.8), for which bar samples were not studied, exhibit an extremely small temperature dependence between 4.2 and 1.4 K and yield very small values

TABLE III. Metallic Si:As samples. *b*, bar sample; *v*, van der Pauw (6 mm diam); and *v**, van der Pauw (6 mm diam) cloverleaf.

| Sample | $r_0$ | $N$ ($10^{18}$ cm$^{-3}$) | $\rho_{RT}$ (mΩ cm) | Measurements |
|---|---|---|---|---|
| C4-31(*b*) | 3.33 | 8.67 | 7.91 | $^3$He |
| C9-X-2(*v*) | 2.94 | 8.78 | 7.85 | $^3$He |
| C8-7(*b*) | 2.66 | 8.91 | 7.76 | $^3$He |
| C13-1(*b*) | 2.48 | 9.06 | 7.46 | $^3$He |
| D9-X-1(*v**) | 2.34 | 9.14 | 7.33 | $^3$He |
| D9-X-2(*v**) | 2.11 | 9.30 | 7.08 | $^3$He |
| E11-22(*b*) | 1.87 | 9.50 | 7.02 | $^3$He |
| F6-X-1(*v*) | 1.47 | 10.2 | 6.48 | $^3$He |
| F6-5(*b*) | 1.38 | 10.4 | 6.58 | $^3$He |
| B1-Y-1(*v*) | 0.64 | 17.9 | 4.22 | $^4$He |
| A1-Y-2(*v*) | 0.60 | 20.9 | 3.44 | $^4$He |
| C1-Y-1(*v*) | 0.58 | 32.8 | 2.30 | $^4$He |

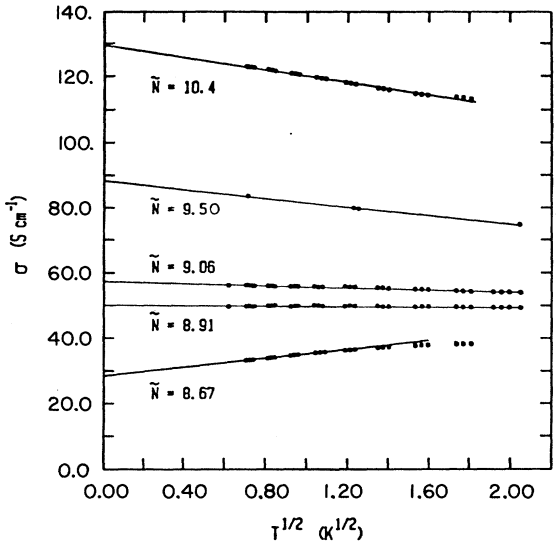


FIG. 8. $\sigma(T)$ vs $T^{1/2}$ for five Si:As bar samples for $0.5<T<4.2$ K. The $\tilde{N}=8.67$ shows a deviation from $T^{1/2}$ behavior for $T>2$ K resulting from the third term in Eq. (5).

of $m(n)$. All three of these samples show a decrease in $\sigma(T)$ with increasing $T$ that becomes more rapid for $20<T<300$ K. In the same temperature range the samples with $N<10.4\times10^{18}/\text{cm}^3$ exhibit an increase in $\sigma(T)$

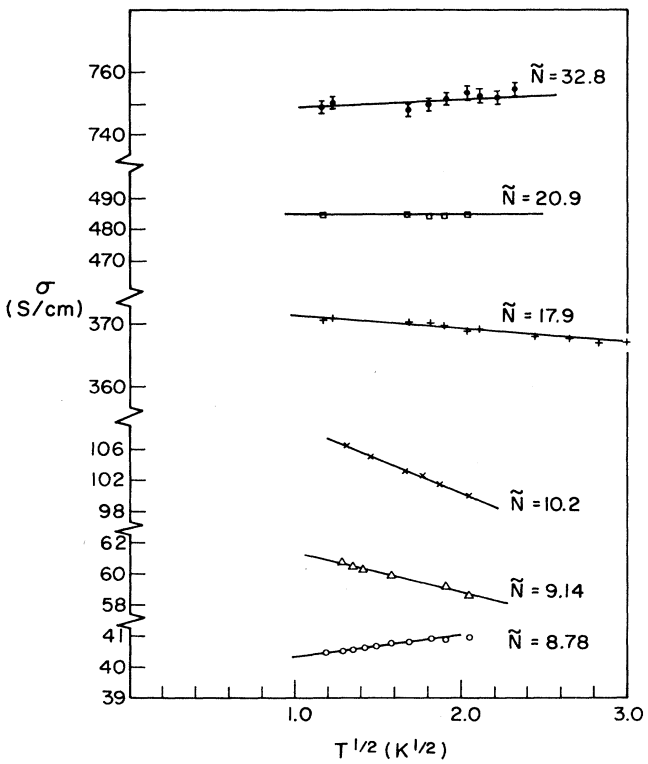


FIG. 9. $\sigma(T)$ vs $T^{1/2}$ for Si:As disk van der Pauw geometry samples.

with $T$ and the results are inferred from the values of $r_0=\sigma(\text{RT})/\sigma(4.2\text{ K})$ given in Table III. The results for the samples with $N<10.4$ for $T>10$ K are very similar to the more extensive higher-temperature Si:As results of Newman and Holcomb.[4]

