Localization in near-monolayer films

Hylton White and Gerd Bergmann
Department of Physics, University of Southern California, Los Angeles, California 90089-0484
(Received 19 June 1989)

Disordered Cu films in the thickness range between 1 and 20 atomic layers and with resistances per square between 40 kΩ and 100 Ω have been investigated. The Cu films were condensed onto a substrate of freshly evaporated, insulating amorphous Sb. The temperature dependence of the conductance and the Hall effect were measured. The results are rather surprising. The conductance shows the universal temperature dependence of $G = G_0 + A(e^2/2\pi^2\hbar)\ln(T)$ in all the films for film resistance up to 50 kΩ, where A is close to unity. This temperature dependence is predicted by the theory of Coulomb interaction in weakly disordered two-dimensional films, which can be extrapolated to high-resistance films by scaling arguments. Although the experimental data for high film resistances can also be described by the theory of hopping conductivity in two dimensions, we doubt that hopping is the underlying mechanism. The Hall coefficient shows a pronounced temperature dependence and even changes sign for a film resistance of 10 kΩ, contrary to the Coulomb theory for weakly disordered metal films.

I. INTRODUCTION

Two-dimensional films showing conductance corrections arising from weak localization are found experimentally in the regime where the resistance per square R is of the order of a few hundred ohms per square (see, for example, the review articles[1–4]). These corrections arise from quantum interference (generally called weak localization) and depend on the long inelastic lifetime of the conduction electrons. This leads to a logarithmic temperature dependence of the conductance (in two dimensions).[5–8] A further contribution to the conductance in this regime arises from the Coulomb anomaly[9–11] (see, for example, the review articles[12,4]). This is due to the fact that the dynamic Coulomb potential in weakly disordered systems is not fully screened by the conduction electrons at finite frequencies.[10,7,11] The Coulomb anomaly also causes a logarithmic temperature dependence of the conductance. The corrections to the conductance from weak localization and the Coulomb anomaly are nevertheless small as compared to the Boltzmann conductance, being of the order of a few percent of the total.

As the resistance per square R is increased, so the electron wave functions are expected to become exponentially localized, and (at $T=0$ K) to exhibit strong Anderson localization.[13] A characteristic resistance for this transition is $\pi\hbar/e^2$ ($\simeq 12.9$ kΩ). (Factors of the order of π or 2π are not well defined for this characteristic resistance.) At $T>0$, in the very-high-resistance regime, the conductance is expected to proceed by a hopping mechanism such as the well-known Miller-Abrahams[14] variable range-hopping process.

Miller-Abrahams hopping does not take the Coulomb interaction between electrons into account. A possible consequence of including the Coulomb interaction is that the density of states (DOS) becomes depleted at the Fermi surface (the so-called "Coulomb gap,"[15] see, for example, Boettger and Brykskin[16]). Such a reduction of the density of states at the Fermi energy has been experimentally observed;[17] for a review see Pollak,[18] and the references therein.

The difficulties of linking or reconciling the weak and strong localization regimes have thus far been almost insurmountable. As pointed out by Abrahams *et al.*,[5] and also Wegner,[19] scaling arguments suggest that two-dimensional systems should always indicate localized behavior (at $T=0$ K). The situation is less clear in the case of strong spin-orbit scattering[20] or large magnetic field, however. In three dimensions a sharp transition between metallic and insulating behavior is expected.

In this paper, we present some results on highly resistive near-monolayer films which cast some light on this question. Metallic films with such a high resistive have been studied experimentally by several groups.[21,17,22–24] There are important differences in the physical properties of relatively thick percolating films (for example thicker than 10 monolayers) as compared to homogeneous thin films. Here we have attempted to prepare homogeneous films by using amorphous Sb as a substrate for the metal films. This permits us to evaporate conducting films of disordered Cu, Au, Bi, and Fe films. These films are rather homogeneous because they become already conducting at a thickness of less than one monolayer. (We investigate in this paper only films with a thickness above one monolayer.) It is possible to control the evaporation so that films with $R > 50$ kΩ can be produced. These highly resistive films would conventionally be expected to lie in the strongly localized regime, and thus it becomes possible to study the crossover between strong and weak localization by increasing the film thickness. In this paper we will discuss the results of Sb-Cu ($1 < d_{Cu} < 23$ atomic layers, where d_{Cu} is the thickness of the Cu layer), as well as the results for Sb-Fe ($1 < d_{Fe} < 2$ atomic layers). In Sec.

II we give a short review of some of the theoretical results in the weakly and strongly localized regime in two dimensions. In Sec. III the experimental results are described and in Sec. IV we discuss these results and try to link them to some of the unexpected properties of such high-resistance homogeneous films.

II. THEORETICAL RESULTS

According to the scaling theory of Abrahams *et al.*[5] a two-dimensional electron system with finite disorder should always be localized at zero temperature. This is independent of the degree of disorder (but applies for vanishing spin-orbit scattering and zero magnetic field). The scaling theory suggests that the localization length is

$$\xi_0 = l \exp(\pi k_F l/2) \tag{2.1}$$

where l is the elastic mean free path and k_F is the Fermi wave number (see, for example, Ref. 3). In finite magnetic field or nonvanishing spin-orbit scattering the character of the localization is not well known.

In experiments, i.e., at finite temperature, the "weak localization" is completely masked by the finite inelastic lifetime of the conduction electrons. This restricts the coherent of the electronic wave function to a distance equal to the inelastic scattering length $\xi_i = (D\tau_i)^{1/2}$, where D is the diffusion constant (in the semiclassical picture) and τ_i is the inelastic lifetime. Experimentally one observes only quantum interference effects in this regime and hence the name "weak localization" is somewhat misleading.

