Stability of two-dimensional Fermi liquids against pair fluctuations with large total momentum

J. W. Serene
Complex Systems Theory Branch, Naval Research Laboratory, Washington, D.C. 20375-5000
(Received 17 May 1989; revised manuscript received 5 July 1989)

Independent pair fluctuations with large total momentum do not cause an instability in a degenerate two-dimensional Fermi liquid with a weak attractive interaction.

I. INTRODUCTION

Understanding the crossover between cooperative Bardeen-Cooper-Schrieffer (BCS) pairing and Bose condensation of independently bound diatomic molecules has assumed new importance on account of evidence suggesting that in the new superconductors the pair radius may be comparable to the mean distance between charge carriers. Most attempts to describe this crossover have employed the BCS mean-field theory, generalized slightly to allow for the change in the chemical potential due to pairing away from the weak-coupling limit.[1-5] As emphasized by Randeria, Duan, and Shieh, the problem in two dimensions has the special feature that the existence of a two-body bound state is a necessary condition for BCS pairing in a dilute Fermi gas. This led Randeria *et al.* to suggest that independently bound pairs might cause anomalous properties in the normal state above the BCS transition temperature.[5] Recently Schmitt-Rink, Varma, and Ruckenstein (SVR) (Ref. 6) proposed tackling this question with a particle-particle t-matrix approximation introduced by Thouless,[7] and previously employed by Nozieres and Schmitt-Rink (NS) in their study of the crossover in three dimensions.[4] The t-matrix approximation goes beyond the BCS mean-field theory by including independent (Gaussian) pairing fluctuations in the normal electron gas.

In the context of this approximation, SVR suggested that the bound state of the two-particle problem might manifest itself in the pair susceptibility and t matrix for q near $2k_F$. To understand the physical picture behind this suggestion, consider the two-particle bound-state wave function in the momentum representation, $\psi(\mathbf{k}+\mathbf{q}/2, -\mathbf{k}+\mathbf{q}/2)$, where $2\mathbf{k}$ is the relative momentum of the particles and $\mathbf{q}$ is the total momentum of the bound state. For a weak attractive interaction, the bound-state radius ξ_0 will be large, and hence the momentum space wave function ψ will be localized near the origin in $\mathbf{k}$, with width $\Delta k \approx 1/\xi_0$. In the many-particle case, if $k_F\xi_0 \gg 1$ then the occupied states inside the Fermi surface will effectively block the construction of the static ($q=0$) bound state, and unless the two-particle binding energy is comparable to the Fermi energy, the Fermi-liquid ground state will be stable against the formation of independently bound pairs (as opposed to Cooper pairs) with small total momentum.

Alternatively, one can imagine applying a Galilean transformation to move the bound state outside of the Fermi surface ($q \gg 2k_F$); for two isolated particles with a Galilean invariant interaction the energy of the bound state is increased by $q^2/4m$, while in the many-particle case the extra cost in free energy is $q^2/4m - 2\mu$. For $q \rightarrow 2k_F$ this free energy cost goes to zero, which might suggest an instability, but for $q - 2k_F < 1/\xi_0$ the occupied states inside the Fermi surface can no longer be neglected, and hence a detailed calculation is needed to determine whether a weak (relative to the Fermi energy) attractive interaction leads to bound states that destabilize the Fermi liquid or significantly influence its properties.

In the remainder of this paper I examine this question within the framework suggested by SVR. In Sec. II, I clarify the formal content of the NS approximation employed by SVR. I show that the NS scheme amounts to calculating the number density to first order in the particle-particle t-matrix self-energy, and I argue that if the fluctuation corrections are significant, a better approach might be to calculate the density from the Green's function obtained by solving Dyson's equation, which includes the self-energy to all orders. In Sec. III, I analyze the pair susceptibility at $T=0$ for imaginary frequencies and $q \approx 2k_F$. I conclude that in this limit the noninteracting pair susceptibility is bounded, and hence in weak coupling the t matrix is also bounded and does not signal a breakdown of the Fermi liquid picture on account of true two-particle bound states.

