Relativistic effects in the electron density of states, specific heat, and the electron spectrum of normal metals

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The correction to the electron density of states due to the interaction between electrons and vector photons in normal metals is investigated. The correction $\delta\nu_{e-\gamma}$ has a nonanalytic energy dependence near the Fermi surface. For small energy $\delta\nu_{e-\gamma}\sim-\varepsilon^{3/2}$ and with increasing energy $\delta\nu_{e-\gamma}\sim-\varepsilon^{2/3}$. The related tunneling conductivity is also considered. It is shown that the correction to the electron specific heat from the electron–vector-photon interaction $\Delta C_{e-\gamma}\sim-T\ln T$ dominates at low temperatures over the correction from the electron-phonon interaction $\Delta C_{e-\mathrm{ph}}\sim-T^3\ln T$. It is also shown that the electron–vector-photon interaction produces resonant states in the electron spectrum near the Fermi surface.

I. INTRODUCTION

It is well known[1,2] that the electron-electron interaction in disordered metal systems results in nontrivial corrections to the electron density of states, thermodynamic quantities, and kinetic coefficients. Consider for example the electron density of states which is defined by the equation

$$\nu(\varepsilon)=-\frac{2}{\pi}\int\frac{d\mathbf{p}}{(2\pi)^3}\mathrm{Im}[G^R(\mathbf{p},\varepsilon)]\ , \tag{1}$$

where $G^R(\mathbf{p},\varepsilon)$ is the retarded electron Green's function which includes the interaction effects. For noninteracting electrons the density of states is practically energy independent and given by $\nu=mp_F/\pi^2$, where m is the electron mass and p_F is the Fermi momentum. In an impure metal $\nu(\varepsilon)$ acquires a nonanalytic correction due to the electron-electron interaction[3]

$$\delta\nu_{e\text{-}e}(\varepsilon)=\frac{\lambda 2^{1/2}}{8\pi^2}\frac{|\varepsilon|^{1/2}}{D^{3/2}}\ , \tag{2}$$

where $D=v_F^2\tau/3$ is the diffusion constant, v_F is the Fermi velocity, and τ is the electron momentum relaxation time due to elastic scattering on impurities.

The result (2) essentially depends on the interaction constant λ. For the exchange correction due to the Coulomb interaction, $\lambda=2$ and the density of states has a square-root cusp near the Fermi surface. In the two-dimensional case the singularity is logarithmic.[4] Such a nonanalytic dependence of the density of states originates from a diffusion singularity in the impurity renormalization of scalar vertices of the electron-electron interaction. The corrections to the electron specific heat in the three- and two-dimensional cases are $\delta C_{e\text{-}e}\sim-T^{3/2}$ and $\delta C_{e\text{-}e}\sim-T\ln T$, respectively.[1]

In this paper we show that similar nonanalytic corrections to the electron density of states and to the specific heat appear in pure metals due to the interaction of electrons with vector photons (transverse electromagnetic fields). The paper is organized as follows. In Sec. II we describe the electron–vector-photon interaction in pure and impure metals and calculate the correction to the electron density of states. In Sec. III we consider the correction to the electron specific heat. This problem was investigated earlier in Ref. 5 by a different method for a pure metal. We consider both pure and impure metals and also low-dimensional systems. In Sec. IV we study the relativistic effects on the electron spectrum and show that they result in appearance of the resonant states near the Fermi surface. In the last Sec. V we summarize and discuss the obtained results.

II. ELECTRON DENSITY OF STATES

The interaction between electrons and the electromagnetic field is described by the Hamiltonian

$$H_{e\text{-}\gamma}=-\sum_{\mathbf{p}}\sum_{\mathbf{q}\neq0}\varphi_{\mathbf{q}}C^{\dagger}_{\mathbf{p}+\mathbf{q},s}C_{\mathbf{p},s}+\sum_{\mathbf{p}}\sum_{\mathbf{q}\neq0}\mathbf{A}_{\mathbf{q}}\cdot\frac{2\mathbf{p}+\mathbf{q}}{2mc}C^{\dagger}_{\mathbf{p}+\mathbf{q},s}C_{\mathbf{p},s}+\frac{1}{2mc^2}\sum_{\mathbf{p}}\sum_{\mathbf{q},\mathbf{q}'\neq0}\mathbf{A}_{\mathbf{q}}\cdot\mathbf{A}_{\mathbf{q}'}C^{\dagger}_{\mathbf{p}+\mathbf{q}+\mathbf{q}',s}C_{\mathbf{p},s}\ , \tag{3}$$