At high resistance (when R is of the order of $\pi\hbar/e^2$) one expects strong localization (similar to three dimensions) with an exponentially decaying wave function at large distances. At sufficiently high disorder the localization length ξ_0 should become smaller than the inelastic scattering length ξ_i. This regime is known as the strongly localized regime. We review briefly the theoretical results in these two regimes.

A. Weakly localized regime

In the regime the conductance of a disordered two-dimensional electron system has two essential corrections, namely the contribution due to weak localization and the Coulomb anomaly. The conductance correction ΔG_{WL} arising from weak localization in two dimensions is given by

$$\Delta G_{\mathrm{WL}} = BG_{00}(\tau_i/\tau_0) = pB \ln(T) \tag{2.2}$$

where $G_{00} = e^2/2\pi^2\hbar$ is a universal conductance and B is a constant.[25] In the case of weak (strong) spin-orbit scattering we have $B = 1$ (or $-\frac{1}{2}$). The inelastic lifetime is assumed to vary as $1/\tau_i \simeq T^p$.

The Coulomb anomaly in the weak localization regime gives rise to a similar temperature dependence in two dimensions.[11,10,26] Here

$$\Delta G_{e\text{-}e} = AG_{00} \ln(T) \tag{2.3}$$

where A is a constant of order unity. In general, weak localization and the Coulomb anomaly coexist at $B = 0$ T. It is, however, well known that the weak localization correction is suppressed upon application of a magnetic field.

The Hall constant provides another criterion for determining whether a film lies in the weakly or strongly localized regime. The Hall coefficient remains unchanged in the presence of weak localization but a correction does arise from the Coulomb anomaly.[7,27] In the weakly localized regime, the correction to the Hall constant C_H arising from the Coulomb anomaly is correlated with the conductance G according to

$$\Delta C_H/C_H = -\zeta \Delta G_{e\text{-}e}/G \tag{2.4}$$

where $\zeta = 2$ for the Coulomb anomaly.

B. Strongly localized regime

In the strongly localized regime the electronic wave functions are localized and generally one assumes that the eigenfunction decays exponentially at large distances from its center. In the absence of electron-phonon and electron-electron interaction such an electronic system should have a zero conductance at finite temperature. This arises from the orthogonality of the different localized electron wave functions, which means that the matrix for a transition between different localization states is zero. In real systems, however, the electrons interact with each other and with the phonons of the system. This yields a reduction of the inelastic lifetime [and, if the inelastic diffusion length L_i (with $L_i^2 = D\tau_i$) becomes shorter than the localization length ξ_0, then we are in the weakly localized regime].

1. Hopping description

In the strongly localized regime the inelastic lifetime is long enough so that the conduction electrons experience confinement due to the disorder. For a transition from one localized state to another, the electrons need a transition matrix element and an energy reservoir since different localized states have, in general, different energies. Both matrix element and energy reservoir can be supplied by phonons and other electrons, allowing the electron to hop. Since the energy change of the hopping electron is of the order of $k_B T$, the electron cannot simply hop to the next closest localized state but has to choose a localized state with $\Delta\epsilon \simeq k_B T$. At lower temperature this requires longer hops, giving rise to the term variable-range hopping.

Generally the variable-range hopping is described in terms of the model of a doped semiconductor. Each donor atom binds an electron in a hydrogenlike wave function but with much larger radius [the electron wave function decays as $\exp(-\alpha r)$]. The wave functions of different donors overlap. One assumes the existence of a hopping matrix element J between different atomic wave functions. In the quantitative calculation, phonons are only needed to supply the energy difference between the different eigenstates. This concept appears to be somewhat questionable, as one can diagonalize the Hamiltonian and would get either delocalized states or localized

states (at the Fermi energy). In the first case the resistance remains finite at zero temperature. In the second case the hopping matrix element between different localized states is zero as long as we ignore the interaction with other electrons or phonons. Therefore the "effective matrix element" between two localized states is temperature dependent. Nevertheless, we review the results of variable-range hopping in the atomic wave-function model (see, for example, Boettger and Brykskin[16]). The electronic transport is governed by the probability that an electron hops from localized site R_m to localized site $R_{m'}$. Electron transport is then determined essentially by the hopping probability $W_{mm'}$ between sites. This is given by the product of the tunneling and phonon terms, i.e.,

$$W_{mm'} \sim \exp(-2\alpha R_{mm'} - \epsilon_{mm'}/k_B T) \tag{2.5}$$

where α is the inverse decay length of the exponentially localized (atomic) wave function which decays as $\exp(-\alpha r)$, $R_{mm'}$ is the absolute value of the distance between sites, and $\epsilon_{mm'}$ is the absolute energy difference between sites. The temperature dependence of the conductance is given by

$$G = G_0 \exp[-(T_0/T)^{(1/3)}] . \tag{2.6}$$

T_0 is a constant given by

$$T_0 = c^3 \alpha^2 / k_B N(E_F) \tag{2.7}$$

where c is a constant of order unity and $N(E_F)$ is the density of states at the Fermi level.

Variable-range hopping gives rise (in the absence of the Coulomb interaction) to a Hall coefficient given by[16]

$$C_H = (k_B T/neJ)(\alpha S^{(-1/3)})^{-2.1} \exp(0.85\alpha S^{(-1/3)}) . \tag{2.8}$$

Here J corresponds to the average energy overlap between states m and m', and S is the site density.

