

## Optical-absorption spectra associated with impurities in a GaAs-(Ga,Al)As quantum well

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(Received 16 June 1989)

The optical-absorption spectra associated with transitions between the  $n = 1$  valence subband and the donor-impurity band and between the acceptor-impurity band and the  $n = 1$  conduction subband were calculated for both the infinite and the finite GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum wells. An absorption edge associated with transitions involving impurities at the center of the well and a peak related with impurities at the edges of the well were the main features observed in the theoretical spectra. Results obtained in the case of an infinite quantum well were rather different from those previously reported in the literature.

### I. INTRODUCTION

In the last 15 years, a considerable amount of work has been devoted to the knowledge of the unique physical properties associated with semiconductor superlattices and heterostructures.<sup>1</sup> Because of the potential device applications of these systems,<sup>2-4</sup> the understanding of the nature of impurity states associated with heterostructures is a subject of considerable technical and scientific relevance. Bastard<sup>5</sup> was the first to treat the problem of hydrogenic impurities in quantum wells. He considered an infinite-barrier quantum well and studied the hydrogenic-impurity states within a variational procedure: the binding energy of hydrogenic impurities was found to vary with the position of the impurity in the well and with the well thickness. This inhomogeneous broadening of the impurity levels results in the formation of an inhomogeneous, localized "impurity band"<sup>5</sup> in the semiconductor quantum well. In the same work, Bastard has also presented a study of the optical properties associated with impurities within the infinite-barrier approximation.

The pioneering work by Bastard<sup>5</sup> on impurity properties in quantum wells was followed by several other more detailed investigations. Mailhot *et al.*<sup>6</sup> and Greene and Bajaj<sup>7</sup> studied the energy spectrum of the ground state and the low-lying excited states for shallow impurities in quantum-well structures consisting of a single slab of GaAs sandwiched between two semi-infinite layers of Ga<sub>1-x</sub>Al<sub>x</sub>As, by taking into consideration the finite size of the potential barrier. The influence of effective-mass and dielectric-constant mismatches at the well interfaces was also considered by Mailhot *et al.*<sup>6</sup> Effects such as the nonparabolicity of the conduction and/or valence bands, the influence of the coupling of the top four valence bands in the case of acceptors, effects of spatially dependent screening, electron-phonon interactions, etc.,

were considered by several other authors.<sup>8-14</sup> Experimentally, Miller *et al.*<sup>15</sup> reported the first observation of impurity-related photoluminescence features attributed to the recombination of  $n = 1$  electrons with neutral acceptors in nonintentionally doped molecular-beam-epitaxy (MBE) -grown GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum wells. Various experimental measurements of the properties of donors in GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum wells have been reported.<sup>16-18</sup> Recent reviews on the subject<sup>19-23</sup> contain a detailed list of theoretical and experimental work on properties of hydrogenic impurities in quantum wells.

In this work we study the optical-absorption spectra associated with transitions between the valence and donor-impurity bands (or between the acceptor-impurity and conduction bands) for a GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As finite quantum well. Section II will be devoted to the presentation of some theoretical aspects and to the calculation of the transition probability per unit time associated with the band $\leftrightarrow$ impurity absorption spectra. Results and discussion are presented in Sec. III and our conclusions are in Sec. IV.

### II. THEORY

In the effective-mass approximation, the Hamiltonian of a shallow hydrogenic impurity in a GaAs quantum well sandwiched between two semi-infinite slabs of Ga<sub>1-x</sub>Al<sub>x</sub>As can be written as

$$H = -\frac{\hbar^2 \nabla^2}{2m^*} - \frac{e^2}{\epsilon_0[\rho^2 + (z - z_i)^2]^{1/2}} + V(z), \quad (2.1)$$

in which the barrier potential  $V(z)$  is taken as a square well of width  $L$  and height  $V_b$ . The size  $V_b$  of the barrier is taken to be 60% (40%) of the band-gap discontinuity  $\Delta E_g$  (eV) = 1.247x in the GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As heterostruc-

ture for the conduction (valence) band.<sup>24,25</sup> In the above expression,  $z_i$  is the position of the impurity with respect to the  $z=0$  origin chosen at the center of the well,  $r = [\rho^2 + (z - z_i)^2]^{1/2}$ , with  $\rho = (x^2 + y^2)^{1/2}$ , is the distance from the carrier to the impurity site,  $\epsilon_0 = 12.58$  is the static dielectric constant, and  $m^*$  is the effective mass of the band under consideration ( $m^* = m_c = 0.0665m_0$  for donors and  $m^* = m_v = 0.30m_0$  for acceptors<sup>26</sup>;  $m_0$  is the free-electron mass).