where $\varphi_{\mathbf{q}}$ $\mathbf{A}_{\mathbf{q}}$ are the Fourier components of the scalar and vector potentials, $C^{\dagger}_{\mathbf{p},s}$ is the creation operator for an electron with momentum $\mathbf{p}$ and spin s, and c is the velocity of light. The absolute value of the electronic charge is absorbed in the definition of the electromagnetic potentials.

We use the same notation as in Ref. 6, where the electron energy relaxation time due to the electron–vector-photon interaction was considered. The vertices corresponding to the interaction of electrons with vector photons, described by the Hamiltonian (3), are

$$\mathbf{a}^1=\frac{1}{mc}\left[\mathbf{p}+\frac{\mathbf{q}}{2}\right], \quad a^{11}=\frac{1}{2mc^2} . \tag{4}$$

The retarded Green's function for the transverse electromagnetic field in the Coulomb gauge (div $\mathbf{A}=0$) and with screening effects taken into account is

$$[V_{11}^R(\mathbf{q},\omega)]_{ij}=V_{11}^R(\mathbf{q},\omega)T_{ij} ,$$
$$V_{11}^R(\mathbf{q},\omega)=\frac{4\pi e^2c^2}{\omega^2-c^2q^2-4\pi e^2c^2P_{11}^R(\mathbf{q},\omega)} , \tag{5}$$

where e is the electron charge and i,j stand for the Cartesian coordinates x,y,z. If the vector $\mathbf{q}$ is directed along the z axis, T_{ij} is given by

$$T_{ij}=\delta_{ij}-\frac{q_iq_j}{q^2} . \tag{6}$$

P_{11} is the polarization operator which is shown in Fig. 1, where the electron Green's function in impure metal is

$$G_0^R(\mathbf{p},\varepsilon)=(\varepsilon-\xi_p+i/2\tau)^{-1}, \quad \xi_p=(p^2-p_F^2)/2m . \tag{7}$$

Calculating $P_{11}^R(\mathbf{q},\omega)$, we have

$$P_{11}^R(\mathbf{q},\omega)=-i\frac{\pi\nu\omega v_F}{4qc^2}, \quad ql\gg1, \quad qv_F\gg\omega, \quad \omega\tau\gg1 \tag{8}$$

$$P^R(\mathbf{q},\omega)=-i\frac{\nu v_F^2\omega\tau}{3c^2}, \quad ql\ll1, \quad \omega\tau\ll1 , \tag{9}$$

where $l=v_F\tau$ is the electron mean free path. Expressions (8) and (9) correspond to two different regimes of screening of electromagnetic fields. The long-wavelength limit $ql\ll1$, $\omega\tau\ll1$ refers to the normal skin effect and the short-wavelength limit $ql\gg1$, $\omega\tau\gg1$ refers to the anomalous skin effect.

The correction to the electron density of states due to the electron–vector-photon interaction is

$$\delta\nu_{e\text{-}\gamma}(\varepsilon)=-\frac{2}{\pi}\mathrm{Im}\int\frac{d\mathbf{p}}{(2\pi)^3}[G_0^R(p,\varepsilon)]^2\Sigma_{e\text{-}\gamma}^R(\mathbf{p},\varepsilon) , \tag{10}$$

where the electron self-energy shown in Fig. 1 is

$$\Sigma_{e\text{-}\gamma}^R(\mathbf{p},\varepsilon)=-\int\frac{d\omega\,d\mathbf{q}}{(2\pi)^3}(a_i^1T_{ij}a_j^1)\{\mathrm{Im}[G_0^A(\mathbf{p}+\mathbf{q},\varepsilon+\omega)]V_{11}^A(\mathbf{q},\varepsilon)\tanh[(\varepsilon+\omega)/2T]$$
$$+G_0^R(\mathbf{p},\varepsilon)\mathrm{Im}[V_{11}^R(q,\omega)]\coth(\omega/2T)\} . \tag{11}$$

