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Optical and magneto-optical absorption in parabolic quantum wells

L. Brey, N. F. Johnson, and B. I. Halperin
Physics Department, Harvard University, Cambridge, Massachusetts 02138
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We show that an n-doped parabolic quantum well absorbs far infrared radiation at the bare harmonic-oscillator frequency ω_0 *independently* of the electron-electron interaction and the number of electrons in the well. In the presence of a magnetic field tilted with respect to the plane of the quantum well, we find that the cyclotron resonance becomes coupled to this ω_0 frequency mode. The absorption then occurs at two frequencies, which are again independent of the electron-electron interaction and the fractional filling of the well.

Wide parabolic quantum wells have been proposed[1–4] as structures in which a high-mobility quasi-three-dimensional electron gas can be realized. A parabolic potential of width W and height Δ_1 is equivalent to the potential created by a uniform slab of positive charge with a thickness W and density $n_0=2\epsilon\Delta_1/W^2e^2\pi$, where e is the electron charge and ϵ is the dielectric constant (taken to be constant in the well). Electrons, which arise from donor impurities located away from the well, enter the well and screen this "fictitious" potential, forming a uniform layer of density n_0 (see Fig. 1). By increasing the number of electrons per unit area n_s between zero and n_0W, the thickness of the electron layer will increase linearly between zero and W, and a thick slab of electrons (>2000 Å) can be achieved.

These wide parabolic quantum wells have recently been grown[2,3] by tailoring the conduction-band edge of a graded $Ga_{1-x}Al_xAs$ semiconductor, and magnetotransport experiments on these systems[2,3] confirm the existence of a thick slab of high-mobility electron gas. In addition Karrai *et al.*[5] have reported measurements[5] of far-infrared magnetotransmission in these parabolic wells. These experiments reveal a coupling between the cyclotron resonance and a frequency Ω_x, as the magnetic field is tilted with respect to the electron-slab plane. The experiments suggest that Ω_x corresponds to the plasma frequency $[\Omega_x=(4\pi n_0e^2/\epsilon m^*)^{1/2}]$ of a three-dimensional electron gas of density n_0, which *by construction* is equal to the frequency of the bare harmonic oscillator potential. This frequency Ω_x is very different from the energy separation between the subbands of the self-consistent Hartree potential. Karrai *et al.*[5] therefore conclude that the three-dimensional character of the electron gas tends to predominate over the two-dimensional properties.[5]

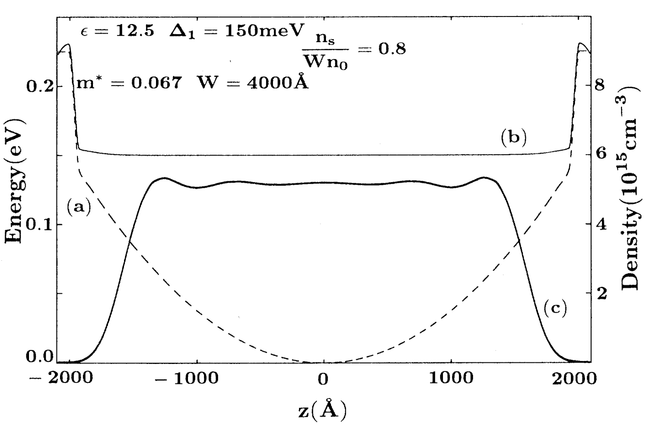

FIG. 1. Calculated electron density profile and self-consistent potential for an 80% full $Al_xGa_{1-x}As$ parabolic well of total width 4000 Å. (a) Potential for the empty well. (b) Self-consistent total potential in the Hartree approximation. (c) Self-consistent charge density profile as obtained in the Hartree approximation. The parameters of the well are given in the figure.

In this paper we study the optical absorption of an n-doped parabolic quantum well. In addition we investigate the coupling between the excitations of this system and the cyclotron energy in the presence of a magnetic field which is tilted with respect to the quantum-well plane. We find that in the ideal parabolic well the absorption spectrum is independent of the electron-electron interaction, and also *independent of the number of electrons in the well.*