We use a variational procedure<sup>5</sup> and assume an impurity trial wave function of the form<sup>26</sup>

$$\Psi(\mathbf{r}) = \begin{cases} e^{k_1(z+L/2)} \Gamma(\rho, z, z_i, \lambda), & z \leq -L/2 \\ \alpha \cos(k_2 z) \Gamma(\rho, z, z_i, \lambda), & -L/2 \leq z \leq L/2 \\ e^{-k_1(z-L/2)} \Gamma(\rho, z, z_i, \lambda), & z \geq L/2 \end{cases} \quad (2.2)$$

where  $\lambda$  is a variational parameter,

$$k_1 = [2m^*(V_b - E_0)]^{1/2} / \hbar, \quad (2.3a)$$

$$k_2 = (2m^*E_0)^{1/2} / \hbar, \quad (2.3b)$$

$$\alpha = 1 / \cos(k_2 L / 2), \quad (2.3c)$$

$$\Gamma(\rho, z, z_i, \lambda) = \exp\{-[\rho^2 + (z - z_i)^2]^{1/2} / \lambda\}, \quad (2.3d)$$

and  $E_0 = E_0(V_b, L)$  is the ground-state energy of the above Hamiltonian without the impurity potential term. The impurity binding energy is given by  $E(L, z_i) = E_0 - \epsilon(L, z_i)$ , where  $\epsilon(L, z_i)$  is the impurity ground-state energy, minimized with respect to  $\lambda$ .

The transition probability per unit time for valence-to-donor transitions (associated with a single impurity located at  $z = z_i$ ) is proportional to the square of the matrix element of the electron-photon interaction  $H_{\text{int}}$  between the wave functions of the initial (valence) and final (impurity) states, i.e.,

$$W = \frac{2\pi}{\hbar} \sum_i |\langle f | H_{\text{int}} | i \rangle|^2 \delta(E_f - E_i - \hbar\omega) \quad (2.4)$$

with  $H_{\text{int}} = C \mathbf{e} \cdot \mathbf{p}$ , where  $\mathbf{e}$  is the polarization vector in the direction of the electric field of the radiation,  $\mathbf{p}$  is the momentum operator, and  $C$  is a prefactor which contains the photon vector potential.<sup>27</sup> The above matrix element may be written as<sup>28</sup>

$$J = \frac{2\pi}{\lambda \beta^4} \left[ 2 + \frac{2 \cos[2(\pi/L)z_i]}{(1+\nu^2)^2} + \frac{z_i \beta \nu^2}{1+\nu^2} e^{-L\beta/2} \sinh(\beta z_i) + \cosh(\beta z_i) e^{-L\beta/2} \left[ -\frac{\nu^2 L \beta}{2(1+\nu^2)} + \frac{2}{(1+\nu^2)^2} - 2 \right] \right], \quad (2.8)$$

with

$$\nu = \nu(k_\perp) = 2\pi / [L\beta(k_\perp)], \quad (2.9)$$

$$\beta = \beta(k_\perp) = (k_\perp^2 + \lambda^{-2})^{1/2}. \quad (2.10)$$

Notice that there are some differences between the above expression for  $J(z_i, \lambda, k_\perp(\omega))$  and the corresponding formula in Bastard's work.<sup>5</sup>

For the GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As finite quantum well, we have

$$\langle f | H_{\text{int}} | i \rangle \simeq C \mathbf{e} \cdot \mathbf{P}_{fi} S_{fi}, \quad (2.5a)$$

with

$$\mathbf{P}_{fi} = \frac{1}{\Omega} \int_{\Omega} d\mathbf{r} u_f^*(\mathbf{r}) \mathbf{p} u_i(\mathbf{r}), \quad (2.5b)$$

and

$$S_{fi} = \int d\mathbf{r} F_f^*(\mathbf{r}) F_i(\mathbf{r}), \quad (2.5c)$$

where  $\Omega$  denotes the volume of the unit cell,  $u_f$  ( $u_i$ ) is the periodic part of the Bloch state for the final (initial) state, and  $F_f$  ( $F_i$ ) is the envelope function for the final (initial) state.