Let us first consider a clean metal. After performing the $\mathbf{p}$ integration we have

$$\delta\nu_{e\text{-}\gamma}(\varepsilon)=\frac{\nu}{4\pi^3c^2}\int_{-\infty}^{\infty}d\omega\int_0^{\infty}dq\,\mathrm{Im}[V_{11}^A(\mathbf{q},\omega)]\tanh\left[\frac{\varepsilon+\omega}{T}\right]+\frac{\nu}{8\pi^2c^2v_F}\int_{-\infty}^{\infty}d\omega\int_0^{\infty}dq\frac{1}{q}\mathrm{Re}[V_{11}^A(\mathbf{q},\omega)]\tanh\left[\frac{\varepsilon+\omega}{T}\right] . \tag{12}$$

Assuming the inequalities $ql\gg1$, $qv_F>\omega$ to be true for the characteristic values of q and ω and using the expressions

$$\int_0^{\infty}\frac{dq\,q}{q^3+ib}=\frac{\pi}{b^{1/3}}\frac{3^{1/2}-i}{3^{3/2}}, \quad \int_0^{\infty}\frac{dq}{q^3+ib}=\frac{\pi}{b^{2/3}}\frac{1-i3^{1/2}}{3^{3/2}} , \tag{13}$$

where

$$b=\frac{\pi\kappa^2\omega v_F}{4c^2}, \quad \kappa^2=4\pi\nu e^2, \quad b^{1/3}l\gg1 ,$$

we get

$$\frac{\delta\nu_{e\text{-}\gamma}(\varepsilon)}{\nu}=\frac{1}{2^{4/3}3^{3/2}\pi^{1/3}}\left[\frac{\kappa}{mc}\right]^{4/3}\left[\frac{T}{\varepsilon_F}\right]^{2/3}\int_0^{\infty}\frac{dx}{x^{1/3}}\left[\tanh\left[x+\frac{\varepsilon}{2T}\right]+\tanh\left[x-\frac{\varepsilon}{2T}\right]-2\right]$$
$$+\frac{1}{2^{5/3}3^{3/2}\pi^{1/3}}\left[\frac{\kappa}{mc}\right]^{2/3}\left[\frac{T}{\varepsilon_F}\right]^{4/3}\int_0^{\infty}dx\,x^{1/3}\left[\tanh\left[x+\frac{\varepsilon}{2T}\right]+\tanh\left[x-\frac{\varepsilon}{2T}\right]-2\right] . \tag{14}$$

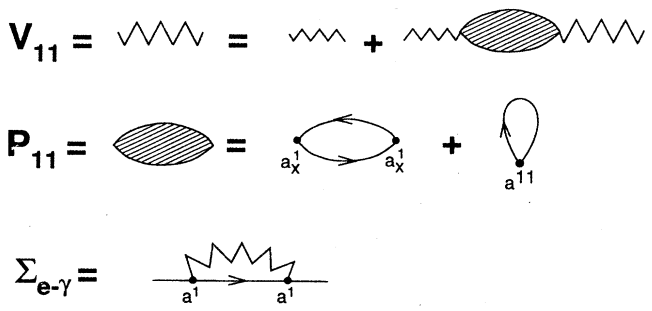

FIG. 1. P_{11} is the polarization operator for the transverse electromagnetic fields and $\Sigma_{e\text{-}\gamma}$ is the electron self-energy describing the electron–vector-photon interaction.

We have subtracted a constant in (14) to avoid the divergence of the integral at the upper limit in the same way as in the Ref. 1. In some limiting cases we have

$$\frac{\delta\nu_{e\text{-}\gamma}(\varepsilon)}{\nu}=-\frac{1}{3^{1/2}4\pi^{1/3}}\left[\frac{\kappa}{mc}\right]^{4/3}\left[\frac{\varepsilon}{\varepsilon_F}\right]^{2/3}, \quad T\ll\varepsilon\ll\varepsilon_0 \tag{15}$$

$$\frac{\delta\nu_{e\text{-}\gamma}(\varepsilon)}{\nu}=-\frac{1-2^{1/3}}{3^{1/2}2\pi^{1/3}}\Gamma(\tfrac{5}{3})\zeta(\tfrac{2}{3})\left[\frac{\kappa}{mc}\right]^{4/3}\left[\frac{T}{\varepsilon_F}\right]^{2/3}, \quad \varepsilon\ll T\ll\varepsilon_0 \tag{16}$$