First, in the absence of any magnetic field, the Hamiltonian of our system (with electrons in the x-y plane) is given within the effective-mass approximation by

$$H=\frac{1}{2m^*}\sum_{i=1}^{N}(p_{i,x}^2+p_{i,y}^2+p_{i,z}^2)+\sum_{i=1}^{N}\frac{\omega_0^2m^*}{2}z_i^2+U\,, \quad (1)$$

where $\mathbf{p}_i$ and $\mathbf{r}_i$ are respectively the momentum and position operators of the ith particle, m^* is the electron effective mass of the host semiconductor, $\omega_0=(8\Delta_1/W^2m^*)^{1/2}$ is the bare harmonic-oscillator frequency of the parabolic well, and U is the interaction between electrons

$$U\equiv\sum_{i<j}u(\mathbf{r}_i-\mathbf{r}_j)\,.$$

We define raising and lowering operators,

$$\hat{c}^{\pm}\equiv\sum_{i=1}^{N}(m^*\omega_0z_i\mp ip_{i,z})\,. \quad (2)$$

It then follows from the quadratic form of the man-made potential that

$$[H,\hat{c}^{\pm}]=\pm\hbar\omega_0\hat{c}^{\pm}\,. \quad (3)$$

If Ψ_n is an eigenstate of H with eigenenergy E_n, Eq. (3) implies

$$H\hat{c}^{\pm}\Psi_n=(\pm\hbar\omega_0+E_n)\hat{c}_{\pm}\Psi_n\,. \quad (4)$$

Defining $\Psi_{n\pm1}\equiv\hat{c}^{\pm}\Psi_n$, it is clear that $\Psi_{n\pm1}$ is an exact eigenstate of our Hamiltonian with energy $E_{n\pm1}=E_n$

$\pm \hbar\omega_0$.

When the system is placed in an electric field E applied in the z direction, the following term must be added to the Hamiltonian:

$$H' = \sum_{i=1}^{N} Ee^{-i\omega t}z_i = E\frac{e^{-i\omega t}}{2m^*\omega_0}(\hat{c}^+ + \hat{c}^-) . \quad (5)$$

This perturbation will only connect the state Ψ_N with the states $\Psi_{N\pm1}$. A sharp peak in the absorption spectra is therefore expected at the frequency ω_0. This result is not affected by the electron-electron interaction U. The parabolic well consequently absorbs light at the frequency ω_0 independently of the number of electrons in the well. We conclude that in the case of a parabolic quantum well the dielectric function diverges at the bare harmonic-oscillator frequency ω_0, and not at the frequencies of the intersubband separation obtained in a Hartree or more sophisticated calculation.

The bare harmonic-oscillator frequency ω_0 is equal *by construction* to the plasma frequency of a three-dimensional electron gas of density n_0.[6] Therefore the appearance in optical-absorption experiments of a peak at this energy does not imply that the system has a three-dimensional behavior, but rather that the bare potential is parabolic. In the three-dimensional limit when the thickness of the electron slab is comparable to the wavelength of incoming light (frequency ω_0), Eq. (5) is no longer adequate to describe the interaction of the light with the system. Retardation effects[7] must be taken into account. In this thick-slab limit the peak at ω_0 in the light absorption spectra disappears and the Drude-type behavior of a bulk three-dimensional electron gas is restored.[8] However, the experimental samples are far from this limit and Eq. (5) holds to a good approximation.

Next we consider the optical absorption of the parabolic quantum well in the presence of a magnetic field $\mathbf{B}=(B\sin\theta,0,B\cos\theta)$ tilted at an angle θ with respect to the z direction. Choosing the gauge $\mathbf{A}=(0,xB\cos\theta-zB\times\sin\theta,0)$, the Hamiltonian of the system becomes

$$H=\frac{1}{2m^*}\sum_{i=1}^{N}\{p_{i,x}^2+[p_{i,y}+(eB/c)(x\cos\theta-z\sin\theta)]^2+p_{i,z}^2\}+\sum_{i=1}^{N}\frac{\omega_0^2 m^*}{2}z_i^2+U . \quad (6)$$

This Hamiltonian can be simplified through a change of coordinates $x\to x'$ and $z\to z'$ corresponding to a rotation of angle α with respect to the y axis,[9,10]

$$H=\frac{1}{2m^*}\sum_{i=1}^{N}(p_{i,x'}^2+p_{i,y}^2+p_{i,z'}^2)+\frac{m^*}{2}\sum_{i=1}^{N}(\omega_1^2x_i'^2+\omega_2^2z_i'^2)+U+\omega_c\sum_{i=1}^{N}[\sin(\alpha-\theta)z_i'p_{i,y}+\cos(\alpha-\theta)x_i'p_{i,y}] . \quad (7)$$