II. THE t-MATRIX APPROXIMATION AND THE NS SCHEME

The particle-particle t-matrix, or ladder diagram, approximation for the thermodynamic potential of a normal Fermi liquid was first studied by Thouless, who showed that for a separable electron-electron interaction with matrix elements

$$\langle \mathbf{k}+\mathbf{q}/2, -\mathbf{k}+\mathbf{q}/2 | V | \mathbf{k}'+\mathbf{q}/2, -\mathbf{k}'+\mathbf{q}/2 \rangle = -g v(\mathbf{k})^* v(\mathbf{k}') \; , \quad (1)$$

the contribution to the thermodynamic potential from the sum of all particle-particle ladder diagrams evaluated with noninteracting Green's functions is given (for free electrons in two dimensions) by

$$\Omega_L = T \sum_m \int \frac{d\mathbf{q}}{(2\pi)^2} \{ \ln[1 - gQ_0(\mathbf{q},\omega_m)] + gQ_0(\mathbf{q},\omega_m) \} \; , \quad (2)$$

where the free-electron "pair susceptibility" $Q_0(\mathbf{q},\omega_m)$ is

$$Q_0(\mathbf{q},\omega_m)=T\sum_n\int\frac{d\mathbf{k}}{(2\pi)^2}|v(\mathbf{k})|^2G_0(\mathbf{k}+\mathbf{q}/2,\varepsilon_n+\omega_m)G_0(-\mathbf{k}+\mathbf{q}/2,\varepsilon_n) \quad (3a)$$

$$=\int\frac{d\mathbf{k}}{(2\pi)^2}|v(\mathbf{k})|^2\frac{\tanh(\xi_{\mathbf{k}+\mathbf{q}/2}/2T)+\tanh(\xi_{\mathbf{k}-\mathbf{q}/2}/2T)}{2(\xi_{\mathbf{k}+\mathbf{q}/2}+\xi_{\mathbf{k}-\mathbf{q}/2}-i\omega_m)}\ , \quad (3b)$$

and $\xi_{\mathbf{k}}=\varepsilon_{\mathbf{k}}-\mu$ is the single-electron energy measured from the chemical potential.[7] The particle-particle t matrix in the ladder approximation is $-g$ times the inverse of the argument of the logarithm in Eq. (2). An equivalent representation of this contribution to the free energy is given by Gaussian fluctuations of the pair field in a functional integral representation of the partition function, as was first pointed out by Langer.[8] The BCS pairing instability corresponds to a singularity of the $q=0$, $\omega_m=0$ term in Ω_L, and the terms with $q\ll k_F$, $\omega_m=0$ were shown by Thouless to yield a fluctuation specific heat proportional to $(T-T_c)^{-1/2}$ in three dimensions, which we now recognize as the expected result for Gaussian fluctuations.

Equations (2) and (3a) can also form the basis of a fully renormalized conserving approximation for the free energy and Green's function.[9,10] If the functional $\Phi[G]$ is defined by

$$\Phi[G]=T\sum_m\int\frac{d\mathbf{q}}{(2\pi)^2}\{\ln[1-gQ(\mathbf{q},\omega_m)]+gQ(\mathbf{q},\omega_m)\}\ , \quad (4)$$

with

$$Q(\mathbf{q},\omega_m)=T\sum_n\int\frac{d\mathbf{k}}{(2\pi)^2}|v(\mathbf{k})|^2G(\mathbf{k}+\mathbf{q}/2,\varepsilon_n+\omega_m)\times G(-\mathbf{k}+\mathbf{q}/2,\varepsilon_n)\ , \quad (5)$$

then the corresponding self-consistent approximation for the free energy is given by the functional

$$\Omega[\Sigma,G]=-2T\sum_n\int\frac{d\mathbf{k}}{(2\pi)^2}\exp(i\varepsilon_n\eta)\{\Sigma(\mathbf{k},\varepsilon_n)G(\mathbf{k},\varepsilon_n)+\ln[-G_0(\mathbf{k},\varepsilon_n)^{-1}+\Sigma(\mathbf{k},\varepsilon_n)]\}+\Phi[G] \quad (6)$$

evaluated at its stationary point with respect to variations of both G and Σ. At this stationary point G, Σ, and Φ are related by

$$G(\mathbf{k},\varepsilon_n)=[G_0(\mathbf{k},\varepsilon_n)^{-1}-\Sigma(\mathbf{k},\varepsilon_n)]^{-1}\ , \quad (7)$$

$$\Sigma(\mathbf{k},\varepsilon_n)=\tfrac{1}{2}\delta\Phi[G]/\delta G(\mathbf{k},\varepsilon_n)\ . \quad (8)$$

