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Critical exponents for ferromagnetic systems with Ruderman-Kittel-Kasuya-Yosida interactions

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The asymptotic critical exponents of a ferromagnetic spherical model on a simple cubic lattice with Ruderman-Kittel-Kasuya-Yosida interactions are calculated. These interactions are of long range but oscillating in sign and therefore lead to the same asymptotic critical behavior as the corresponding nearest-neighbor model.

It is well known that the critical behavior of d-dimensional systems with long-range interactions decaying as $1/r^{d+\sigma}$ ($\sigma>0$) may be different from the one of systems with nearest-neighbor interactions. For instance, both the exact calculations within the framework of the spherical model[1] and the renormalization-group theory[2] for the isotropic n-vector model have shown that in three dimensions the susceptibility exponent γ attains its short-range value for $\sigma>2$, its mean-field value of $\gamma=1$ for $\sigma<\frac{3}{2}$, and values varying continuously with σ for $\frac{3}{2}<\sigma<2$.

Experimentally, considerable deviations from the Heisenberg values towards the mean-field values were found[3-5] for the ferromagnetic alloys Fe_3Pt and $FePd_3$. For these materials the so-called s-d model has been applied[6] in which it is assumed that there are fairly well-defined $3d$ local moments on Fe atoms that couple with itinerant electrons of Pd or Pt, giving rise to Ruderman-Kittel-Kasuya-Yosida (RKKY) indirect s-d exchange couplings[7] which vary in space as

$$J_{ij}(R_{ij})\sim\frac{\sin x - x\cos x}{x^4},\quad x=2k_FR_{ij}\,. \tag{1}$$

Here k_F denotes the Fermi wave vector and R_{ij} is the distance between the moments localized at sites i and j of a lattice. Because the term dominating the long-range behavior decays as $1/R_{ij}^3$, it was suggested[4] that the critical behavior of the systems will resemble the one of a system with long-range interactions decaying as $1/r^{d+\sigma}$ with $\sigma=0$, which would explain why the exponent values of Fe_3Pt and $FePd_3$ are modified towards the mean-field values.

However, it is also possible to argue that the oscillating behavior of the RKKY interaction with plus and minus signs may cause, in effect, a short-range behavior. In the present paper we therefore consider the ferromagnetic spherical model with RKKY interactions, which may be solved exactly in the asymptotic critical regime. It will be shown that this system exhibits the same asymptotic critical exponents as the corresponding nearest-neighbor model. Because of the close similarity of the results for the $1/r^{d+\sigma}$ model between the calculations for the spherical model and the isotropic n-vector model, we hope that our results will also be valid for the more realistic n-vector model. Our calculations proceed on a line totally equivalent to the one of Joyce[1] for the spherical model with $1/r^{d+\sigma}$ interactions. We therefore discuss only the modifications due the RKKY interactions and refer to this classical paper for all details.

The derivations of Joyce which constitute a generalization of the nearest-neighbor ferromagnetic spherical model of Berlin and Kac[8] to further distant interactions are valid for all types of interaction $J_{ij}(R_{ij})$, providing that the lattice sums

$$\phi(\underline{\omega})=\sum_{l_1=-\infty}^{+\infty}\cdots\sum_{l_d=-\infty}^{+\infty}{}' J_{l_1\ldots l_d}\cos(\underline{\omega}\underline{l})\,, \tag{2}$$

are all positive with $\phi(\underline{\omega})<\phi(\underline{\omega}=0)$ (see also Ref. 9). In Eq. (2) the quantity $J_{l_1\ldots l_d}$ is obtained from J_{ij} by taking the ith spin as an origin for the coordinates $\underline{l}=(l_1a,\ldots,l_da)$ of the jth spin (a is the lattice spacing). In Joyce's paper this restriction is fulfilled by considering positive J_{ij} monotonically decreasing with R_{ij}. However, this restriction is also fulfilled for the oscillating RKKY interaction on a simple cubic lattice, for which the lattice sum is given by[7]

$$\phi(\underline{\omega})=\tfrac{1}{2}\phi(\underline{\omega}=0)D(\omega)\,, \tag{3}$$

with

$$D(\omega)=1+\frac{4k_F^2-\omega^2}{4k_F\omega}\ln\frac{2k_F+\omega}{2k_F-\omega}\,. \tag{4}$$

The function $D(\omega)$ indeed decreases monotonically with increasing ω. Furthermore, $\phi(\underline{\omega}=0)$ is positive for small values of $2k_Fa$. We have shown by numerical calculations for a system with 100^3 spins that $\phi(\underline{\omega}=0)$ is positive for $0\le 2k_Fa\lesssim 4$ and also for other intervals with larger values of $2k_Fa$. In the limit $2k_Fa\to 0$ the system certainly exhibits a ferromagnetic ground state, whereas for larger values more complicated ground states may occur. For example, our numerical calculations show that for $T=0$ K the ferromagnetic state is energetically more favorable than the antiferromagnetic state for $0\le 2k_Fa\lesssim 5$ (and for other intervals). Therefore, in the following we consider only values of $2k_Fa$ for which the ground state is ferromagnetic and for which $\phi(\underline{\omega}=0)$ is positive.