In the strongly localized regime the Coulomb interaction has even a stronger effect than in the weakly localized regime. It introduces a so-called "Coulomb gap" in the density of states[28,15,18,29] (see, for example, the review article[30]).

As pointed out by Pollak[29] the one-electron hopping probability is much reduced by the Coulomb gap. In Eq. (2.5) for the hopping probability one has to replace $\epsilon_{mm'}$ by the Coulomb gap E_g. Nevertheless, according to Pollak, a multiple hop can have a nonvanishing transition rate in the present of electron-electron interactions.[29] This is in direct contrast to the case where the Coulomb interaction is absent, and multihops occur with a vanishing transition rate. If multihopping is taken into account (due to the Coulomb interaction), then it can be shown that the conductance is given by an expression similar to Eq. (2.6).[16]

It is interesting to note that the Hall coefficient arises from cascading two-electron hops, with single-electron hops giving only a vanishingly small contribution.

2. *Scaling theory for strong electron-electron interaction*

The interacting electron gas was first studied to the first-order perturbation, with respect to both disorder and the Coulomb interaction [yielding the $\ln(T)$ dependence of the conductance]. It is, however, well known that the Coulomb interaction is not a small perturbation but of the order of one. There is a large body of literature which discusses the Coulomb interaction to all orders (see, for example Refs. 31–33), where the theoretical difficulties are extreme. The extrapolation to high disorder is then attempted by scaling theory. In the presence of long-range Coulomb interaction, scaling arguments[34] (see also Ref. 3) suggest in two dimensions at finite temperature that the conductance is given by

$$G = G_0 + \frac{e^2}{2\pi^2\hbar} \ln(k_B T\tau/\hbar) . \tag{2.9}$$

This result has been obtained in the limit of either strong magnetic or spin-orbit scattering and in the presence of a magnetic field to suppress the crossed diagrams.

III. EXPERIMENTAL RESULTS

The films were prepared by quench condensation of Sb, Cu, and Fe onto a quartz substrate held at 6.5 K in a vacuum of 10^{-11} Torr. The experimental apparatus is described elsewhere.[35] A computer-controlled electronic shutter is opened at the start of the evaporation, and then closed when the resistance of the film reaches the desired value (at 6.5 K). After the evaporation, the films are annealed to 35 K for several minutes. The temperature and magnetic field dependence, as well as the Hall resistance, are then measured.

In the first experiment, 8.4 atomic layers of Sb were evaporated onto the quartz substrate. This film was annealed to 40 K and found to be insulating ($R > 50$ MΩ). (It was found that the Sb underlayers remained insulating even if annealed to room temperature.) A Cu film was then evaporated onto the Sb with R (6.5 K)=41 kΩ. The thickness of this Cu film was measured to be 1.24 atomic layers. The film first became continuous at approximately 0.9 atomic layer. Many successive Cu layers were evaporated onto this layer to give resistances of 24.6, 15.8, 5.11, 1.98, and 1.12 kΩ, and then 489, 194, and 96 Ω. The final total thickness of the Cu was 22.5 atomic layers. For the purposes of analyses, it is best to begin with the 96-Ω film (which lies in the weakly localized regime) and then to proceed to successively higher resistances. The 96-Ω film shows behavior characteristic of the weak localization regime, as a function of both temperature and magnetic field. In particular, the resistance shows the typical $\ln(T)$ Coulomb anomaly at $B = 7$ T [experimentally $\Delta G = 1.09 \ln(T)$—see Fig. 1], and the magnetoresistance data can be described by the theory of Hikami *et al.*[8] As the resistance was increased, it was found that the $\ln(T)$ Coulomb anomaly remained present even up to $R = 15.8$ kΩ. Above this resistance, slight deviations from the $\ln(T)$ law are observed (see Fig. 1). Nevertheless, it is interesting to note that the constant A [in Eq. (2.3)] remains almost the same ($A \simeq 1$) up to 41 kΩ. The

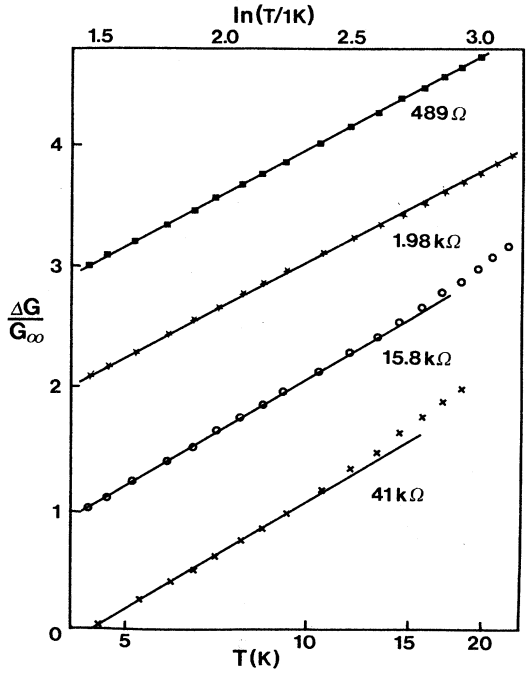

FIG. 1. Conductance change as a function of $\ln(T)$ for some of the films. The conductance scale has been shifted to permit films ranging from 500 Ω to 41 kΩ to be shown on a single plot.

coefficients A are collected in Table I together with the thicknesses and resistances of the films.

Hall effect measurements proved to be quite difficult in the high- (>10 kΩ) resistance range because the Hall angle is only about $(2-5)\times10^{-6}$. Small inhomogeneities in the thinnest films cause much larger deformations in the equipotential lines. Hence it was not possible to measure the Hall effect for all the films. The Hall constant of the thinner films was also an order of magnitude less than the bulk value for Cu.