SVR have suggested that at temperatures above the BCS transition temperature the presence of true bound pairs in the normal Fermi liquid might be apparent in contributions to the number density from pair fluctuations with $q\approx 2k_F$. The electron density as a function of T and μ is given in terms of the thermodynamic potential by

$$n(T,\mu)=-\partial\Omega/\partial\mu\ , \quad (9)$$

and if one calculates $n(T,\mu)$ from $\Omega[\Sigma,G]$ and exploits the stationary properties of this functional, one immediately obtains the expected result,

$$n(T,\mu)=2T\sum_n\int\frac{d\mathbf{k}}{(2\pi)^2}G(\mathbf{k},\varepsilon_n)\exp(i\varepsilon_n\eta)\ , \quad (10)$$

with G given by Eq. (7).

If G and Σ are not calculated self-consistently, then one obtains two different results for $n(T,\mu)$, depending on whether G is replaced by G_0 in $\Omega[\Sigma,G]$ before or after differentiating with respect to μ. Nozieres and Schmitt-Rink approximate the fluctuation correction to the density by $-\partial\Omega_L/\partial\mu$, which is equivalent to approximating $\Omega[\Sigma,G]$ by $\Omega[0,G_0]$ before differentiating. In practice, NS replace the frequency sum in Ω_L by the standard contour integral representation, and deform the contour to lie along the real axis before differentiating (numerically) with respect to μ. An alternative (but formally equivalent) procedure is to retain the explicit frequency sum, and to evaluate the derivative

$$-\partial\Omega_L/\partial\mu=-\partial\Phi[G_0]/\partial\mu$$

by first differentiating Φ with respect to G_0 using Eq. (8), and then differentiating G_0 with respect to μ. The result is

$$n(T,\mu)=n_0+2T\sum_n\int\frac{d\mathbf{k}}{(2\pi)^2}G_0(\mathbf{k},\varepsilon_n)\Sigma_0(\mathbf{k},\varepsilon_n)G_0(\mathbf{k},\varepsilon_n)\ , \quad (11)$$

where $\Sigma_0(\mathbf{k},\varepsilon_n)$ is the particle-particle t-matrix self-energy, evaluated with the free-particle Green's function,

$$\Sigma_0(\mathbf{k},\varepsilon_n)=\tfrac{1}{2}\delta\Phi[G_0]/\delta G_0(\mathbf{k},\varepsilon_n)\ . \quad (12)$$

The other approach is to approximate G by G_0 in the self-consistent result for $n(T,\mu)$ given by Eqs. (7), (8), and (10). In this case one finds

$$n(T,\mu)=2T\sum_n\int\frac{d\mathbf{k}}{(2\pi)^2}\exp(i\varepsilon_n\eta)\times[G_0^{-1}(\mathbf{k},\varepsilon_n)-\Sigma_0(\mathbf{k},\varepsilon_n)]^{-1}\ . \quad (13)$$

Equations (11) and (13) differ because Ω_L is not a conserving approximation, as pointed out long ago by Baym.[10] In fact, the NS result is just Eq. (13) expanded to first order in the self-energy Σ_0. The two approximation are equivalent when the self-energy corrections are small, but if the corrections are not small, Eq. (13) might yield a sensible result when the NS approach does not. Physically, Eq. (13) sums all repeated scatterings of an electron by independent pair fluctuations, but omits vertex corrections and interactions between fluctuations. NS of course omit the latter, but they also miss all the repeated scatterings by independent fluctuations. Whatever the merit of (13) relative to (11), it is clear from the latter that the NS scheme is only meaningful when it yields small corrections to the unperturbed density or chemical potential.