where

$$\varepsilon_0=\varepsilon_F\left[\frac{\kappa}{mc}\right],$$

$\Gamma(x)$ is the gamma function and $\zeta(x)$ is the Riemann zeta function. Expressions (15) and (16) originate from the first term in (14). For higher frequencies $\varepsilon>\varepsilon_0$ the second term in (14) dominates and $\delta\nu_{e\text{-}\gamma}\sim\varepsilon^{4/3}$ for $\varepsilon_0<T<\varepsilon$. We however will not consider such frequencies further.

Considering next the correction $\delta\sigma$ to the tunneling conductivity, we assume as in Ref. 1 that the density of states in one of the metallic electrodes in the junction is constant, and in the second electrode it has a correction $\delta\nu$. Then

$$\frac{\delta\sigma_{e\text{-}\gamma}(\varepsilon)}{\sigma}=-\int_{-\infty}^{\infty}d\varepsilon\frac{\partial n(\varepsilon-eV)}{\partial\varepsilon}\frac{\delta\nu_{e\text{-}\gamma}(\varepsilon)}{\nu}, \tag{17}$$

where V is the voltage and $n(\varepsilon)$ is the Fermi distribution function. In the low- and high-temperature limits, the conductivity correction is

$$\frac{\delta\sigma_{e\text{-}\gamma}}{\sigma}=\frac{\delta\nu_{e\text{-}\gamma}(eV)}{\nu}, \quad T\ll eV \tag{18}$$

$$\frac{\delta\sigma_{e\text{-}\gamma}}{\sigma}=b_1\left[\frac{\kappa^2T}{m^2c^2\varepsilon_F}\right]^{2/3}, \quad eV\ll T \tag{19}$$

where

$$b_1=-\frac{1}{2^{1/3}3^{1/2}\pi^{1/3}}\times\int_0^{\infty}dy\,y^{2/3}\int_{-\infty}^{\infty}\frac{dx}{\cosh^2x\cosh^2(x+y)}. \tag{20}$$

We now discuss the validity of inequalities $ql\gg1$, $\omega\tau\gg1$, and $qv_F\gg\omega$. For $\delta\nu_{e\text{-}\gamma}$ the characteristic values of q and ω are

$$q_0=\left[\frac{\pi\kappa^2\omega_0v_F}{4c^2}\right]^{1/3}, \quad \omega_0=\max(T,|\varepsilon|). \tag{21}$$

Thus to satisfy the required inequalities we must have

$$\omega_1\ll\max(T,\varepsilon)\ll\left[\frac{v}{c}\right]\kappa v_F,$$

$$\omega_1=\frac{1}{\tau}\max\left[1,\left[\frac{c}{v_F^2\kappa\tau}\right]^2\right]. \tag{22}$$

Now consider the long-wavelength limit $ql\ll1$, $\omega\tau\ll1$, which is valid for $\max(|\varepsilon|,T)<\omega_1$. Using expression (9) we get

$$\frac{\delta\nu_{e\text{-}\gamma}}{\nu}=-\frac{\kappa^3\tau^{5/2}}{3^{3/2}2^{1/2}mp_F}\left[\frac{v_F}{c}\right]^3\times\int_0^{\omega_1}d\omega\,\omega^{1/2}[n(\omega+\varepsilon)+n(\omega-\varepsilon)]. \tag{23}$$

In some limiting cases $\delta\nu_{e\text{-}\gamma}$ is given by

$$\frac{\delta\nu_{e\text{-}\gamma}}{\nu}=-\frac{2^{1/2}}{3^{5/2}}\left[\frac{v_F}{c}\right]^3\frac{\kappa^3\tau}{mp_F}(\varepsilon\tau)^{3/2}, \quad T\ll\varepsilon\ll\omega_1 \tag{24}$$

$$\frac{\nu_{e\text{-}\gamma}}{\nu}=-\frac{2(2^{1/2}-1)}{3^{3/2}}\Gamma(\tfrac{5}{2})\zeta(\tfrac{3}{2})\left[\frac{v_F}{c}\right]^3\frac{\kappa^3\tau}{mp_F}(T\tau)^{3/2}, \quad \varepsilon\ll T\ll\omega_1. \tag{25}$$

As for the tunneling conductivity in the long-wavelength limit, the expression (18) is valid when $T\ll eV\ll\omega_1$.