The experimental results for the $m(n)$ coefficient of the $T^{1/2}$ term are shown in Fig. 10(a) versus $(n/n_c-1)^{1/2}$ for all the metallic Si:As samples (except $N=9.3\times10^{18}$ cm$^{-3}$) listed in Table III. The results show the same type of dependence on $(n/n_c-1)^{1/2}$ found for Si:P[3] with the difference that we find $m(n)$ changing sign a second time and becoming positive for $(n/n_c-1)^{1/2}>1.2$. The Si:P results only extended to $(n/n_c-1)^{1/2}\sim1$ but did show $m(n)$ headed toward zero. This second reversal in sign to positive values of $m(n)$ at larger values of $n/n_c$ has also been reported for uncompensated Ge:As by Ionov *et al.*[7] Since the Hartree correction [$\propto 2F$ in Eq. (6) or $3/2\tilde{F}$] becomes less important at high densities for $n/n_c\gg1$ one excepts $m(n)$ to become positive again for sufficiently high values of $n/n_c$. Using Eq. (8) and the Si:As $m(n)$ results in Fig. 10(a) we have calculated $A(F)$ for two different values of $\tau$, one for $l(n_c)=0.55d_c$ ($d_c=n_c^{-1/3}$ and $0.55d_c$ is the mean pair distance for a Poisson distribution) and secondly for $l(n_c)=1.01d_c$ corresponding $(k_f l)_{n_c}=\sqrt{3}$. The $A(F)$ values versus $(n/n_c-1)^{1/2}$ are shown in Fig. 10(b). $A(F)$ is negative for $0.2<(n/n_c-1)^{1/2}<1.2$ indicating the dominance of the Hartree interaction in this range. The maximum negative values for $A(F)$ are too large to be explained by the single-valley results [$A(F)=\frac{4}{3}-2F$, $A(F)=\frac{4}{3}-3\tilde{F}/2$, $F_{\max}=1$]. Bhatt and Lee[60] have explained the Si:P results with the moderate anisotropy expression [$A(F)=\frac{4}{3}-4F$] for negligible intervalley scattering. For the Si:As results in Fig. 10(b) $|A(F)|>\frac{8}{3}$ and the moderate anisotropy case will not explain the more negative $A(F)$ values for Si:As. The intervalley scattering should be more important for Si:As but Bhatt and Lee[60] find intervalley scattering reduces the importance of the Hartree term to values below that of the single-valley case. The second and more troubling problem is the very rapid variation of $A(F)$ with density $n$. For $x=(2k_F/K)^2\propto n^{1/3}$, $F(x)$ varies much too slowly to explain the rapid increase in $A(F)$ with $n$. Even with $x\propto n^2$ one cannot explain the rapid variation of $A(F)$. The fact that $A(F)>\frac{4}{3}$ for the sample at $(n/n_c-1)^{1/2}=1.68$ suggests there might be another contribution to the temperature dependence in this density range, perhaps from inelastic scattering. We note that Si:P and Ge:Sb also show a faster variation of $A(F(n))$ than predicted by the *e-e* interaction theory of $m(n)$.

As $n\rightarrow n_{c+}$, $m(n)$ increases dramatically for Si:P and Si:As shows the same tendency although $n\sim1.01n_c$ is the closest $n_c$ has been approached for Si:As. Paalanen *et al.*[3] have explained this as a breakdown of the Thomas-Fermi screening with $F\rightarrow0$, $A(F)\rightarrow\frac{4}{3}$, and $m(n)\propto[D(n)^{-1/2}]\propto(n/n_c-1)^{-\mu/2}$. The rapid increase in $m(n)$ for $n/n_c-1<0.04$ for both Si:P and also for Si:As appear to support the dominance of the *e-e* interaction contribution to $m(n)$ as $n\rightarrow n_{c+}$ and argues against