III. THE t MATRIX FOR $q \approx 2k_F$

The properties of the particle-particle t matrix and the free energy in the ladder-diagram approximation are governed by the pair susceptibility $Q_0(\mathbf{q},\omega_m)$, given by Eq. (3). In particular, an unconditional breakdown of adiabatic continuity with the noninteracting electron gas requires that Q_0 diverge as a function of at least one of its arguments, the classic case being the BCS instability, which appears as a $\ln(T)$ singularity in $Q_0(0,0)$.

To study the possibility of anomalous behavior near $q=2k_F$ it will suffice to approximate the "form factor" $v(\mathbf{k})$ by a simple cutoff for $\varepsilon_k > \varepsilon_c$, with $\varepsilon_c \sim k_F^2/2m$. The precise form of the cutoff is unimportant, because the possible anomalies of interest here are cutoff independent. Since the free-electron density of states in two dimensions is a constant, $N_0=m/2\pi$, with this choice for $v(\mathbf{k})$, Eq. (3) simplifies to

$$Q_0(\mathbf{q},\omega_m)=N_0\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{\varepsilon_c}d\varepsilon_{\mathbf{k}}\frac{\tanh(\xi_{\mathbf{k}+\mathbf{q}/2}/2T)+\tanh(\xi_{\mathbf{k}-\mathbf{q}/2}/2T)}{2(\xi_{\mathbf{k}+\mathbf{q}/2}+\xi_{\mathbf{k}-\mathbf{q}/2}-i\omega_m)}\ , \tag{14}$$

with $\cos(\phi)=\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}$. With the free-electron dispersion relation, the denominator becomes

$$\xi_{\mathbf{k}+\mathbf{q}/2}+\xi_{\mathbf{k}-\mathbf{q}/2}-i\omega_m=2(\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2}-i\omega_m/2)\ , \tag{15}$$

while standard identities applied to the numerator yield

$$\tanh(\xi_{\mathbf{k}+\mathbf{q}/2}/2T)+\tanh(\xi_{\mathbf{k}-\mathbf{q}/2}/2T)=2\tanh[(\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2})/T]/\{\cosh[kq\cos(\phi)/2mT]\operatorname{sech}[(\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2})/T]+1\}\ . \tag{16}$$

Equation (15) alone suggests that $Q_0(\mathbf{q},\omega_m)$ might have logarithmic singularities for $\omega_m=0$ associated with the vanishing of this denominator, and integrating (14) by parts does yield a singular contribution to the integrated part,

$$(N_0/2)\ln(\xi_{\mathbf{q}/2}-i\omega_m/2)\tanh(\xi_{\mathbf{q}/2}/2T)\ , \tag{17}$$

which one might be tempted to identify as the singular part of the pair susceptibility.

I will show that this identification is incorrect, and that $Q_0(\mathbf{q},\omega_m)$ is bounded for all ω_m and all T when $q\neq 0$. I first observe that nonanalytic behavior of $Q_0(\mathbf{q},\omega_m)$ associated with the vanishing of the denominator of (14) can only occur for $\omega_m=0$. Furthermore, from (15) and (16) it is obvious that the integrand of (14) is always positive for $\omega_m=0$, and hence

$$|Q_0(\mathbf{q},\omega_m)|\le Q_0(\mathbf{q},0)\ ,$$

as first pointed out by Thouless. Therefore it is sufficient to consider $Q_0(\mathbf{q},0)$. For $T>0$ the singularity of (17) occurs for

$$\xi_{\mathbf{q}/2}\to 0\ (q\to 2k_F)\ ,$$

and is of the form

$$(\xi_{\mathbf{q}/2}/2T)\ln(|\xi_{\mathbf{q}/2}|)\ .$$

But whenever

$$|\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2}|\ll T\ ,$$

the factor

$$\tanh[(\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2})/T]$$

in (16) cancels the potentially troublesome denominator, so that there can be no singularity of $Q_0(\mathbf{q},0)$ in this limit, not even one as mild as the $x\ln(|x|)$ implied by (17). This still leaves open the possibility of a singularity of $Q_0(\mathbf{q},0)$ for $\xi_{\mathbf{q}/2}\to 0$ at $T=0$; in this limit the singularity of (17) is of the form $\operatorname{sgn}(x)\ln(|x|)$. The $T=0$ limit of the occupation factor given by Eq. (16) is easily seen to be