The rest of the calculation is exactly in line with the procedure of Joyce.[1] The critical exponents α, β, and γ of the specific heat at $T>T_c$, the spontaneous magnetiza-

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tion, and the zero-field susceptibility for $T > T_c$, are calculated from Eqs. (4.2), (4.9), and (4.12) of Ref. 1. The saddle point z_s entering these equations is calculated from the saddle-point equation (2.9) and is for $T \approx T_c$ and small magnetic field H determined by the lowest-order term in $\underline{\omega}$ of the expansion for the function $\phi(\underline{\omega})$. Because this term is proportional to ω^2 both for the RKKY interactions and for the nearest-neighbor interactions,[8] we yield the exponents of the short-range spherical model, i.e., $\alpha = -1$, $\beta = \frac{1}{2}$, and $\gamma = 2$.

The question arises whether the asymptotic short-range behavior may be really observed in an experiment or whether there are "residual" long-range features of the system resulting in an apparent mean-field-like behavior of the observed effective exponents. One way to check this would be to calculate analytically within the present model the correction to scaling terms up to a certain order in the Wegner series.[10] However, what the experimentalist then needs to know is the temperature range where such a truncated series is valid, which requires full control of the convergence behavior of the series. Instead, we therefore have calculated numerically without any approximation the quantity[11]

$$\gamma(T) = (T - T_c)\chi \frac{d\chi^{-1}}{dT} = \frac{\partial \ln \chi^{-1}}{\partial \ln(T - T_c)} , \qquad (5)$$

using Eq. (4.12) of Ref. 1 with the saddle point z_s determined from Eq. (2.9), inserting our Eqs. (3) and (4), and performing the integration numerically. The exponent $\gamma(T)$ is constructed in such a way that it approaches the asymptotic critical value for $T \rightarrow T_c$ and the mean-field value of $\gamma = 1$ for very high T. Figure 1 represents $\gamma(T)$ for various values of $2k_F a$. For $2k_F a \rightarrow 0$ the quantity $\gamma(T)$ decreases very rapidly from $\gamma = 2$ to $\gamma = 1$, so that the experimentalist will not be able to observe the asymptotic short-range behavior. This is easy to understand because for $2k_F a \rightarrow 0$ the exchange interaction $J_{ij}(R_{ij})$ according to Eq. (1) changes sign for the first time at a large value of R_{ij}, say R_{ij}^1, and the system therefore exhibits a residual long-range behavior as soon as the correlation length $\zeta(T)$ is smaller than R_{ij}^1. In contrast, for reasonable values of $2k_F a$ as in Fig. 1 the quantity $\gamma(T)$ decreases very gradually with increasing T, and experiments like those of Refs. 3 and 4 should easily be able to resolve the asymptotic short-range behavior.

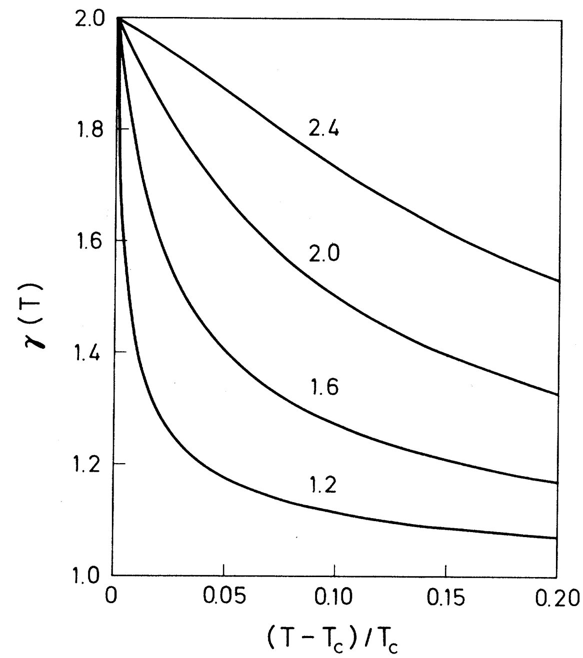

FIG. 1. Temperature dependence of $\gamma(T)$ for various values of $2k_F a$.

To conclude, we have shown that the simple cubic ferromagnetic spherical model with RKKY interactions, which are of long range but oscillating in sign, exhibits the same asymptotic critical exponents as the corresponding short-range model.

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