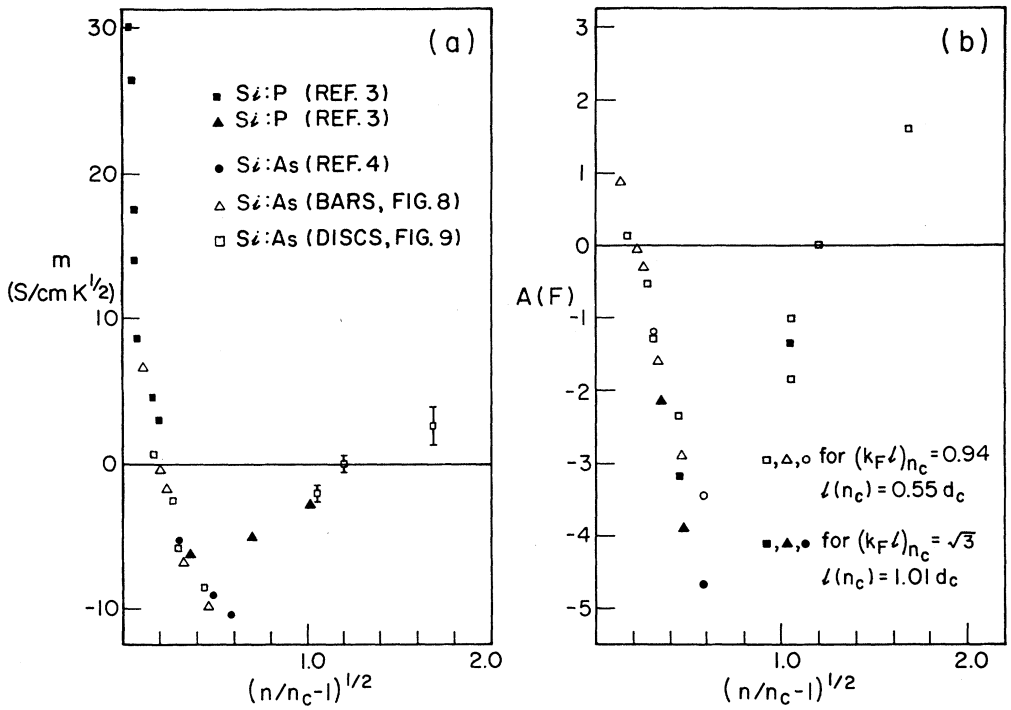


FIG. 10. (a) $m(n)$ vs $(n/n_c-1)^{1/2}$ for Si:P and Si:As samples. The large positive increase in $m(n)$ for $(n/n_c-1)^{1/2}<0.12$ is predicted by Eqs. (6) and (8). (b) $A(F)$ vs $(n/n_c-1)^{1/2}$ obtained from Eq. (8) for two different elastic scattering times corresponding to $(k_F l)_{n_c}=0.94$ and $(k_F l)_{n_c}=\sqrt{3}$.

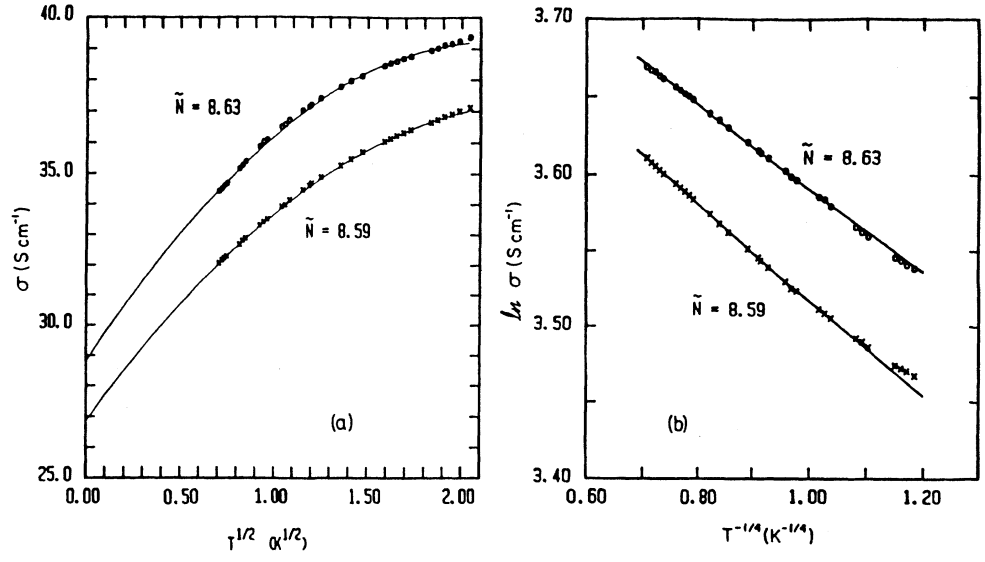


FIG. 11. (a) $\sigma(T)$ vs $T^{1/2}$ for two bar Si:As samples within 1% of $n_c$. (b) $\ln\sigma(T)$ vs $T^{-1/4}$ for the same two samples. Despite the apparently better fit to the Mott law the temperature is not low enough to accurately determine $\sigma(N, T\rightarrow 0)$.

an important role for inelastic scattering (localization correction) very close to $n_c$.

### C. Results for two samples very close to $n_c$

Two samples with $N=8.59$ and 8.63 were measured between 4.2 and 0.5 K and it was not possible to conclude whether these samples were insulating or metallic without measurements to much lower temperatures. In Fig. 11(a) $\sigma(T)$ is shown versus $T^{1/2}$ and there is considerable curvature which would require a negative $B$ in Eq. (5). Figure 11(b) shows the same data plotted as $\ln\sigma(T)$ versus $T^{-1/4}$ giving a relatively good fit with $T_0 \sim 10$ mK for the $N=8.59$ sample and $T_0 \sim 5$ mK for the $N=8.63$ sample. Even though the Mott law is the better fit one should not conclude these samples are insulating since Fig. 11(a) could be viewed as suggesting a finite conductivity for these samples as $T \to 0$. Both of these samples are within 1% of $n_c$, a region where sample doping inhomogeneities can play a significant role in determining $\sigma(n,T)$. For these two samples $\delta\sigma(n,T)$ is comparable to $\sigma(n,T=0)$ or even larger and only measurements to 50 mK or lower would reliably determine $\sigma(n,T=0)$. It seems likely that the third term in Eq. (5) is important at much lower temperatures for these samples than for Si:As samples with $N \geq 8.67\times10^{18}/\text{cm}^3$.

