Comments

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Comment on "Spin excitations in multilayered ferromagnetic electron gases"

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In a recent paper Zhou and Gong [Phys. Rev. B **38**, 4936 (1988)] determined the spin-wave spectrum of a multilayered ferromagnetic electron gas. They found an explicit influence of the anisotropy of the Coulomb interaction matrix element on the spin-wave stiffness. It will be shown in the present Comment that this is an artifact of their approximation.

The starting point for the description of the multilayered ferromagnetic electron gas is the Hamiltonian[1,2]

$$H=\sum_{\mathbf{K}\sigma}\left(\frac{\hbar^2k^2}{2m^*}-\frac{\Delta}{2}\cos k^z d\right)c^\dagger_{\mathbf{K}\sigma}c_{\mathbf{K}\sigma}+\tfrac{1}{2}\sum_{\substack{\mathbf{K}\mathbf{K}'\\ \mathbf{Q}\\ \sigma\sigma'}}\frac{2\pi e^2}{\Omega q}\frac{d\sinh qd}{\cosh qd-\cos q^z d}c^\dagger_{\mathbf{K}\sigma}c^\dagger_{\mathbf{K}'\sigma'}c_{\mathbf{K}'+\mathbf{Q}\sigma'}c_{\mathbf{K}-\mathbf{Q}\sigma}\,, \tag{1}$$

which is a slight modification of models which have been investigated by several authors before.[3-5] $c^\dagger_{\mathbf{K}\sigma}(c_{\mathbf{K}\sigma})$ is the creation (annihilation) operator for an electron in a single-particle state of the energy

$$\varepsilon(\mathbf{K})\equiv\hbar^2k^2/2m^*-(\Delta/2)\cos k^z d$$

and spin orientation σ, where $\mathbf{K}\equiv(\mathbf{k},k^z)\equiv(k^x,k^y,k^z)$. Δ is the width of the band which is connected with electron tunneling from one plane to a nearest-neighbor plane, d is the distance between adjacent planes, and Ω is the volume of the crystal.[6] The strong anisotropy of the system is expressed in both the electron dispersion relation $\varepsilon(\mathbf{K})$ and the Coulomb interaction matrix element (the Fetter model[5])

$$V(\mathbf{Q})=\frac{2\pi e^2}{\Omega q}\frac{d\sinh qd}{\cosh qd-\cos q^z d}\,. \tag{2}$$

The quantity of central interest, which has to be determined from the Hamiltonian (1), is the retarded spin-density commutator Green's function[6,7]

$$\langle\langle\hat{S}^+(\mathbf{Q});\hat{S}-(-\mathbf{Q})\rangle\rangle_\omega=|F(q^z)|^2\sum_{\mathbf{K},\mathbf{K}'}\langle\langle c^\dagger_{\mathbf{K}\uparrow}c_{\mathbf{K}+\mathbf{Q}\downarrow};c^\dagger_{\mathbf{K}'\downarrow}c_{\mathbf{K}'-\mathbf{Q}\uparrow}\rangle\rangle_\omega\,. \tag{3}$$

Zhou and Gong obtained this function on the basis of their approximate Eq. (14b). In the Green's-function language their Eq. (14b) means

$$\langle\langle c^\dagger_{\mathbf{P}\uparrow}c_{\mathbf{P}+\mathbf{Q}\downarrow};S^-(-\mathbf{Q})\rangle\rangle_\omega=\frac{\langle n_{\mathbf{P}\uparrow}\rangle-\langle n_{\mathbf{P}+\mathbf{Q}\downarrow}\rangle}{\hbar\omega-\varepsilon_{\mathbf{P}+\mathbf{Q}\downarrow}+\varepsilon_{\mathbf{P}\uparrow}}\left(1-\sum_{\mathbf{P}'}V(\mathbf{P}-\mathbf{P}')\langle\langle c^\dagger_{\mathbf{P}'\uparrow}c_{\mathbf{P}'+\mathbf{Q}\downarrow};S^-(-\mathbf{Q})\rangle\rangle_\omega\right), \tag{4}$$

where

$$\varepsilon_{\mathbf{K}\sigma}\equiv\varepsilon(\mathbf{K})-\sum_{\mathbf{K}'}V(\mathbf{K}-\mathbf{K}')\langle n_{\mathbf{K}'\sigma}\rangle\,, \tag{5}$$

and

$$\hat{S}^\pm(\mathbf{Q})=F(q_z)S^\pm(\mathbf{Q})\,. \tag{6}$$

It should be noted that Eq. (4) may be derived easily from the exact equation of motion