Now we make some estimates. In a clean metal with $\varepsilon_F\tau\sim10^4$ we have $\omega_1\sim10^{-3}$ K. The absolute value of the correction to the density of states for $\omega_1<\max(|\varepsilon|,T)$ in a typical metal is very small; however, as indicated in (15) and (16) $\delta\nu_{e\text{-}\gamma}$ grows when the electron density n_e is decreased. For example, for $n_e=10^{21}$ cm^{-3} and $\max(|\varepsilon|,T)=10$ K we have $\delta\nu_{e\text{-}\gamma}/\nu\sim10^{-4}$. The correction $\delta\nu_{e\text{-}e}$ has the same value for $\varepsilon_F\tau\sim10$. Note also that the usual correction to the density of states from the Fermi distribution function is of order $[\max(|\varepsilon|,T)/\varepsilon_F]^2$ and is small in comparison with $\delta\nu_{e\text{-}\gamma}$.

Now consider a film with thickness $d<q_1^{-1}$, where

$$q_1=\frac{\kappa v_F}{3c}[\tau\max(|\varepsilon|,T)]^{1/2}. \tag{26}$$

Such a film may be considered two dimensional. In this case $\delta\nu_{e\text{-}\gamma}$ is proportional to $\max(|\varepsilon|,T)$, but the value of the correction is small, unlike the correction $\Delta\nu_{e\text{-}e}$ from the Coulomb interaction. Therefore in a dirty and low-

dimensional electron system the Coulomb interaction correction dominates: $|\Delta\nu_{e\text{-}e}| \gg |\Delta\nu_{e\text{-}\gamma}|$.

III. ELECTRON SPECIFIC HEAT

The correction to the thermodynamic potential from the electron–vector-photon interaction is

$$\frac{\Delta\Omega_{e\text{-}\gamma}}{v_0} = \int_{-\infty}^{\infty} d\mathbf{q}\, d\omega \frac{1}{(2\pi)^4} \coth(\omega/2T) \times \mathrm{Im}(\ln\{1-[V_{11}^0(\mathbf{q},\omega)]^R P_{11}^R(\mathbf{q},\omega)\}) , \tag{27}$$

$$[V_{11}^0(\mathbf{q},\omega)]^R = \frac{4\pi e^2 c^2}{(\omega+i0)^2 - c^2 q^2} \tag{28}$$

and v_0 is the volume of the electron system.

Using the expressions (8) and (9) we have

$$\frac{\Delta\Omega_{e\text{-}\gamma}}{v_0} = \frac{1}{4\pi^3} \int_0^{\infty} d\omega\, N(\omega) \int_0^{\infty} dq\, q^2 \arctan[\varphi(q,\omega)] , \tag{29}$$

where $N(\omega)$ is the Bose distribution function and

$$\varphi(q,\omega) = \frac{\pi\kappa^2 v_F \omega}{4q(\omega^2 - q^2c^2)}, \quad ql \gg 1, \ \omega\tau \gg 1 \tag{30}$$

$$\varphi(q,\omega) = \frac{\kappa^2 v_F^2 \omega\tau}{3(\omega^2 - q^2c^2)}, \quad ql \ll 1, \ \omega\tau \ll 1 . \tag{31}$$

The main contribution to $\Delta\Omega_{e\text{-}\gamma}$ originates from the region where $|\varphi(q,\omega)| \ll 1$, and hence for $ql \gg 1$ we have the inequality

$$l^{-3} \ll \frac{\pi\kappa^2 v_F \omega}{4c^2} \ll q^3 . \tag{32}$$

Thus with the logarithmic accuracy we have the following correction to the electron specific heat:

$$\Delta C_{e\text{-}\gamma} = -\frac{\pi^2}{48} \left(\frac{\kappa}{mc}\right)^2 \frac{T}{\varepsilon_F} \ln\left(\frac{T}{T_2}\right), \quad \omega_1 = T_1 < T \tag{33}$$