## V. DISCUSSION OF RESULTS

### A. Metallic results

The values of $\sigma(n,0)$ obtained from extrapolating the $\sigma(T)$ versus $T^{1/2}$ data to $T=0$ in Figs. 8 and 9 are plotted versus $n/n_c-1$ for $n_c=8.55\times10^{18}/\text{cm}^3$ in Fig. 12. From the relation $\sigma(n)=\sigma_0(n/n_c-1)^{\mu}$ one obtains for the data $\sigma_0=376\pm5$ S/cm and $\mu=0.60\pm0.05$. These results compare with the earlier results of Newman and Holcomb,[4] who obtained $\sigma_0=381$ and $\mu=0.64^{+0.20}_{-0.15}$ and the results on the bar samples only[71] of $\sigma_0=356$ and $\mu=0.61\pm0.05$. It is not at all clear that the scaling result with a constant $\sigma_0$ and a single exponent $\mu$ should be valid over the entire range $0.015 < n/n_c < 3.8$ as suggested in Fig. 12. Although the $\mu$ obtained is slightly larger than $\mu=0.5$ obtained for uncompensated Si:P[3] it seems unlikely that $\mu$ could approach the localization prediction of one found for compensated systems[13,72,73] and for the amorphous alloys.[8-12] Although it is still possible that data much closer to $n_c$ $[(n/n_c-1)\ll0.01]$ would yield a much different exponent than $\mu=0.60\pm0.05$ this seems unlikely since the data in Figs. 8 and 9 are well into the $T^{1/2}$ regime and there is no other known temperature dependence of $\sigma(n>n_c,T)$ at sufficiently low temperatures.

The value of $n_c=8.55\times10^{18}/\text{cm}^3$ used for the fit in Fig. 12 was not obtained solely from the metallic sample data discussed above. Results on the divergence of the dielectric constant $\epsilon'(n)$,[18] the scaling of the Mott characteristic temperature[37] $T_0(N)$ with $1-N/N_c$ to be discussed more below, and the minimum in the electron-spin-resonance (ESR) linewidth[74] suggest a value of $n_c$ such that $8.55 < n_c < 8.60$, but the smallest ESR linewidth yet observed was obtained for $n \sim 8.57$. In addition a better fit of the scaling of $T_0(N)$ with $1-N/N_c$ was obtained for 8.6 than for 8.55 (see Fig. 13). Thus this evidence actually suggests $8.55 < n_c < 8.60$. A slight increase in $n_c$ above the 8.55 value used in Fig. 12 has the effect of decreasing $\mu$ slightly. To improve upon the above results by getting closer to $n_c$ will require DR data and the uniaxial stress tuning of $n_c$ as accomplished for Si:P by Paalanen *et al.*[3]

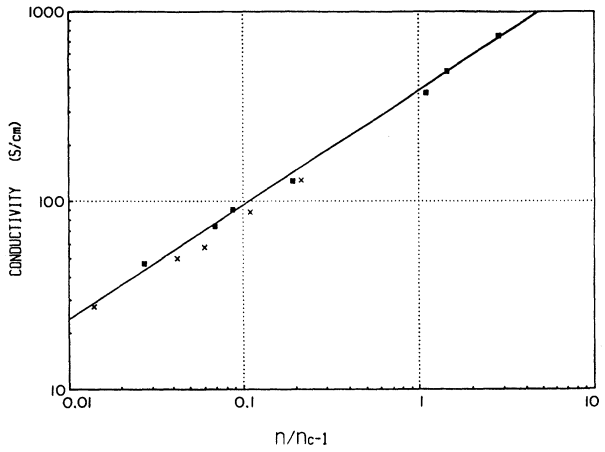


FIG. 12. $\sigma(n,T=0)$ vs $n/n_c-1$ for Si:As based on $N_c=8.55\times10^{18}$ cm$^{-3}$. Squares are from van der Pauw samples and crosses from bar samples. The straight line is the fit $\sigma(n,T=0)=\sigma_0(n/n_c-1)^{\mu}$, $\mu=0.60$ and $\sigma_0=376$ S/cm.

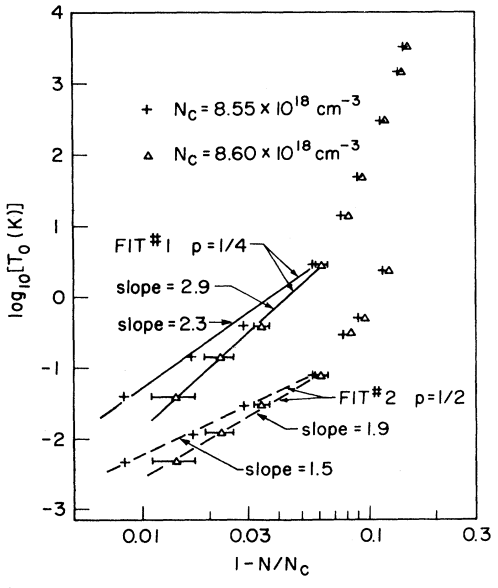


FIG. 13. $\log_{10}T_0(N)$ vs $1-N/N_c$. From the $T_0 \propto (1-N/N_c)^{3\nu}$ relationship one obtains $\nu \sim 0.97$ for $N_c=8.60\times10^{18}$ cm$^{-3}$ and $\nu \sim 0.77$ for $N_c=8.55\times10^{18}$ cm$^{-3}$ for $1-N/N_c < 0.07$.