For the Hall effect measurements, we evaporated 9.9 atomic layers of Sb and then 1.59 atomic layers of Cu to give $R=4.90$ kΩ. The temperature dependence of the Hall coefficient and resistance were measured, and then more Cu evaporated to give $d_{Cu}=2.61$ atomic layers and $R=1.96$ kΩ. The same measurements were then done again. As before, both the 1.96- and 4.90-kΩ films apparently show the $\ln(T)$ Coulomb conductance anomaly at $B=7$ T. It was also found (see Fig. 2) that $C_H \propto \ln(T)$ for both films.

To investigate the higher-resistance regime, 10.3 atomic layers of Sb was evaporated to serve as the substrate, and then 1.13 atomic layers of Cu with $R=10.4$ kΩ was evaporated and measured. As can be seen from Fig. 2, the absolute value of C_H decreases with increasing resistance, making accurate Hall constant measurements even more difficult above 10 kΩ. It is also interesting to note that the Hall coefficient for this film changes sign at 9.4 K [see Fig. 2(b)]. The temperature dependence of C_H shows deviations from the logarithmic temperature dependence expected from the Coulomb anomaly.

The microstructure of the film must be addressed at this stage. Unfortunately, it is not possible to study quench condensed films at room temperature (in an elec-

TABLE I. Thickness, resistance per square, Hall coefficient, and Coulomb slope for several investigated Cu films on an Sb substrate.

Thickness (at. layers)	R(6.5 K) (Ω)	Hall coeff. (10^{-11} m^{3}C^{-1})	A
1.24	41 000		1.38
1.50	15 000		1.06
3.87	1980		1.10
8.51	489		1.12
1.59	4900	−1.1	
2.61	1960	−1.3	
1.13	10 400	+0.65	

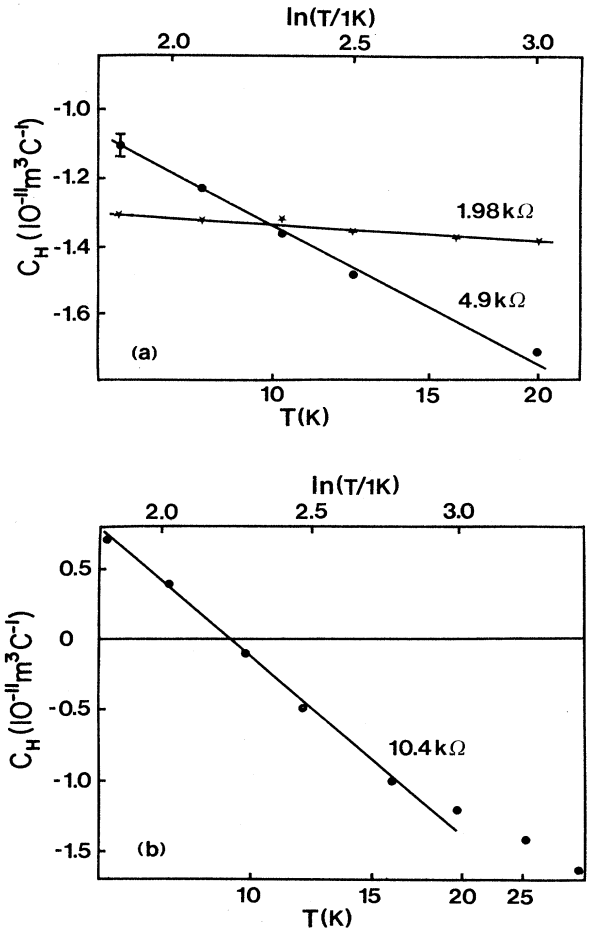

FIG. 2. (a) Hall coefficient as a function of $\ln(T)$ for a 1.96- and a 4.9-kΩ film. The curve is merely to guide the eye. (b) Hall coefficient vs $\ln(T)$ for the 10.4-kΩ film.

tron microscope, for example) as major irreversible structural changes take place upon warming. There are at least three possibilities for the structure: (i) A smooth continuous metallic film on an insulating substrate, (ii) a several monolayer thick alloy of Cu-Sb, or (iii) a percolating island structure with the Sb serving as a poorly (but nevertheless present) conducting bridge.

The anomalous Hall effect (AHE) (see, for example, for a review Ref. 36) can be used to distinguish between cases (i) and (ii), and case (iii). To do this we investigated the magnetic nature of ultrathin Fe films evaporated onto a 10.6-atomic-layer Sb substrate. The Hall resistance R_{xy} of a 4.57-kΩ film ($d_{Fe}=1.08$ atomic layers), and a 2.0-kΩ film ($d_{Fe}=2.15$ atomic layers) is shown as a function of applied field and temperature in Fig. 3. If the film deposits with the initial formation of islands, the islands would be expected to show superparamagnetic behavior,[36] and in particular the initial slope of R_{xy} as a function of B would then vary as $1/T$. This is not observed for either film, with the initial slope being approximately constant with varying temperature. This behavior is more characteristic of ferromagnetism, suggesting that the film is continuous. It is still possible however that the film is deposited in the form of large "flat" islands, with a demagnetization coefficient similar to that of a complete film.