where

$$T_2 = \frac{4q_2^3 c^2}{\pi\kappa^2 v_F} . \tag{34}$$

Here we have introduced the photon boundary wave vector q_2 to cut off the q integral at the upper limit in (29), $q_2 \sim p_F$. Comparison of the correction $\Delta C_{e\text{-}\gamma}$ from (33) with the correction $\Delta C_{e\text{-ph}}$ from the electron-phonon interaction,[7,8] given by

$$\Delta C_{e\text{-ph}} \sim -\frac{T^3}{\varepsilon_F (p_F u)^2} \ln\left(\frac{T}{p_F u}\right) , \tag{35}$$

where u is the sound velocity, shows that at low temperatures the correction from the electron–vector-photon interaction dominates. As was mentioned earlier, for $\varepsilon_F\tau \sim 10^4$ we have $T_1 \sim 10^{-3}$. Thus for $T \sim 1$ K we have $\ln(T/T_2) \sim 20$ and $\Delta C_{e\text{-}\gamma} \sim 10^{-3} C_e$, where C_e is the specific heat of the free electron gas.

In the long-wavelength limit the inequality $|\varphi(q,\omega)| \ll 1$ is valid when

$$\frac{\kappa^2 v_F^2 \omega\tau}{3c^2} \ll q^2 \ll l^{-2} . \tag{36}$$

Calculating the correction to the specific heat under condition (36), we find

$$\Delta C_{e\text{-}\gamma} = \frac{\pi}{24} \left(\frac{\kappa}{mc}\right) \frac{T}{\varepsilon_F}, \quad T < T_1 . \tag{37}$$

A film with the thickness d such that $d^{-1} > \kappa v_F (\omega\tau)^{1/2}/c$ may be treated as two dimensional, and hence

$$\Delta C_{e\text{-}\gamma} = -\frac{\pi}{12} \left(\frac{\kappa}{mc}\right)^2 \frac{T\tau}{p_F d} \ln\left(\frac{T}{T_1}\right), \quad T < \{T_1, T_3\} \tag{38}$$

where

$$T_3 = \frac{1}{\tau} \left(\frac{c}{d\kappa v_F}\right)^2 . \tag{39}$$

Note that in the long-wavelength limit $\Delta C_{e\text{-}\gamma}$ diminishes with increasing disorder, while the correction from the Coulomb interaction[1] is enhanced by disorder. Therefore, in impure and low-dimensional electron systems the correction from the Coulomb interaction dominates.

Results qualitatively similar to expression (33) were obtained earlier by a different method in Ref. 5. The main difference between our work and Ref. 5 is in considering an impure metal from the very beginning. This approach allows us not only to get natural cutoffs of the logarithmic terms (in Ref. 5 the logarithmic terms were not cut off at all), but also to study two different regimes of screening (normal and anomalous skin effects) and to consider the two-dimensional case.

IV. ELECTRON SPECTRUM

The spectrum of electron excitations is obtained from the electron spectral functions $A(\mathbf{p},\varepsilon) = -2\,\mathrm{Im}[G^R(\mathbf{p},\varepsilon)]$, where

$$[G^R(\mathbf{p},\varepsilon)]^{-1} = \varepsilon - \xi_p - \mathrm{Re}\Sigma(\varepsilon) - i\,\mathrm{Im}\Sigma^R(\varepsilon) . \tag{40}$$

To calculate $\Sigma(\varepsilon)$ for $T=0$, we start from Eq. (11) and in the short-wavelength limit we get[9]

$$\mathrm{Re}\Sigma_{\varepsilon\text{-}\gamma}(\varepsilon) = -\frac{1}{8\pi^2 v_F} \left(\frac{v_F}{c}\right)^2 \int_0^{\varepsilon} d\omega \int_0^{\infty} dq\, q\, \mathrm{Re}[V_{11}(\mathbf{q},\omega)] = -\alpha_1 \varepsilon \ln\left(\frac{|\varepsilon|}{\varepsilon_1}\right) , \tag{41}$$

$$\mathrm{Im}\Sigma_{\varepsilon\text{-}\gamma}^R(\varepsilon) = -\frac{\pi}{2}\alpha_1 \varepsilon , \tag{42}$$

where

$$\alpha_1=\frac{1}{24}\left[\frac{\kappa}{mc}\right]^2, \quad \varepsilon_1=\frac{\varepsilon_F}{3\pi\alpha_1} . \tag{43}$$