### B. Critical behavior of VRH data

The density dependence of the characteristic temperature $T_0(N)$ [or $T_0'(N)$] is determined from the slopes in Fig. 2 (corresponding to fit 1 in Table II for $p=\frac{1}{4}$ or from fit 2 in Table II for $p=\frac{1}{2}$ for the ES VRH case). Note that for $N<7.57$ the data show deviations from Mott VRH behavior for $T<2$ K. The scaling behavior of the Mott temperature $T_0(N)$ versus $1-N/N_c$ for two different values of $N_c$, 8.55 and 8.60, is shown in Fig. 13. This figure shows the Mott $T_0$ decreasing by 5 orders of magnitude as $N$ increases from 7.30 to 8.48, a surprisingly rapid decrease for a 14% increase in donor density and a far more rapid decrease of $T_0$ with $N$ than that found for Si:P from 400-MHz data by Hess *et al.*[54] The behavior of $T_0(N)$ versus $1-N/N_c$ shows two distinct regions with the crossover between these regions at $1-N/N_c \approx 0.07$. For $1-N/N_c<0.07$ the slope depends sensitively on the chosen value of $N_c$, and the better fit to a straight line is obtained for $N_c=8.60$. Since $T_0 \propto [N(E_F)\xi^3]^{-1} \propto (1-N/N_c)^{3\nu}$ the slope for $1-N/N_c<0.01$ yields $\nu=0.77$ for $N_c=8.55$ and $\nu=0.97$ for $N_c=8.60$. The resulting $\nu$ is closer to 1 than to the conductivity exponent of $\mu \approx 0.5$–$0.6$ and is in better agreement with the localization prediction of $\nu \simeq 1$. This was explained by Shafarman and Castner[37] as resulting from a smooth DOS $N(E)$ in the vicinity of $E_F$ because of the thermal filling of the Coulomb gap. It was also noted that empty sites near $E_F$ from the hopping could act analagously to the compensated case in which $\nu=\mu \sim 1$ was observed. However, from the discussion in Sec. II one expects Mott VRH for $\bar{E}_{\rm hop}(T) \gg \Delta_{\rm Cg}$ and ES VRH for $E'_{\rm hop}(T) \ll \Delta_{\rm Cg}$. This translates into $T > T_0/1170$ for Mott VRH and $T \ll T_0/2000 = T_0'/25$ for ES VRH. Figure 13 shows that $T/T_0 > 10^{-3}$ until $N$ is decreased to 7.39 or less. For $N \le 7.39$ deviations from Mott VRH are observed for $T<2$ K. Hence, using estimates based on Eq. (4) for $1-N/N_c<0.12$ one finds $\bar{E}_{\rm hop}(T) > \Delta_{\rm Cg}$ and one thereby expects Mott VRH *independent* of whether the Coulomb gap is thermally filled in or not. Thus, if one closely approaches $N_c-$, $T_0$ will become so small that it is not possible to reach small enough temperatures to see deviations from Mott VRH. The discussion here based on Eq. (4) is limited to the very weakly compensated case since it is well known that $\Delta_{\rm Cg}$ increases rapidly with increasing compensation $K$. If, on the other hand, one plots the $T_0'$ values from Table II for fit 2 ($p=\frac{1}{2}$) versus $1-N/N_c$ one obtains $\nu=0.63$ for $N_c=8.6$ for $1-N/N_c<0.07$ which is in reasonable agreement with the conductivity exponent $\mu=0.60\pm0.05$. From fit 2 $T_0'$ varies from 2.3 K for $N=7.57$ to 4.5 mK for $N=8.48$. But in order to observe ES VRH one requires $T/T_0' < \frac{1}{25}$ and this condition is not satisfied. Furthermore, if one uses $kT_0'=2.8/4\pi N(E_F)\xi^3$ to obtain an $N(E_F)$ one obtains values of $N(E_F)$ that are more than 2 orders of magnitude larger than the free electron value of $N(E_F)$. Thus, although fit 2 with $p=\frac{1}{2}$ is almost as good as the Mott fit with $p=\frac{1}{4}$, the parameters obtained are unrealistic.

In Table IV various parameters characterizing Mott VRH and activated conduction ($E_{\rm act}$ determined from Fig. 2 for $T>40$ K) are shown as $N$ approaches $N_c-$. Table IV shows that $\bar{R}_{\rm hop}(T=1\ {\rm K})$ becomes smaller than the localization length $\xi(N)$ for $N \ge 7.79$. As $N \to N_c-$, $\xi(N)$ diverges and $\bar{R}_{\rm hop}(T=1\ {\rm K})$ also increases, but only as $\xi^{1/4}$. The hopping length $\bar{R}_{\rm hop}(T)$ reaches a minimum near $1-N/N_c \sim 0.06$. The regime where one has $\bar{R}_{\rm hop}(T) < \xi(N)$ in the critical regime is an unfamiliar regime for hopping theory and will require new theoretical consideration. The fact that $E_{\rm act}$, which is normally associated with the $\epsilon_2$ process, is always much larger than $\bar{E}_{\rm hop}(T)$ at sufficiently low temperatures explains why one must observe VRH conduction at low enough temperatures. For $\epsilon_2=E_c-E_F$, where $E_c$ is the mobility edge energy, we conclude that in the critical regime $\bar{E}_{\rm hop}(T)$ is very much smaller than the energy required to excite an electron to extended states. This also explains why it is possible to observe VRH behavior of the Hall coefficient[66] as $N \to N_c-$ for temperatures such that $kT \ll E_{\rm act}$ since the contribution to $R_H$ from carriers in extended states is negligible since $n \propto \exp(-E_{\rm act}/kT)$.