$$\begin{aligned}[\hbar\omega-\varepsilon(\mathbf{P}+\mathbf{Q})+\varepsilon(\mathbf{P})]&\langle\langle c^\dagger_{\mathbf{P}\uparrow}c_{\mathbf{P}+\mathbf{Q}\downarrow};S^-(-\mathbf{Q})\rangle\rangle_\omega\\ &=\langle n_{\mathbf{P}\uparrow}\rangle-\langle n_{\mathbf{P}+\mathbf{Q}\downarrow}\rangle+\sum_{\mathbf{K}'\mathbf{P}'}V(\mathbf{P}')[\langle\langle c^\dagger_{\mathbf{P}\uparrow}\underline{c_{\mathbf{P}+\mathbf{Q}-\mathbf{P}'\downarrow}c^\dagger_{\mathbf{K}'\downarrow}}c_{\mathbf{K}'+\mathbf{P}'\downarrow}-c^\dagger_{\mathbf{K}'\uparrow}\underline{c_{\mathbf{K}'+\mathbf{P}'\uparrow}c^\dagger_{\mathbf{P}+\mathbf{P}'\uparrow}}c_{\mathbf{P}+\mathbf{Q}\downarrow};S^-(-\mathbf{Q})\rangle\rangle_\omega\\ &\quad+\langle\langle\underline{c^\dagger_{\mathbf{P}\uparrow}c_{\mathbf{P}+\mathbf{Q}-\mathbf{P}'\downarrow}c^\dagger_{\mathbf{K}'\uparrow}}c_{\mathbf{K}'+\mathbf{P}'\uparrow}-\underline{c^\dagger_{\mathbf{K}'\downarrow}c_{\mathbf{K}'+\mathbf{P}'\downarrow}c^\dagger_{\mathbf{P}+\mathbf{P}'\uparrow}}c_{\mathbf{P}+\mathbf{Q}\downarrow};S^-(-\mathbf{Q})\rangle\rangle_\omega]\,,\end{aligned} \tag{7}$$

decoupling the higher-order Green's function on the right as indicated by means of the brackets below the lines. The problem of Eq. (4) is in the last term on the right. Zhou and Gong introduced in this place the approximation

$$V(\mathbf{P}-\mathbf{P}') \approx G(\mathbf{Q})V(\mathbf{Q}) , \tag{8}$$

which has been proposed originally by Hubbard[8] for the homogeneous electron gas. They further assumed that the local-field correction G takes the form

$$G(\mathbf{Q}) = \frac{q}{q+q_s} , \tag{9}$$

where q_s is an "appropriate" screening wave number. Using (8) and (9), Eq. (4) may now be solved exactly, giving the results (16), (17), and (26) in their paper. There are several objections to this procedure. So the factorization approximation (8) is unjustified and in the (also unjustified) local-field correction $G(\mathbf{Q})$ [Eq. (9)] the third dimension is neglected completely, which leads to an overestimation of the anisotropy of the Coulomb interaction. Further, in our earlier paper[2] another solution of the basic Eq. (7) has been derived using methods of the dynamical local-field theories as first introduced by Devreese *et al.*[9-11] and later used by many other authors as well.[12-15] In general, the results of the dynamical local-field theory should be more reliable than those of the factorization approximation since the latter corresponds to neglecting certain terms in Wick's theorem, whereas the former takes all contributions into account. The resulting spin-wave equation of that approach is given by[2]

$$1+\frac{1}{\chi_0^{+-}(\mathbf{Q},\omega)}\sum_{\mathbf{K},\mathbf{P}}\frac{V(\mathbf{K}-\mathbf{P})(\langle n_{\mathbf{K}\uparrow}\rangle-\langle n_{\mathbf{K}+\mathbf{Q}\downarrow}\rangle)(\langle n_{\mathbf{P}\uparrow}\rangle-\langle n_{\mathbf{P}+\mathbf{Q}\downarrow}\rangle)}{[\hbar\omega-\varepsilon(\mathbf{P}+\mathbf{Q})+\varepsilon(\mathbf{P})](\hbar\omega-\varepsilon_{\mathbf{K}+\mathbf{Q}\downarrow}+\varepsilon_{\mathbf{K}\uparrow})}=0 , \tag{10}$$

where

$$\chi_0^{+-}(\mathbf{Q},\omega)=\sum_{\mathbf{K}}\frac{\langle n_{\mathbf{K}\uparrow}\rangle-\langle n_{\mathbf{K}+\mathbf{Q}\downarrow}\rangle}{\hbar\omega-\varepsilon(\mathbf{K}+\mathbf{Q})+\varepsilon(\mathbf{K})} . \tag{11}$$