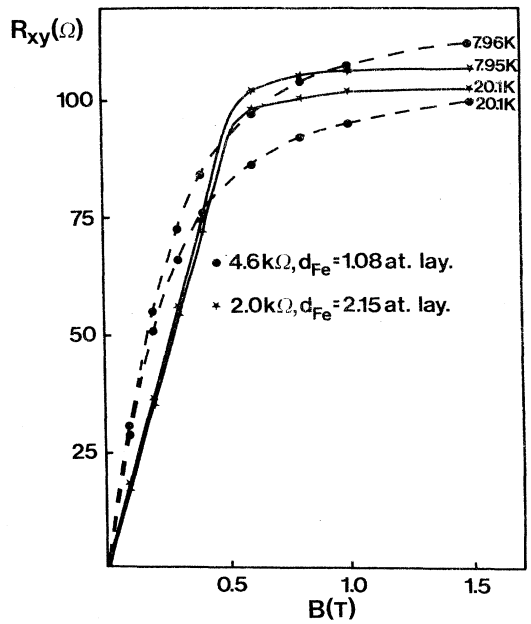

FIG. 3. The Hall resistance as a function of applied magnetic field for the thin Fe films. The * represents a 2.15-atomic-layer film. The initial slope of the curve is proportional to the magnetic susceptibility and is independent of temperature. The ● represents a 1.08-atomic-layer film. Here the curves are more rounded, but the initial slopes are still apparently independent of temperature.

IV. DISCUSSION

A. Model of the electronic structure of the film

From the change of the conductance during the evaporation of the metal on the Sb substrate, from the change of the normal Hall effect and the inelastic lifetime which we measured in a different experiment (to be published) we suggest the following model.

The Cu transforms the first layer of the amorphous Sb into a metal with five conduction electrons per Sb atom. These Sb electrons participate in the conduction process. The deeper-lying layers of Sb remain essentially insulating. Although it is difficult to show that this is the only possible model for this system, it appears to be consistent with the experimental findings.

B. The conductance

The most striking result of our measurements is the fact that the conductance shows $\ln(T)$ dependence for films with resistances varying from 100 to 50 000 Ω. The prefactor of the $\ln(T)$ term is essentially universal $e^2/(2\pi^2\hbar)$ (see Table I). Only in the weakly localized regime ($R \ll \pi\hbar/e^2 \simeq 12.9$ kΩ) this result has been theoretically derived for two-dimensional systems. Higher-resistance films are expected to lie in the strongly localized regime. It is thus necessary to compare the experimental data with Eq. (2.3) for variable-range hopping. To check this, the resistance data for a Cu film (on Sb) with $R=60$ kΩ ($d_{Cu}=0.83$ atomic layer), and a Cu film with $R=4.9$ kΩ ($d_{Cu}=1.59$ atomic layers) have been plotted in the form $\ln(G)$ versus $(1/T)^{(1/3)}$ in Fig. 4. The surprising result is that these plots also yield good approximations to a straight line; obviously the data are consistent with variable-range hopping. The 4.9-kΩ film does, however, yield a poorer straight-line fit than the 60-kΩ film. T_0 can be determined from Eq. (2.6): In the case of the 4.9-kΩ film we find $T_0=0.056$ K, while for the 60-kΩ film we have $T_0=95$ K. This is to be contrasted with the values of T_0 found in impurity band conduction (see for example Boettger and Brykskin[16]), where values of order 10^8 K are found. The exponential decay length $\xi_0=1/\alpha$ of the localized site wave functions can also be calculated. We find values of 39 and 3.3 nm for α for the 4.9- and 60-kΩ films, respectively.

Let us assume that the $\ln(T)$ dependence of the conductance for $R_0 \ll R_{00}$ is due to the Coulomb anomaly. Going to higher resistance, i.e., lower conductance, does not change the temperature dependence of the conductance. These data can also be well described at low conductance by variable-range hopping. These two mechanisms have, however, nothing in common. It appears to be extremely unlikely that our observed universal temperature dependence connects these two independent mechanisms.

At present, we prefer to regard the Coulomb interaction as the dominant mechanism determining the temperature dependence of the conductance (after we have suppressed weak localization by a magnetic field of 7 T). Scaling theory in the presence of long-range Coulomb in-

teraction at finite temperature yields the observed universal $\ln(T)$ dependence of the conductance [Eq. (2.9)]. This result has been obtained in the limit of either strong magnetic or spin-orbit scattering and in the presence of a magnetic field to suppress the crossed diagrams. Both conditions are fulfilled in our experiments: Firstly the logarithmic temperature dependence was obtained in the presence of a magnetic field of 7 T perpendicular to the film, and secondly the magnetoresistance was positive up to the highest temperature. The spin-orbit scattering was found to be strongly enhanced over that of Cu on quartz. This is due to the heavier underlying Sb nuclei. Therefore our measurements are in agreement with the scaling arguments.