TABLE IV. Hopping parameters.

| $N$ ($10^{18}$ cm$^3$) | $1-N/N_c$ | $T_0(k)$[a] | $\bar{R}_{\rm hop}(T=1\ {\rm K})$ (Å) | $\xi(N)$[b] (Å) | $\bar{E}_{\rm hop}(T=1\ {\rm K})$ (meV) | $E_{\rm act}$ (meV) |
|---|---|---|---|---|---|---|
| 6.98 | 0.188 | 17 500 | 377 | 82 | 0.24 | 5.9 |
| 7.30 | 0.155 | 2530 | 289 | 102 | 0.15 | 4.9 |
| 7.39 | 0.141 | 1470 | 270 | 109 | 0.13 | 3.9 |
| 7.57 | 0.120 | 296 | 214 | 129 | 0.087 | 3.8 |
| 7.79 | 0.094 | 45.4 | 171 | 164 | 0.055 | |
| 7.90 | 0.081 | 12.9 | 146 | 191 | 0.040 | 3.4 |
| 8.07 | 0.062 | 2.74 | 128 | 249 | 0.027 | |
| 8.30 | 0.035 | 0.37 | 138 | 441 | 0.016 | |
| 8.41 | 0.022 | 0.14 | 171 | 702 | 0.013 | 2.6 |
| 8.48 | 0.014 | 0.036 | 192 | 1104 | 0.009 | |

[a]Based on fit 1 in Table II for $p=\frac{1}{4}$ and $s=0$.

[b]Based on $\xi_0=15.45$ Å and $\nu=1$.

The results discussed above have documented Mott VRH much closer to the MIT than other studies. We note that Finlayson *et al.*[49] have observed Mott VRH with $p=0.25$, and $T_0=0.82$ K for a $3.3\times10^{16}/\text{cm}^3$ insulating In:P sample measured between 4.2 K and 50 mK. For a more dilute In:P sample with $9.6\times10^{15}/\text{cm}^3$ carriers (from $R_H$) which is further from the MIT, Finlayson and Mason[47] observed ES VRH with $p=\frac{1}{2}$ and $T_0'=10.4$ K. Both of these samples have significant compensation. These In:P results are consistent with discussion above that as $N$ approaches $N_c-$ and $T_0$ becomes small one will observe Mott VRH for $T>T_0/1170$. Recently, Biskupski *et al.*[50] have demonstrated that a $4.8\times10^{16}/\text{cm}^3$ ($K=0.6$) InP sample exhibited Mott VRH for the magnetic field $4<B<6$ T and ES VRH for $B>6$ T. These results are consistent with the magnetic field tuning of $n_c$ and with the above discussion. In this case $1-N/N_c$ increases with increasing magnetic field.

Other data for $n$-type Si showing the scaling of $T_0$ with $1-N/N_c$ are those of Hess *et al.*[54] for Si:P obtained at 400 MHz in the temperature range 20–120 mK. These authors plotted $(4\pi\chi')^{-1/1.15}$ versus $T_0$ assuming $T_0\propto N_c/N-1$ which differs from the relation $T_0\propto(1-N/N_c)^{3\nu}$ expected for the Mott temperature. We have replotted their four $T_0$ values versus $1-N/N_c$ and obtained $\nu\sim0.5$ consistent with the conductivity exponent $\mu\approx0.5$ found for Si:P. We emphasize that the two larger $T_0$ values ($T_0>10$ K) obtained by Hess *et al.*[54] for $1-N/N_c>0.1$ are orders of magnitude smaller than $T_0$ values obtained by Sasaki[39] for Si:P with dc measurements in the same range of $1-N/N_c$. This is a surprising difference, but we emphasize values of $T_0$ for Si:As (Ref. 37) and Si:P (Ref. 39) obtained from dc measurements agree rather well for $1-N/N_c>0.15$. The explanation for this discrepancy is not clear because even for the lower-temperature data of Hess *et al.* one still has $T>T_0/1170$ leading one therefore to expect Mott VRH conduction to dominate for a temperature less than a fixed temperature (8 K for Si:As). It is clearly important to extend the microwave measurements[75] for Si:As to the dilution refrigerator range to compare with the Si:P results of Hess *et al.*[54] The 400-MHz Si:P data are to date the only data reporting Mott VRH conduction in a regime where the photon and phonon energies are comparable in magnitude. It is not at all clear that one should obtain results that are identical to the dc case when $\hbar\omega$ is comparable to $kT$. We are unaware of any theory of VRH for this case.

The extension of Miller-Abrahams phonon-assisted hopping between a pair of donor sites to the critical regime has been outlined in the Appendix. It is concluded that only the prefactor $\sigma_0(T)$ [$\sigma_0(T)\propto(1/T)^s$] is altered when $\bar{q}\bar{R}_{\text{hop}}<1$ and a term neglected by MA is kept. The extended pair theory still does not fit the experimental result of $s\approx0$. The MA approach neglects the change in longer-range Coulomb interactions or the possibility of multielectron correlated hopping.[52,53] There is currently a need to consider the theory of multielectron correlated VRH conduction in the regime where $\xi\gg d_c\sim n_c^{-1/3}$ and $\xi>\bar{R}_{\text{hop}}(T)$.