For a GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum well of width  $L$ , the transition probability per unit time for valence-to-donor transitions (associated with a single impurity located at  $z = z_i$ ) is therefore given by

$$W_L(z_i, \omega) = W_0 \frac{1}{2} \left[ \frac{\alpha_0^*}{\alpha_0} \right]^2 \left[ \frac{m_v}{m_0} \right] (N_b^2 \alpha_0^*) \frac{N^2 J^2}{\alpha_0^{*3}} \Big|_{k_\perp} Y(\Delta), \quad (2.6)$$

where  $\alpha_0$  is the Bohr radius,  $\alpha_0^* = \hbar^2 \epsilon_0 / m^* e^2$  is the effective Bohr radius,  $Y(\Delta)$  is the step function,  $N = (1 / \langle \Psi | \Psi \rangle)^{1/2}$  is the normalization factor for the impurity wave function, and  $N_b$  is the normalization factor for the wave function associated with the first valence subband. In the above expression, we have

$$\Delta = \hbar\omega - \epsilon_g + E(L, z_i), \quad (2.7a)$$

$$k_\perp = (2m_v \Delta / \hbar^2)^{1/2}, \quad (2.7b)$$

$$\epsilon_g = E_g + E_{n=1}^c + E_{n=1}^v, \quad (2.7c)$$

$$W_0 = \frac{4m_0}{\hbar^3} \alpha_0^2 |C|^2 |\mathbf{e} \cdot \mathbf{P}_{fi}|^2, \quad (2.7d)$$

in which  $E_g$  is the bulk GaAs gap and  $E_{n=1}^c$  ( $E_{n=1}^v$ ) is the bottom (top) of the first conduction (valence) subband. Notice that the above expressions are for both finite and infinite GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum wells.

In the case of the infinite GaAs quantum well,<sup>5</sup>  $N_b^2 = 2/L$ , and  $N^2$  is given in Bastard's work.<sup>5</sup> We have, therefore, for the function  $J = J(z_i, \lambda, k_\perp(\omega))$  in Eq. (2.6),

$$J = 2\pi(L/2)^4 \frac{1}{\lambda\mu^3} \left[ \Phi - \mu \frac{\partial \Phi}{\partial \mu} \right], \quad (2.11)$$

where

$$\Phi = e^{-\mu \cosh(\mu x_i)} \left[ \frac{2}{\gamma_+ + \mu} + \alpha\alpha_b \frac{-\mu \cos(\gamma_+) + \gamma_+ \sin(\gamma_+)}{\gamma_+^2 + \mu^2} + \alpha\alpha_b \frac{-\mu \cos(\gamma_-) + \gamma_- \sin(\gamma_-)}{\gamma_-^2 + \mu^2} \right] + \alpha\alpha_b \left[ \frac{\mu \cos(\gamma_+ x_i)}{\gamma_+^2 + \mu^2} + \frac{\mu \cos(\gamma_- x_i)}{\gamma_-^2 + \mu^2} \right], \quad (2.12)$$

and

$$\mu = \mu(k_{\perp}) = \beta L / 2, \quad (2.13a)$$

$$\gamma_{\pm} = (k_2 \pm k_{2b})L / 2, \quad (2.13b)$$

$$\gamma_1 = (k_1 + k_{1b})L / 2, \quad (2.13c)$$

$$x_i = 2z_i / L. \quad (2.13d)$$

In the above expression  $k_{1b}$ ,  $k_{2b}$ , and  $\alpha_b$  are given by Eqs. (2.3a), (2.3b), and (2.3c) with the effective mass and barrier potential corresponding to the valence band.

Finally, for a homogenous distribution of impurities,<sup>5</sup> and assuming that the quantum-well thickness is much larger than the lattice spacing, one has for the total transition probability per unit of time

$$W_L(\omega) = \frac{1}{L} \int_{-L/2}^{L/2} dz_i W_L(z_i, \omega). \quad (2.14)$$

The case of transitions from an acceptor-impurity band to the first conduction subband is obtained by performing the change  $m_v \leftrightarrow m_c$  — and by exchanging the barrier potentials associated with the valence and conduction subbands — through Eqs. (2.6)–(2.14).