The characteristic values of the wave vector q_0 and frequency ω_0 are the same as in (21), and thus Eqs. (41) and (42) hold given the validity of (22). Expressions (41) and (42) result in the following analytical structure of the spectral function:

$$A(\mathbf{p},\varepsilon)=\frac{\pi\alpha_1\varepsilon}{\left[\varepsilon+\alpha_1\varepsilon\ln\left[\frac{|\varepsilon|}{|\varepsilon_1|}\right]-\xi_p\right]^2+\left[\frac{\pi}{2}\alpha_1\varepsilon\right]^2} . \tag{44}$$

In particular for $\xi_p=0$ the spectral function has singularities at frequencies,

$$\varepsilon_{m1}=0, \quad \varepsilon_{m2,3}=\pm\varepsilon_1\exp\left[-\frac{1}{\alpha_1}\right], \quad \alpha_1\ll 1 . \tag{45}$$

The first frequency ε_{m1} corresponds to a usual quasiparticle state at the Fermi level. The others correspond to the resonant electron states near the Fermi level. In some aspects these resonances are similar to the Abrikosov-Suhl resonance in the Kondo problem.[10] As in the Kondo problem a careful treatment of the weak relativistic interaction to higher orders in the perturbation expansion results in the resonant states near the Fermi surface due to the singularity in electron screening. The magnitude of ε_{m2} is very small, $\varepsilon_{m2}\sim 10^{-7}$ K, and the left inequality in (22) cannot be fulfilled even in very pure metals. In an impure metal we should include in $\mathrm{Im}\Sigma^R$ the contribution from the electron-impurity scattering which smears the resonant states near the Fermi surface. For this reason we do not discuss the structure of the electron density of states at such low frequencies and the corresponding correction to the electron specific heat.

It is interesting to consider as an example an electron system of a neutron star, where electrons may be considered as relativistic, $v_F=c$. In such a system the resonances in the electron spectrum exist at

$$\varepsilon_r=\pm cp_F\exp\left[-\frac{1}{\alpha}\right], \quad \alpha=\frac{e^2}{4\pi c}\ll 1 . \tag{46}$$

In conclusion we show that in spite of the fact that $\mathrm{Re}V_{11}(\mathbf{q},\omega)$ is negative for small q the relativistic electron-electron interaction does not result in superconducting state in a metal at low temperatures. To prove it we note that in the Eliashberg equation for the complex order parameter in the Nambu representation the Coulomb potential $V_{00}(q)$ and the phonon Green's function $D(\mathbf{q},\omega)$ appears in the combination $\hat{\tau}_3V_{00}(q)\hat{\tau}_3$ and $\hat{\tau}_3D(\mathbf{q},\omega)\hat{\tau}_3$ (see, for example, Ref. 11), where $\hat{\tau}_i$ are the Pauli matrices. On the other hand the vector photon Green's function appears in the combination $\hat{\tau}_0a_i^1[V_{11}(\mathbf{q},\omega)]_{ij}a_j^1\hat{\tau}_0$ due to the symmetry of vertex $\mathbf{a}^1$ under the transformation $\mathbf{p}\rightarrow-\mathbf{p}$. Consequently both V_{00} and V_{11} give positive contributions in the kernel of the integral equation for the order parameter and thus the electron-vector photon interaction does not support superconductivity.

V. SUMMARY

We have shown that the relativistic electron-electron interaction leads to a nonanalytic behavior of the electron density of states near the Fermi surface, the logarithmic temperature dependence of the electron specific heat, and the appearance of resonance states near the Fermi surface. The effects are small but as for the density of states and the specific heat, they are in principle measurable. Nevertheless the obtained results are important for several reasons. First, they show that even in a such harmless system as the electron gas without a periodical potential there are some deviations from the traditional Fermi liquid theory. It is also worth mentioning that, as was recently shown by Lee,[12] some strongly correlated electron systems may be described by means of the interaction between electrons and some gauge fields. Thus the electron–vector-photon interaction may be considered as a simplest model for such interactions, and in such electron systems the effects discussed above may not be small.

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