## VI. CONCLUSIONS

The data on metallic Si:As samples show $T^{1/2}$ behavior at sufficiently low temperatures and the density-dependent coefficient $m(n)$ is in good qualitative agreement with the predictions of electron-electron interaction theory. $m(n)$ shows a second crossover to small positive values for $n/n_c>2.4$ as expected from the diminishing importance of the Hartree term $F$ for larger densities. The critical behavior $\sigma(n,T=0)$ has extended the results of Newman and Holcomb[4] to lower temperatures and over a wider range of $n/n_c$. The results for the conductivity exponent $\mu$ have been narrowed to $\mu=0.60\pm0.05$. This apparently places Si:As in the same category as Si:P[3] with $\mu$ close to $\frac{1}{2}$. The Si:As data have only reached $n/n_c-1\approx0.014$ compared to 0.001 for Si:P with stress tuning. Furthermore, the Si:As data have not been extended to the DR range for the zero-field metallic results. Despite these deficiencies it seems highly unlikely that uncompensated Si:As could be moved to the $\mu\approx1$ category. As for Si:P, the Si:As metallic results seem to disagree with the weak localization prediction.

On the other hand, the VRH data of barely insulating Si:As samples yield a result, if interpreted as Mott VRH, that is in better agreement with localization theory results for the scaling of the Mott characteristic temperature $T_0$ with reduced density. A single parameter (exponential) fit to our VRH data for $N/N_c>0.8$ gives a strong preference to the Mott exponent $p=\frac{1}{4}$ over the Shklovskii-Efros exponent of $p=\frac{1}{2}$. With a two-parameter fit incorporating a temperature-dependent prefactor $(1/T)^s$ in addition to the exponential term it is much more difficult to choose between the two cases. However, the Mott case with $s\approx0$ is the simpler explanation compared with the ES case, which requires a negative density-dependent $s$ that increases in magnitude with decreasing donor density. We have given criteria to determine in which temperature ranges one should observe either the Mott case or the ES case. In the critical regime the Mott $T_0$, the ES $T_0'$, and the Coulomb gap width temperature $T_{Cg}$ all scale to zero as $N\rightarrow N_c-$. Using these criteria and the magnitudes of the various parameters [$T_0$, $T_0'$, and the DOS $N(E_F)$] we conclude that our VRH data in the critical regime are best fit by the Mott explanation.

The pair hopping probability of Miller and Abrahams needs to be modified in the critical regime where $\bar{q}\bar{R}_{\text{hop}}<1$ and where the term omitted by Miller and Abrahams can become the dominant term. The applicability of the pair approximation in the critical regime has been seriously questioned. Multiple-electron correlated hopping is a very probable candidate to dominate in the critical regime. However, whichever theoretical model is correct it must produce Mott VRH conduction in the critical regime. The interpretation of the parameters such as $\bar{R}_{\text{hop}}(T)$, $T_0$, etc. may change with a more complex theory but the idea of a temperature-dependent conductivity of the Mott form as $N\rightarrow N_c-$ seems well established in the temperature range of our measurements when one is close enough to $N_c$. If one could reach very

much lower temperatures for small $1-N/N_c$ one might expect to see the crossover to ES VRH conduction which would confirm the dominance of *e-e* interactions for $N<N_c$ as $T\rightarrow 0$.

## ACKNOWLEDGMENTS

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## APPENDIX: PAIR HOPPING IN THE CRITICAL REGIME

The pair hopping transition probability calculated by Miller and Abrahams (MA) was calculated in the dilute limit using a dilational deformation approach for the electron-phonon interaction. MA calculated the electron-phonon matrix in the dilute limit where the approximation $qR_{ab}\gg 1$ is valid where $q$ is the longitudinal phonon wave number and $R_{ab}$ is the distance between the sites $a$ and $b$ making up the pair. In the critical regime where $\bar{R}_{\rm hop}$ is given by Eq. (3a) and $\bar{q}=\bar{E}_{\rm hop}/\hbar c_s$ with $\bar{E}_{\rm hop}$ given by Eq. (3b) one obtains

$$\bar{q}\bar{R}_{\rm hop}\simeq 0.1(k/\hbar c_s)\xi T^{1/2}T_0^{1/2} \propto T^{1/2}/\{[N(E_F)]\xi\}^{1/2} . \tag{A1}$$

For sufficiently low temperatures for a given $T_0$ and $\xi$ one gets into a regime with $\bar{q}\bar{R}_{\rm hop}<1$. For *n*-type Si, $0.1k/\hbar c_s\sim 1.5\times 10^4$ cm K. Using the results from Table IV for the $8.07\times 10^{18}$ cm$^{-3}$ sample one finds $\bar{q}\bar{R}_{\rm hop}\sim 0.062T^{1/2}$, while for the $8.48\times 10^{18}$ cm$^{-3}$ sample $\bar{q}\bar{R}_{\rm hop}\sim 0.031T^{1/2}$. For the more insulating $7.39\times 10^{18}$ cm$^{-3}$ sample with a much larger $T_0$ one obtains $\bar{q}\bar{R}_{\rm hop}=0.63T^{1/2}$. These results show that for samples sufficiently close to $N_c$ it is possible to achieve $\bar{q}\bar{R}_{\rm hop}\ll 1$ in the temperature regime of our measurements.