### III. RESULTS AND DISCUSSION

In this section we present the main results of our calculation. First, a schematic representation of a GaAs quantum well doped with an homogeneous distribution of donor impurities is shown in Fig. 1. There we show the edges for optical absorption from the first valence subband to the donor-impurity band ( $\hbar\omega_1$ ) and to the first conduction subband ( $\epsilon_g$ ). The transition  $\hbar\omega_2$  corresponds to absorption to an impurity level associated with impurities at the edges of the quantum well. In Fig. 1 we also show schematically the densities of states for the two first valence subbands and for the impurity band.

Figure 2 shows some of the properties of the transition probability per unit of time  $W_L(z_i, \omega)$  which is used in (2.14) to obtain the total probability  $W_L(\omega)$ . Results shown are for an infinite GaAs quantum well of width  $L = 100 \text{ \AA}$ . In Fig. 2(a) the absorption probabilities  $W_L(z_i, \omega)$  are shown for some fixed values of the impurity position and as a function of the difference between the photon energy  $\hbar\omega$  and the GaAs bulk gap  $E_g$ . It is clear that there is no absorption for  $\hbar\omega - E_g < E_1 = \hbar\omega_1 - E_g$  and that for a given photon frequency between  $\omega_1$  and  $\omega_2$  only part of the donor-impurity band will contribute to

the absorption. For a given  $z_i$ , the maxima of the absorption probability per unit time — maxima of each curve in Fig. 2(a), which occur at  $\omega_i = \omega_i(z_i)$  — are shown in Fig. 2(b) as functions of the impurity position.

In Fig. 3(a), the absorption probability  $W_L(z_i, \omega)$  for valence-to-donor transitions is shown as a function of the impurity position, for some fixed values of  $\omega$  and for an infinite GaAs quantum well of width  $L = 100 \text{ \AA}$ . Note

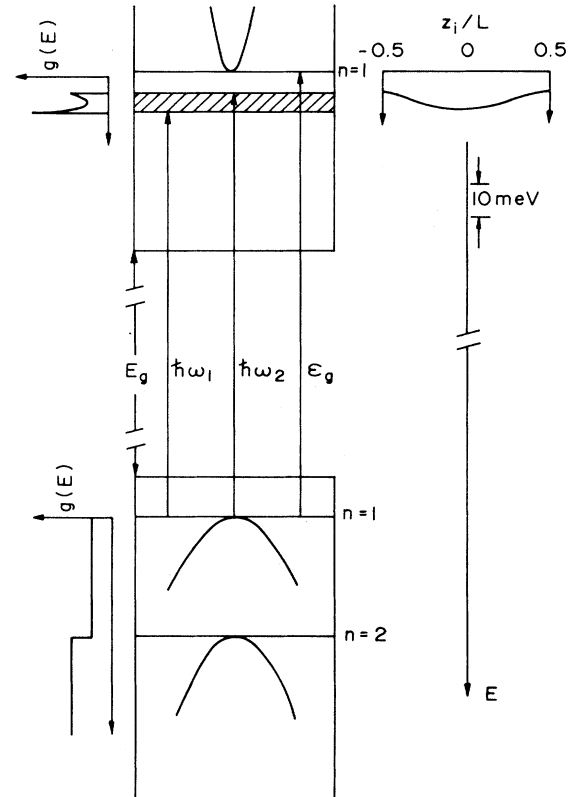


FIG. 1. Schematic representation of some possible absorption transitions in a  $L = 100 \text{ \AA}$  GaAs infinite quantum well with a donor-impurity band. The densities of states,  $g(E)$ , that result from the  $n=1$  and  $n=2$  valence bands and from the positional-dependent donor binding energy are shown schematically on the left. The dependence of the binding energy as a function of the donor position is shown schematically on the right. The parabolas represent a pictorial view of the planar ( $k_{\perp}$  dependence) dependence relation of the first conduction and valence minibands.