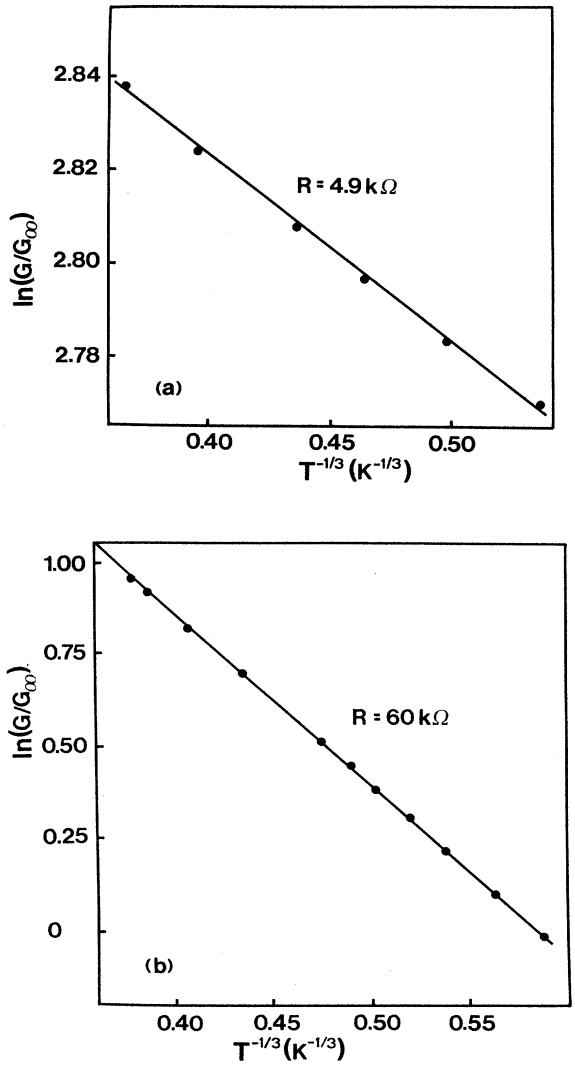

FIG. 4. Logarithmic conductance vs $(1/T)^{(1/3)}$. The value of T_0 [in Eq. (2.7)] is determined from the slope. (a) $R=4.9$ kΩ and (b) $R=60$ kΩ.

Unfortunately scaling arguments tell us little about the physics involved. Why is the temperature dependence of the conductance universal with increasing resistance? What happens when the resistance approaches the universal value R_{00}?

The Coulomb anomaly is (similar to weak localization) a noninstant (nonlocal in time) effect. This means that when we shift the Fermi surface by κ at a time $t=0$ (for example, by a pulse of an electrical field), the Fermi surface does not directly relax to the unshifted position within the elastic relaxation time τ, but "interference effects" cause a displacement of the Fermi surface even long after the relaxation time τ. In both weak localization and the Coulomb anomaly, this displacement is proportional to $1/t$, where t is the time after the pulse, and is given by

$$\kappa(t)=-(1/2\pi E_F t)\kappa \ . \tag{4.1}$$

In weak localization this counter current lasts from τ until τ_i. In the case of the Coulomb interaction, the smearing of the electron phase by the thermal broadening yields a cutoff of the counter current after a time $\tau_T=\hbar/(k_B T)$ (apart from a possible factor of π or 2π in the denominator). In perturbation calculations one assumes a displacement of the Fermi surface by $\kappa=eE\tau/\hbar$ due to the electrical field E, and then calculates the total counter current by integrating its contribution from τ to τ_T (or τ_i in the case of weak localization).

There are several reasons why this result should not extend to much-higher-resistance films.

(1) The calculation is not self-consistent.

(2) The small number of electron states in the diffusion area $\pi D\tau_T$ do not yield thermal smearing.

We will discuss both in the following.

1. *A self-consistent treatment of the Coulomb anomaly*

A consistent calculation would include the contribution of the counter current into the displacement of the Fermi surface κ. Then the condition for κ is the following:

$$\frac{d\kappa}{dt}=\frac{eE}{\hbar}-\frac{\kappa}{\tau}-\frac{1}{2\pi E_F}\int_{t-\tau_T}^{t-\tau}dt'\frac{\kappa(t')}{t'} \ . \tag{4.2}$$

In the stationary case this yields for κ

$$\hbar\kappa=\frac{eE\tau}{1-1/2\pi E_F\ln(k_B T\tau/\hbar)} \ . \tag{4.3}$$

As a consequence we find the $\ln(T)$ anomaly in the resistance instead of the conductance:

$$R=R_0-R_0^2 G_{00}\ln(k_B T\tau/\hbar) \tag{4.4}$$

where R_0 is the Boltzmann resistance $R_0=m(ne^2\tau)$. If the correction is small then this result is equivalent to Eq. (2.3). However, for large resistances the two equations differ strongly and we do not expect a $\ln(T)$ correction for the conductance. This result applies for weak localization too; here it means that one includes the echo of the

echo, etc. in the calculation. We would like to emphasize that this calculation does not take other higher-order terms of the Coulomb interaction into account, but only the Fock diagram consistently.

2. *Lack of electrons in highly resistive monolayers*

In the following we refer in part to a physical interpretation of the Coulomb anomaly which was given by one of us in a former paper.[37] The conduction electron diffuses after the action of the electric field in real space, experiences a Coulomb interaction, and contributes after the time τ_T to the electrical current. The diffusion time is limited to τ_T because of the thermal smearing of the conduction electrons. During the time τ_T the area of diffusion is about $\pi D\tau_T$. The above consideration breaks down when the number of conduction electrons in the energy range k_BT in the area $\pi D\tau_T$ is of the order of one or less because then there are no states over which the energy could be smeared. The density of states in the area $\pi D\tau_T$ is given by $n\pi D\tau_T/E_F$ (n is the two-dimensional density of electrons) and therefore the energy between these states is

$$\Delta E = mk_BT/(n\pi\tau\hbar) . \tag{4.5}$$

We are interested in the case when there is only 1 state per energy k_BT in this area, i.e., $\Delta E = k_BT$. This yields for τ the value