The rotation angle α is obtained from

$$\tan(2\alpha)\equiv\frac{\omega_c^2\sin2\theta}{\omega_c^2\cos2\theta-\omega_0^2} . \quad (8)$$

The frequencies $\omega_{1,2}$ are

$$\omega_{1,2}\equiv[\tfrac{1}{2}(\omega_c^2+\omega_0^2)\pm\tfrac{1}{2}(\omega_c^4+\omega_0^4-2\omega_0^2\omega_c^2\cos2\theta)^{1/2}]^{1/2} , \quad (9)$$

and $\omega_c=eB/m^*c$ is the cyclotron frequency. Defining the raising and lowering operators

$$\hat{a}^{\pm}\equiv\sum_{i=1}^{N}\left[m^*\omega_1x_i'\mp ip_{i,x'}+\frac{\omega_c}{\omega_1}\cos(\alpha-\theta)p_{i,y}\right] ,$$
$$\hat{b}^{\pm}\equiv\sum_{i=1}^{N}\left[m^*\omega_2z_i'\mp ip_{i,z'}+\frac{\omega_c}{\omega_2}\sin(\alpha-\theta)p_{i,y}\right] , \quad (10)$$

it follows from Eq. (7) that

$$[\hat{a}^{\pm},\hat{b}^{\pm}]=0 ,$$
$$[H,\hat{a}^{\pm}]=\pm\hbar\omega_1\hat{a}^{\pm} , \quad (11)$$
$$[H,\hat{b}^{\pm}]=\pm\hbar\omega_2\hat{b}^{\pm} .$$

If Ψ_{n_1,n_2} is an eigenstate of H with eigenvalue E_{n_1,n_2}, then

$$H\hat{a}^{\pm}\Psi_{n_1,n_2}=(\pm\hbar\omega_1+E_{n_1,n_2})\hat{a}^{\pm}\Psi_{n_1,n_2} ,$$
$$H\hat{b}^{\pm}\Psi_{n_1,n_2}=(\pm\hbar\omega_2+E_{n_1,n_2})\hat{b}^{\pm}\Psi_{n_1,n_2} . \quad (12)$$

Defining $\Psi_{n_1\pm1,n_2}\equiv\hat{a}^{\pm}\Psi_{n_1,n_2}$ and $\Psi_{n_1,n_2\pm1}\equiv\hat{b}^{\pm}\Psi_{n_1,n_2}$, we see that $\Psi_{n_1\pm1,n_2}$ and $\Psi_{n_1,n_2\pm1}$ are both exact eigenstates with energies $E_{n_1\pm1,n_2}=E_{n_1,n_2}\pm\hbar\omega_1$ and $E_{n_1,n_2\pm1}=E_{n_1,n_2}\pm\hbar\omega_2$, respectively. When illuminated with long wavelength light, polarized in the z direction (x direction), the optical absorption of the parabolic well will present two peaks at ω_1 and ω_2 with intensities proportional to $\sin^2\alpha$ ($\cos^2\alpha$) and $\cos^2\alpha$ ($\sin^2\alpha$), respectively. Once again this result is independent of the electron-electron interaction and is a direct consequence of the parabolic form of the bare quantum-well potential. If the term corresponding to the parabolic well is dropped from the Hamiltonian equation (6), then $\omega_1=\omega_c$, $\omega_2=0$ and we recover the Kohn result[11] that the cyclotron resonance ω_c in a bulk three-dimensional gas is not affected by the electron-electron interaction U.

In the experimental samples the width of the parabolic well is finite, and the well is confined by an additional barrier (see Fig. 1). This extra confinement will not affect our results as long as the number of electrons in the well is small enough so that the self-consistent charge-density profile remains essentially unchanged. We have checked this numerically in the case of no magnetic field by calculating the dynamical conductivity[12,13] for the potential profile shown in Fig. 1. For a density of electrons per unit area n_s bigger than $0.9Wn_0$, we find that small satellites appear in the optical absorption close to ω_0. Of course, impurities or other imperfections in the well can lead to additional absorption even at smaller values of n_s. Variation of m^* across the parabolic well will also give corrections to these results.

In conclusion, we have shown that an ideal parabolic well absorbs light at the bare harmonic-oscillator frequency $\omega_0 = (8\Delta_1/W^2m^*)^{1/2}$ *independently* of the electron-electron interaction. In addition, the presence of a tilted magnetic field couples the cyclotron frequency with ω_0 and *not* with the frequency corresponding to the self-consistent intersubband separation.

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