that, for a given  $\omega$ ,  $W_L(z_i, \omega)$  corresponds to the integrand in Eq. (2.14), and it is clear therefore that, for  $\omega_1 < \omega < \omega_2$ , only a fraction of the impurity band contributes to the absorption. Also, it is apparent from Fig. 3(a) (cf. curve I) that the total absorption  $W_L(\omega)$  goes continuously to zero as  $\omega$  approaches  $\omega_1$  from above [see Fig. 3(b) and the Appendix], a behavior which is *not* obtained by Bastard<sup>5</sup> (notice that in Figs. 6 and 7 of his work he obtained a steplike behavior for the total absorption

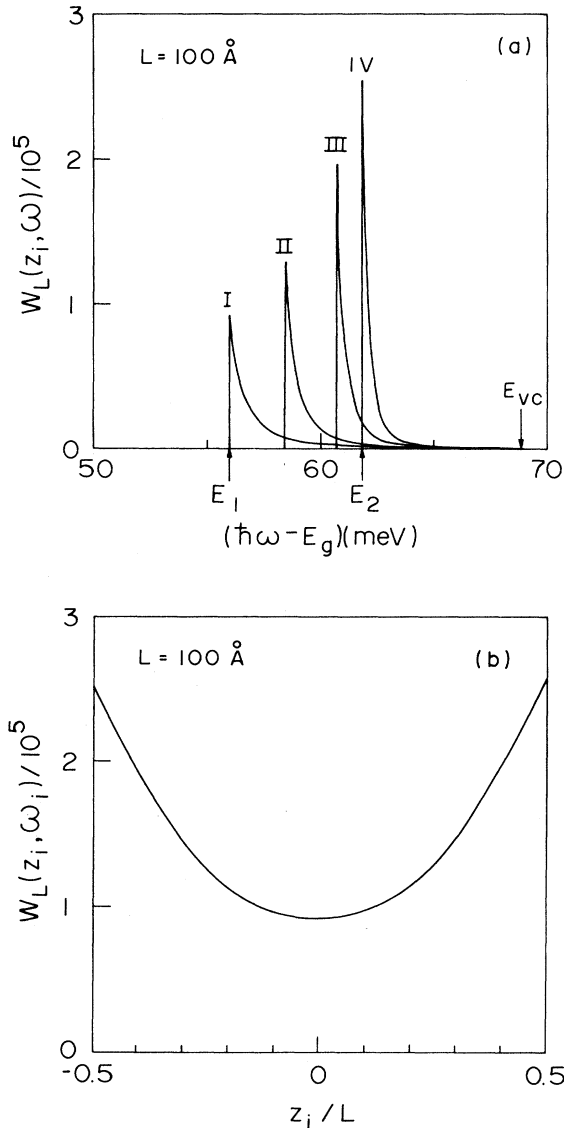


FIG. 2. Absorption probability per unit time  $W_L(z_i, \omega)$  (in units of  $W_0$ ; see text) for valence-to-donor transitions for a  $L=100 \text{ \AA}$  GaAs infinite quantum well. In (a) curves I–IV correspond to  $W_L(z_i, \omega)$  as a function of  $\hbar\omega - E_g$  for fixed  $z_i/L=0, 0.25, 0.4,$  and  $0.5$ , respectively.  $E_g$  is the bulk GaAs gap and  $E_{vc}$  indicates the onset of valence-to-conduction-band absorption. The maxima of the transition probability per unit time for a given  $z_i$ —which occur at  $\omega_i = \omega_i(z_i)$ —are shown in (b) as functions of the impurity position in the quantum well.

probability). Our result for the total absorption probability is shown in Fig. 3(b). Note that the value of  $E_2=61.82 \text{ meV}$  [see curve V in Fig. 3(a)] corresponds to the onset of transitions to the upper edge of the donor-impurity band and to a peak in the total transition probability per unit time. Our Fig. 3(b) (for  $L=100 \text{ \AA}$ , or  $L/\alpha_0^* \approx 1$ ) should be compared with the corresponding curve in Fig. 6 of Bastard's work.<sup>5</sup>