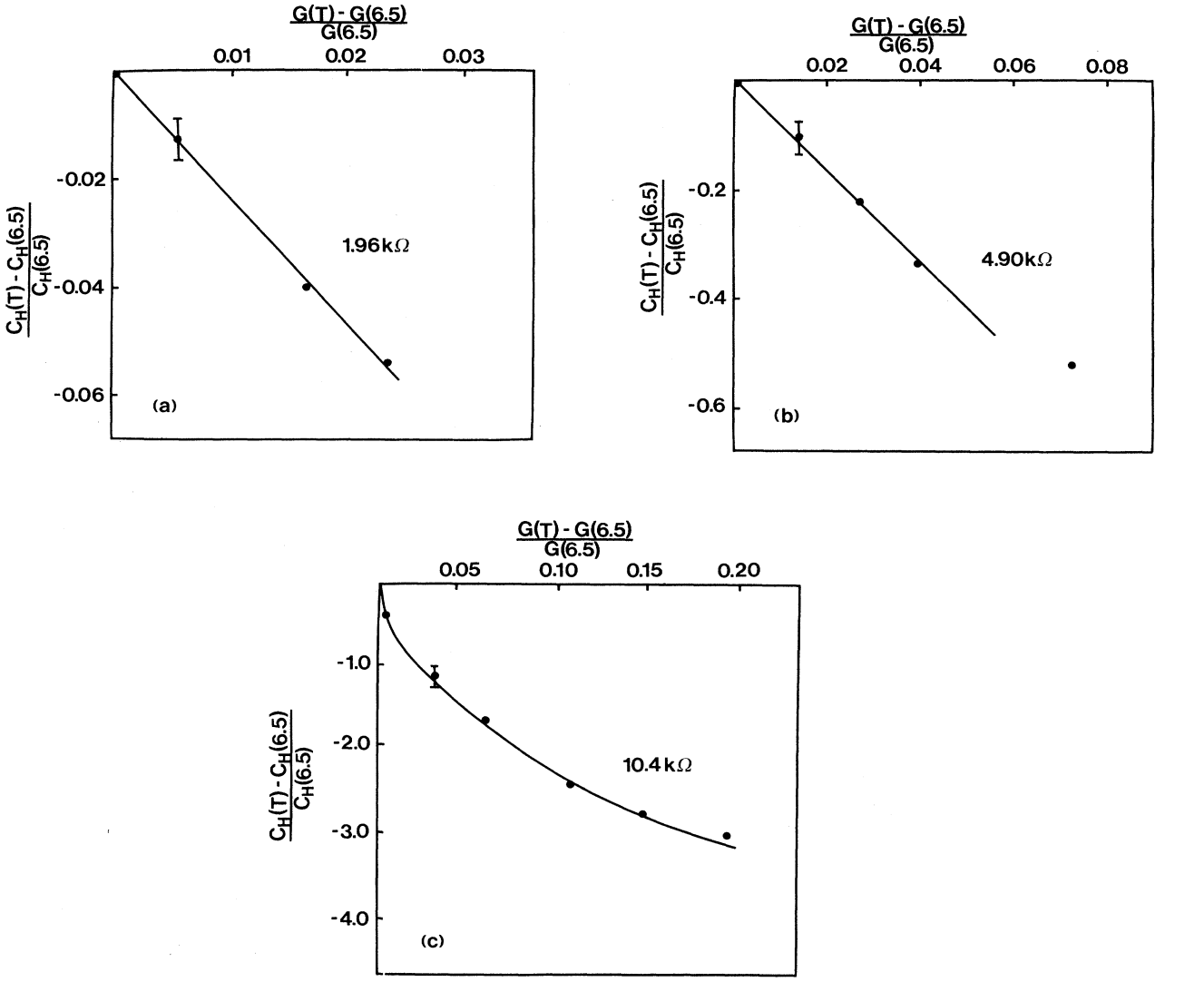

FIG. 5. $\Delta C_H(T)/C_H$ vs $\Delta L(T)/L$ for some of the films. (a) 1.96 kΩ, (b) 4.90 kΩ, and (c) 10.4 kΩ. The respective slopes are 2.3, 7.9, and $\simeq 10$.

$$\tau = m/(\pi\hbar n) . \quad (4.6)$$

This corresponds to a resistance of the film of

$$R_{00} = m/(ne^2\tau) = \pi\hbar/e^2 . \quad (4.7)$$

This is the universal resistance (besides possible factors of π or 2π). It is the same resistance that one obtains from Thouless' argument.[38] Here one splits the film into small areas and calculates the energy splitting and the energy broadening due to a diffusive escape. If the broadening becomes smaller than the energy splitting, then the electrons become localized. The consideration yields the value R_{00} for the characteristic resistance.

When there is only one conduction electron in the energy range $k_B T$ and the diffusion area $\pi D\tau_T$, the "thermal coherence length" should become longer than $(D\tau_T)^{1/2}$ and thus the effect of the Coulomb anomaly should be enhanced.

C. The Hall effect

We found experimentally that even a film with 5 kΩ resistance shows a $\ln(T)$ temperature dependence of the Hall constant. This temperature dependence is in agreement with the theory for the Coulomb interaction. In addition, the theory predicts (in the weakly localized regime) that the relative change of the Hall constant is twice the change of the resistance [see Eq. (2.4)]. To compare directly with Eq. (2.4), Fig. 5 shows a plot of $[C_H(T) - C_H(6.5\ \text{K})/C_H\ (6.5\ \text{K})$ versus $[G(T) - G(6.5\ \text{K})]/G(6.5\ \text{K})$. It can be seen that this plot yields (within the accuracy of the data) a straight line for the two films with a resistance of 1.96 and 4.9 kΩ. The proportionality constant $\zeta = 2.3$ for the 1.96-kΩ film, and $\zeta = 7.9$ for the 4.90-kΩ film. This demonstrates that the experimental data cannot be described by the perturbation result for the Coulomb interaction for films above 2 kΩ

According to Eq. (2.2) variable-range hopping yields a linear temperature dependence of the Hall constant. Such a linear temperature dependence is not found experimentally. This should exclude variable-range hopping (not including the Coulomb interaction) as a reasonable description of the transport properties of these films. To the best of our knowledge there are no results for the Hall coefficient derived from scaling theory.