$$2\operatorname{sgn}(\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2})\Theta(|\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2}|-|kq\cos(\phi)/2m|)\ , \tag{18}$$

and in this case the energy integrations in Eq. (14) can be carried out analytically. To simplify notation, I introduce dimensionless variables,

$$u=(\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2})/\varepsilon_F,\quad u_c=(\varepsilon_c+\xi_{\mathbf{q}/2})/\varepsilon_F\ ,$$
$$\delta=\xi_{\mathbf{q}/2}/\varepsilon_F,\quad \alpha=|q\cos(\phi)/k_F|\ , \tag{19}$$
$$y=\omega_m/2\varepsilon_F\ ,$$

in terms of which the pair susceptibility becomes

$$Q_0(\mathbf{q},\omega_m)=(N_0/2)\int_0^{2\pi}\frac{d\phi}{2\pi}I(\delta,\alpha,y)\ , \tag{20}$$

$$I(\delta,\alpha,y)=\int_\delta^{u_c}du\frac{\operatorname{sgn}(u)\Theta[g(u)]}{u-iy}\ , \tag{21}$$

with

$$g(u)=|u|-\alpha(u-\delta)^{1/2}\ . \tag{22}$$

The integrals in (21) are elementary, and the only challenge is keeping track of the limits of integration and sign changes implied by the numerator. In particular, one needs to know the zeros of $g(u)$, the argument of the Θ function. These zeros occur at either

$$u_+=(\alpha^2/2)[1+(1-4\delta/\alpha^2)^{1/2}] \tag{23}$$

or

$$u_-=(\alpha^2/2)[1-(1-4\delta/\alpha^2)^{1/2}]\ , \tag{24}$$

and the appropriate root (if any) can easily be identified graphically. In the limit of interest, $\delta\to 0$, these roots behave as

$$u_+ \to \alpha^2 = [q\cos(\phi)/k_F]^2 \ , \tag{25a}$$

$$u_- \sim \delta \to 0 \ . \tag{25b}$$

First consider the case $\delta < 0$ ($q < 2k_F$). In this case,

$$I(\delta,\alpha,y) = I^{(+)}(\delta,\alpha,y) - I^{(-)}(\delta,\alpha,y) \ , \tag{26}$$

where

$$I^{(+)}(\delta,\alpha,y) = \int_0^{u_c} du \frac{\Theta[g(u)]}{u-iy} \ , \tag{27}$$

and

$$I^{(-)}(\delta,\alpha,y) = \int_\delta^0 du \frac{\Theta[g(u)]}{u-iy} \ . \tag{28}$$

To evaluate $I^{(+)}(\delta,\alpha,y)$, observe that

$$g(0) = -\alpha|\delta|^{1/2} \le 0 \ ,$$

and that $g(u)$ changes sign at u_+. Thus the effective lower limit of the integral is u_+, and one has

$$\mathrm{Re}I^{(+)}(\delta,\alpha,y) = \tfrac{1}{2}\ln\frac{u_c^2+y^2}{u_+^2+y^2} \ , \tag{29}$$

$$\mathrm{Im}I^{(+)}(\delta,\alpha,y) = \tan^{-1}(u_c/y) - \tan^{-1}(u_+/y) \ . \tag{30}$$

These are both well behaved for all y when $\delta\to 0-$. At the lower limit of integration in $I^{(-)}(\delta,\alpha,y)$, $g(\delta)=|\delta| > 0$, and hence $g(u)$ must change sign between the limits of integration; the appropriate root must be u_-, which thus becomes the effective upper limit of the integral, so that

$$\mathrm{Re}I^{(-)}(\delta,\alpha,y) = \tfrac{1}{2}\ln\frac{u_-^2+y^2}{\delta^2+y^2} \ , \tag{31}$$

$$\mathrm{Im}I^{(-)}(\delta,\alpha,y) = \tan^{-1}(u_-/y) - \tan^{-1}(\delta/y) \ . \tag{32}$$

These are also well behaved for all y when $\delta\to 0-$, as is evident from the limiting form of u_- given in Eq. (25b).