The matrix element (Eq. II-11 from MA) takes the form

$$\langle H'\rangle = iE_1(\hbar q n_q/2\rho_0 V c_S)^{1/2}\left\langle \frac{W}{\Delta}\left[\int \phi_b^* e^{i\mathbf{q}\cdot\mathbf{r}}\phi_b\, d\tau - \int \phi_a^* e^{i\mathbf{q}\cdot\mathbf{r}}\phi_a\, d\tau\right] + \left[1-\left[\frac{W}{\Delta}\right]^2\right]\int \phi_a^* e^{i\mathbf{q}\cdot\mathbf{r}}\phi_b\, d\tau\right\rangle , \tag{A2}$$

where the last overlaplike term is the term omitted by MA, which is certainly valid in the dilute limit where $qR_{ab}\gg 1$. In Eq. (A2) the prefactor quantities are defined in MA. Even in the critical regime we have used the approximation that $W/\Delta\ll 1$ where $W$ is the resonance energy and $\Delta$ is the site energy difference $E_a-E_b$. Analysis of the MA expression for $W$ shows that it decreases rapidly as $N\rightarrow N_c-$ and one obtains $W/\Delta\ll 1$ even in the critical regime. The first term in Eq. (A2) yields a contribution to $|\langle H'\rangle|^2$ given from MA by

$$|\langle H'\rangle|^2\propto (W/\Delta)^2[1-\cos(\mathbf{q}\cdot\mathbf{R}_{ab})]/[1+(q\xi/2)^2]^4 . \tag{A3}$$

For $qR_{ab}<1$ and $(W/\Delta)\ll 1$ this contribution is very much smaller than the contribution from the third term which is proportional $|S_{ab}(q)|^2$ where $S_{ab}=\int \phi_a^* e^{i\mathbf{q}\cdot\mathbf{r}}\phi_b\, d\tau$. For $q=0$,

$$S_{ab}(q=0)=[1+R_{ab}/\xi+\tfrac{1}{3}(R_{ab}/\xi)^2]e^{-R_{ab}/\xi} ,$$

which is the standard overlap integral. From the oscillatory nature of $S_{ab}(q)$ it is clear that $|S_{ab}(q)|<S_{ab}(0)$. From some analysis one finds for $qR_{ab}\ll 1$

$$|S_{ab}(q)|^2=S_{ab}^2(0)\left[1-\left[\frac{q^2R_{ab}^2}{4}\right]f\left[\frac{R_{ab}}{\xi}\right]+\frac{(q_x^2+q_y^2)R_{ab}^2}{40}+O(q^4)\right] . \tag{A4}$$

The function $f(R_{ab}/\xi)$ varies slowly in the range 1–5 for $R_{ab}/\xi$ near unity, but does increase rapidly for $R_{ab}/\xi\ll 1$. Nevertheless, $|S_{ab}(q)|^2$ and the transition probability for phonon absorption from just the third term Eq. (A2) is

$$U_{b\rightarrow a}\leq \frac{E_1^2S_{ab}^2(0)\Delta^3 n_\Delta}{2\pi\rho\hbar^4c_s^5} . \tag{A5}$$

Using Eq. (A4) in a Mott-like derivation of the Mott VRH law leads to the standard result in Eq. (1) but with a prefactor dependence, which will be of the form

$$\sigma_0\left[\frac{1}{T}\right]^s\propto\left\langle\left[1+\frac{R_{ab}}{\xi}+\frac{1}{3}\left[\frac{R_{ab}}{\xi}\right]^2\right]^2\Delta^3\right\rangle . \tag{A6}$$

Using $\langle\Delta\rangle=\bar{E}_{\rm hop}\propto T^{3/4}T_0^{1/4}$ and

$$\langle R_{ab}\rangle/\xi=\bar{R}_{\rm hop}/\xi=\bar{R}_{\rm hop}/\xi=0.4(T_0/T)^{1/4}$$

one observes that Eq. (A6) will yield a value of $s$ in the range $-\frac{9}{4}<s<-\frac{5}{4}$ depending on the relative magnitude of $\bar{R}_{\rm hop}/\xi$. This value for $s$ is much more negative than the $s$ found by Mott ($s=\frac{1}{2}$) and others and is in poor agreement with the experimental results suggesting $s\sim 0$. However, Eq. (A6) does predict a density-dependent $s(N)$. If Eq. (A6) is applied to the ES VRH case where $\bar{E}_{\rm hop}\propto(TT_0')^{1/2}$ and $\bar{R}'_{\rm hop}/\xi\propto(T_0'/T)^{1/2}$ then one obtains $-\frac{3}{2}<s<\frac{1}{2}$ with $s$ decreasing to more negative values as $\bar{R}'_{\rm hop}/\xi$ increases as is expected with decreasing

donor density. This result is actually in qualitative agreement with fit 2 in Table II. However, as has been discussed in Sec. V, the ES fit yields parameters that do not seem to be consistent with other results or with the criterion established in Sec. II from the magnitudes of $T$, $T_0$, $T_0'$, and $T_{Cg}$. One also notices the density dependence of $\sigma_0(N)$ predicted from Eq. (A6) is an increase in $\sigma_0(N)$ with decreasing density $N$. This is in qualitative agreement with the data for $\sigma_0(N)$ in Fig. 5 but the experimental increase is very much slower than predicted by Eq. (A6), which yields a dependence faster than $T_0^{3/4}$ for Mott VRH.

In view of the serious questions concerning the applicability of the pair model as $N \rightarrow N_c -$ it is unwise to take the above discussion too seriously. Such important questions as correlated many electron hopping and the possible fractal nature of the localized electron wave functions as $N \rightarrow N_c -$ all may significantly affect the nature of VRH as $N \rightarrow N_c -$. It has been well recognized that different models of VRH lead to the same exponential $[\ln\sigma(T) \propto -(T_0/T)^p]$ behavior, but lead to different behaviors of the preexponential factors. We suggest that a consideration of correlated many electron hopping and also of the effects of the fractal behavior of the wave function will primarily affect the preexponential factors of the VRH expressions for $\sigma(T)$.

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