V. CONCLUSIONS

Highly disordered electron systems present a difficult theoretical challenge. The large disorder causes localization which is in itself very demanding. Together with the strong Coulomb interaction, the theoretical problem becomes extremely complicated. We have investigated experimentally the transport properties of two-dimensional highly disordered films in the thickness range down to a monolayer. The experimental results for the temperature dependence of the conductance are rather simple and essentially described by a universal law $\Delta G(T) = [e^2/(2\pi^2\hbar)] \ln(k_B T\tau/\hbar)$. This appears to confirm scaling arguments. For resistances larger than $R_{00} = \pi\hbar/e^2 \simeq 12.9$ kΩ, the experimental data can also be described by electron hopping. But it is doubtful that hopping is the physical mechanism of conductance. For example, the Hall effect does not support the simple hopping model. Our experiment raises several questions. In particular, the problem has to be addressed whether there is any hopping conductance at all in these high-resistance films or whether the Coulomb interaction dominates the mechanism of conductance.

Finally we want to point out that experimentally it is very desirable to extend the measurements to lower temperatures. It is obvious that the $\ln(T)$ dependence of the conductance has to break down when the correction equals the unperturbed term G_0, as for lower temperatures the conductance would become negative. Such experiments are in preparation.

ACKNOWLEDGMENTS

This research was supported by NSF Grant No. DMR-8521662.

[1]B. L. Al'tshuler, A. G. Aronov, D. E. Khmelnitskii, and A. I. Larkin, in *Quantum Theory of Solids,* edited by I. M. Liftshits (MIR, Moscow, 1982), p. 130.
[2]G. Bergmann, Phys. Rep. **107**, 1 (1984).
[3]P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[4]H. Fukuyama, in *Modern Problems in Condensed Sciences,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p. 155.
[5]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[6]L. P. Gor'kov, A. I. Larkin, and D. E. Khmelnitskii, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].
[7]B. L. Al'tshuler, D. Khmelnitskii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).
[8]S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).
[9]B. L. Al'tshuler and A. G. Aronov, Solid State Commun. **36**, 115 (1979).
[10]B. L. Al'tshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).
[11]H. Fukuyama, J. Phys. Soc. Jpn. **48**, 2169 (1980).
[12]B. L. Altshuler and A. G. Aronov, in *Modern Problems in Condensed Sciences,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p. 1.
[13]P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
[14]A. Miller and B. Abrahams, Phys. Rev. **120**, 745 (1960).
[15]A. L. Efros and B. I. Shklovskii, J. Phys. C **8**, L49 (1975).
[16]H. Boettger and V. V. Brykskin, *Hopping Conduction in Solids* (VCH, Deerfield Beach, Florida, 1985).
[17]R. C. Dynes, A. E. White, J. M. Graybeal, and J. P. Garno, Phys. Rev. Lett. **57**, 2195 (1986).
[18]M. Pollak, Philos. Mag. B **42**, 781 (1980).
[19]F. J. Wegner, Z. Phys. **35**, 207 (1979).
[20]A. MacKinnon, in *Localization, Interaction, and Transport*

Phenomena, edited by B. Kramer, G. Bergmann, and Y. Bruynseraede, Vol. 61 of *Springer Series in Solid State Sciences* (Springer-Verlag, Berlin, 1985).
[21]R. C. Dynes, J. P. Garno, and J. M. Rowell, Phys. Rev. Lett. **40**, 479 (1978).
[22]D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. **62**, 2180 (1989).
[23]D. J. Bishop, D. C. Tsuei, and R. C. Dynes, in *Ordering in Two Dimensions,* edited by S. K. Sinha (Elsevier, North-Holland, Amsterdam, 1980).
[24]Y. Fehr, S. May-tal, and R. Rosenbaum, Phys. Rev. B **33**, 6631 (1986).
[25]S. Hikami, Phys. Rev. B **24**, 2671 (1981).
[26]R. Oppermann and K. Jüngling, Phys. Lett. **76A**, 449 (1980).
[27]H. Fukuyama, J. Phys. Soc. Jpn. **49**, 644 (1980).
[28]M. Pollak, Proc. R. Soc. London, Ser. A **325**, 383 (1971).
[29]M. Pollak, J. Phys. C **14**, 2977 (1981).
[30]A. L. Efros and B. I. Shklovskii, in *Modern Problems in Condensed Sciences,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p. 155.
[31]A. M. Finkelstein, Zh, Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys.—JETP **57**, 97 (1983)].
[32]C. Di Castro, in *Proceedings of the International Symposium on Anderson Localization, Tokyo, Japan, 1987,* edited by T. Ando and H. Fukuyama (Springer-Verlag, Berlin, 1987), p. 96.
[33]C. Castellani, C. DiCastro, P. A. Lee, M. Ma, S. Sorella, and E. Tabet, Phys. Rev. B **30**, 1596 (1984).
[34]B. L. Al'tshuler and A. G. Aronov, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 349 (1983) [JETP Lett. **37**, 410 (1983)].
[35]G. Bergmann, Phys. Rev. B **7**, 4850 (1973).
[36]G. Bergmann, Phys. Rev. Lett. **41**, 264 (1978).
[37]G. Bergmann, Phys. Rev. B **35**, 4205 (1987).
[38]D. J. Thouless, Phys. Rev. Lett. **39**, 1167 (1977).