Finally, consider the case $\delta > 0$. Then $u > 0$ throughout the range of integration, and the energy integral is simply

$$I(\delta,\alpha,y) = \int_\delta^{u_c} du \frac{\Theta[g(u)]}{u-iy} \ . \tag{33}$$

Now at the lower limit $g(\delta)=\delta > 0$, and $g(u)$ is clearly also positive for large u. If $\alpha^2 < 4\delta$, then $g(u)$ remains positive for all $u > \delta$, while if $\alpha^2 > 4\delta$, then $g(u)$ is negative between u_- and u_+, and this interval must be removed from the range of integration. Hence the energy integrations yield

$$\mathrm{Re}I(\delta,\alpha,y) = \tfrac{1}{2}\left[\ln\frac{u_c^2+y^2}{\delta^2+y^2} + \Theta(\alpha^2-4\delta)\ln\frac{u_-^2+y^2}{u_+^2+y^2}\right] \ , \tag{34}$$

$$\mathrm{Im}I(\delta,\alpha,y) = \tan^{-1}(u_c/y) - \tan^{-1}(\delta/y) + \Theta(\alpha^2-4\delta)[\tan^{-1}(u_-/y) - \tan^{-1}(u_+/y)] \ . \tag{35}$$

At first sight it appears that for $y=0$ Eq. (34) will lead to a singularity in $Q_0(\mathbf{q},0)$ of the form $\log(\xi_{\mathbf{q}/2})$, consistent with Eq. (17) for $\xi_{\mathbf{q}/2} > 0$, but there is actually no divergence, because $u_- \sim \delta$ for $\delta\to 0$. After combining the divergent parts of both terms in (34), what remains of the divergence is

$$\Theta(2\delta^{1/2}-\alpha)\ln(\delta) \ , \tag{36}$$

and hence for this contribution the integration over ϕ in Eq. (20) for $Q_0(\mathbf{q},0)$ is restricted to

$$|\cos(\phi)| < (8m\xi_{\mathbf{q}/2})^{1/2}/q \ . \tag{37}$$

For $\delta\to 0+$ this condition is satisfied for $\phi\approx\pi/2$ and $\phi\approx 3\pi/2$, and each neighborhood makes the same contribution to $Q_0(\mathbf{q},0)$. For $\phi\approx\pi/2$, in terms of the angle θ defined by $\phi=\pi/2+\theta$, condition (37) becomes simply

$$|\theta| < (2m\xi_{\mathbf{q}/2})^{1/2}/q \ ,$$

and hence for $\delta\to 0+$ the potentially divergent part of $Q_0(\mathbf{q},0)$ is

$$(N_0/\pi)(\xi_{\mathbf{q}/2}/\varepsilon_F)^{1/2}\ln(\xi_{\mathbf{q}/2}/\varepsilon_F) \ . \tag{38}$$

Thus $Q_0(\mathbf{q},\omega_m)$ is bounded for $q\approx 2k_F$.

There is a simple physical picture behind the contribution to $Q_0(\mathbf{q},0)$ given by Eq. (38). Consider the two momenta $\mathbf{q}/2+\mathbf{k}$ and $\mathbf{q}/2-\mathbf{k}$ for $\mathbf{q}$ approaching the Fermi surface from above. Except when $\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}=0$, for any fixed $k>0$, as $q\to 2k_F$ either $\mathbf{q}/2+\mathbf{k}$ or $\mathbf{q}/2-\mathbf{k}$ will pass through the Fermi surface, and thus be blocked from participating in a bound state. For a given $q > 2k_F$, the directions of $\mathbf{k}$ for which neither $\mathbf{q}/2+\mathbf{k}$ nor $\mathbf{q}/2-\mathbf{k}$ ever (no matter what the length of $\mathbf{k}$) intersects the Fermi surface all lie within a wedge whose angular width is given precisely by Eq. (37). For $q\to 2k_F$ the momenta within this wedge give singular contributions to $Q_0(\mathbf{q},0)$ proportional to $\ln(\xi_{\mathbf{q}/2}/\varepsilon_F)$, but the angular width of the wedge vanishes like $(\xi_{\mathbf{q}/2}/\varepsilon_F)^{1/2}$ to give, finally, the weak singularity in Eq. (38).