If the approximation of Zhou and Gong [i.e., Eqs. (8) and (9)] are substituted into (10), their spin-wave equation [Eq. (17) of Ref. 1] is obtained:

$$1+V(\mathbf{Q})G(\mathbf{Q})\sum_{\mathbf{k}}\frac{\langle n_{\mathbf{K}\uparrow}\rangle-\langle n_{\mathbf{K}+\mathbf{Q}\downarrow}\rangle}{\hbar\omega-\varepsilon_{\mathbf{K}+\mathbf{Q}\downarrow}+\varepsilon_{\mathbf{K}\uparrow}}=0 . \tag{12}$$

In the same way it may be shown that the result for the spin-density function of Zhou and Gong [Eq. (16) of Ref. 1] is an approximation of the expression (12) of Ref. 2. In other words, the results of Ref. 2 are more comprehensive than those of Ref. 1. Finally, we regard the spin-wave dispersion relations in the long-wavelength limit. From Eq. (10) one obtains the solution

$$\hbar\omega(Q)=V_0\sum_{\mathbf{K}}\frac{1}{\sum_{\mathbf{P}}V(\mathbf{K}-\mathbf{P})N_{\mathbf{P}\uparrow\downarrow}(0)}\left[(\langle n_{\mathbf{K}\uparrow}\rangle+\langle n_{\mathbf{K}\downarrow}\rangle)\left(\frac{\hbar^2}{2m}\sin^2\theta+\frac{\Delta d^2}{4}\cos^2\theta\cos k^z d\right)\right.$$
$$\left.-\frac{\langle n_{\mathbf{K}\uparrow}\rangle-\langle n_{\mathbf{K}\downarrow}\rangle}{\sum_{\mathbf{P}'}V(\mathbf{K}-\mathbf{P}')N_{\mathbf{P}'\uparrow\downarrow}(0)}\left(\frac{\hbar^4k^2}{2m^2}\sin^2\theta+\frac{\Delta^2d^2}{4}\cos^2\theta\sin^2k^z d\right)\right]Q^2 , \tag{13}$$

where

$$V_0=\left[\sum_{\mathbf{K}}N_{\mathbf{K}\uparrow\downarrow}(0)\Big/\left[\sum_{\mathbf{P}}V(\mathbf{K}-\mathbf{P})N_{\mathbf{P}\uparrow\downarrow}(0)\right]\right]^{-1} , \tag{14}$$

and

$$N_{\mathbf{K}\uparrow\downarrow}(\mathbf{Q})\equiv\langle n_{\mathbf{K}\uparrow}\rangle-\langle n_{\mathbf{K}+\mathbf{Q}\downarrow}\rangle . \tag{15}$$

θ is the angle between the three-dimensional vector $\mathbf{Q}=(\mathbf{q},q^z)$ and the z axis which is the axis of the stack. From (13) one may obtain the spin-wave stiffness of Ref. 1 also, if the approximations (8) and (9) are used again. This shows that the explicit dependence of the spin-wave stiffness on the anisotropy of the Coulomb interaction matrix element as shown by Zhou and Gong is not a true property of the system but only a result of the approximations (8) and (9). Expression (13) shows that only integrals of the Coulomb interaction matrix elements of the type (14) influence the stiffness.

In conclusion, it should be noted that the multilayered ferromagnetic electron gas is still a hypothetical system. However, spin waves should certainly exist in a multilayered electron gas if a strong external magnetic field is applied to this system. This case has been investigated in a different paper.[16]

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$$\langle\langle \hat{S}^+(\mathbf{Q});\hat{S}^-(-\mathbf{Q})\rangle\rangle_\omega=(V/\hbar)D_R^{+-}(\mathbf{Q},\omega)$$

and

$$\langle\langle c^\dagger_{\mathbf{P}\uparrow}c_{\mathbf{P}+\mathbf{Q}\downarrow};S^-(-\mathbf{Q})\rangle\rangle_\omega = A(\mathbf{P})\chi(\mathbf{P}) = \frac{(\langle n_{\mathbf{P}\uparrow}\rangle-\langle n_{\mathbf{P}+\mathbf{Q}\downarrow}\rangle)\chi(\mathbf{P})}{\hbar\omega-\varepsilon(\mathbf{P}+\mathbf{Q})+\varepsilon(\mathbf{P})-\sum_{\mathbf{P}'}V(\mathbf{P}-\mathbf{P}')(\langle n_{\mathbf{P}\uparrow}\rangle-\langle n_{\mathbf{P}'+\mathbf{Q}\downarrow}\rangle)}.$$

[7]The symbols $\langle\langle A;B\rangle\rangle_\omega$ stand for the Fourier transform with respect to time of the retarded commutator Green's function

$$\langle\langle A(t);B(t')\rangle\rangle=-(i/\hbar)\theta(t-t')\langle[A(t),B(t')]\rangle,$$

where $\langle\cdots\rangle$ denotes the mean value of the grand canonical ensemble.
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