Calculating $Q_0(\mathbf{q},0)$ for $q\to 2k_F$ is thus a subtle matter, and requires a careful treatment of the angular integrals in Eq. (14). A numerical integration can easily give spurious results, as can an approximate treatment of the angular dependence of the integrand. For example, if one were to replace $\xi_{\mathbf{k}+\mathbf{q}/2}$ and $\xi_{\mathbf{k}-\mathbf{q}/2}$ by $\varepsilon_{\mathbf{k}}+\xi_{\mathbf{q}/2}$ in the numerator of Eq. (14), this would be equivalent to assuming that $\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}=0$ always, while this is actually the one point in the ϕ integral where the occupation factors never enter to block the singularity. The result of this seemingly innocuous approximation is thus to replace Eq. (38) by an unmodified $\ln(\xi_{\mathbf{q}/2}/\varepsilon_F)$ singularity, as can be seen formally by noting that this amounts to assuming that for all ϕ, $I(\delta,\alpha,y)$ is given by Eq. (34) with $\alpha=0$.

These results do not suggest that bound pairs with $q > 2k_F$ lead to anomalous properties of a degenerate

Fermi liquid with a weak attractive interaction. In particular, there is no weak-coupling instability of the normal Fermi liquid at or near $T=0$ due to pair fluctuations with $q>2k_F$. These fluctuations will of course contribute to the low-temperature thermodynamic properties at some order in T, but this does not indicate a breakdown of Fermi liquid theory unless the fluctuation contributions enter at the same or lower order in T as do the quasiparticle contributions. The situation here is not obviously different than for particle-hole excitations such as zero sound in a neutral liquid or incoherent spin fluctuations. I hope to return to the delicate problem of the leading corrections to thermodynamic properties due to large-q pair fluctuations in a subsequent paper. I also emphasize that the calculations reported here are relevant to the weak-coupling limit, and have no direct bearing on the crossover between BCS pairing and true Bose condensation when the pair binding energy becomes comparable to the Fermi energy.

After the original version of this paper was submitted, SVR showed that the $\omega=0$ pair susceptibility is not only bounded, but analytic, for all $q\neq 0$.[11] Their result is fully consistent with the calculations and conclusions reported here. In particular, after evaluating the remaining (explicitly nondivergent) contributions to the susceptibility from Eq. (34), I find the same result as SVR, up to cutoff-dependent terms, which are treated consistently in my calculation.

ACKNOWLEDGMENTS

I have benefited from discussions with N. E. Bickers, T. Kopp, and J. A. Sauls. This work was begun during a visit to the Institute for Theoretical Physics, Santa Barbara, supported in part by the National Science Foundation under Grant No. PHY82-17853, supplemented by funds from the National Aeronautics and Space Administration.

[1]D. M. Eagles, Phys. Rev. **186**, 456 (1969).
[2]A. J. Leggett, in *Modern Trends in the Theory of Condensed Matter,* edited by A. Pekalski and J. Przystawa (Springer-Verlag, Berlin, 1980).
[3]K. Miyake, Prog. Theor. Phys. **69**, 1794 (1983).
[4]P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
[5]M. Randeria, J.-M. Duan, and L.-Y. Shieh, Phys. Rev. Lett. **62**, 981 (1989).
[6]S. Schmitt-Rink, C. M. Varma, and A. E. Ruckenstein, Phys. Rev. Lett. **63**, 445 (1989).
[7]D. J. Thouless, Ann. Phys. (N.Y.) **10**, 553 (1960).
[8]J. S. Langer, Phys. Rev. **134**, 553 (1964).
[9]J. M. Luttinger and J. C. Ward, Phys. Rev. **118**, 1417 (1960).
[10]G. Baym, Phys. Rev. **127**, 1391 (1962).
[11]S. Schmitt-Rink, C. M. Varma, and A. E. Ruckenstein (private communication).