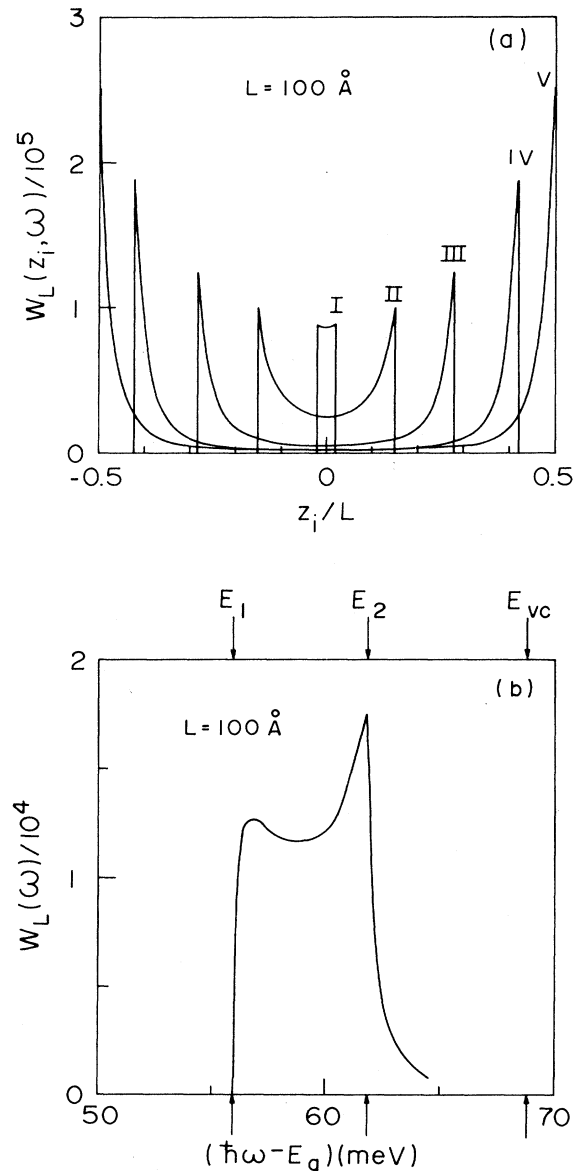


FIG. 3. (a)  $W_L(z_i, \omega)$  in units of  $W_0$  shown as a function of the impurity position for fixed values of  $\omega$  and for  $L=100 \text{ \AA}$ . Curves I–V correspond to values of  $\hbar\omega - E_g = 56, 57, 59, 61,$  and  $61.82 \text{ meV}$ , respectively. The value of  $E_2=61.82 \text{ meV}$  corresponds to the onset of transitions to the upper edge of the donor-impurity band and to a peak in the total transition probability per unit time shown in (b). Note that  $W_L(\omega)$  in (b) is given by the integral of the corresponding—fixed  $\omega$ —curve in (a).  $E_1$  indicates the onset of transitions from the first valence subband to the lower edge of the impurity band.

The absorption probabilities per unit time  $W_L(\omega)$  for valence-to-donor transitions and for acceptor-to-conduction transitions are shown in Figs. 4 and 5, respectively, for infinite GaAs quantum wells of different widths. In all curves one could note that there are two special structures, one associated with transitions involving impurities at the center of the well ( $E_1 = \hbar\omega_1 - E_g$ ) and the other with transitions associated to impurities at the edge of the well ( $E_2 = \hbar\omega_2 - E_g$ ). Also, it is clear that the structure near the edge  $E_1$  becomes more relevant with respect to the  $E_2$  structure as the size  $L$  of the GaAs quantum well increases with respect to the effective im-

purity Bohr radius  $\alpha_0^*$  (notice that  $\alpha_0^* \approx 100 \text{ \AA}$  for donors and  $\approx 22 \text{ \AA}$  for acceptors). In the case of acceptor-to-conduction transitions, for  $L = 1000 \text{ \AA}$ , the  $E_2$  structure has essentially disappeared. This may be understood in terms of the density of impurity states<sup>5,11</sup> which has a decreasing weight for impurities at the edges of the well as  $L/\alpha_0^*$  increases. Our results in Figs. 4 and 5 should be compared with Figs. 6 and 7 of Bastard's work<sup>5</sup> or Fig. 3 of the review by Delalande.<sup>21</sup>

Figures 6 and 7 present the  $W_L(\omega)$  absorption probabilities for transitions from the first valence subband to the donor-impurity band and from the acceptor-impurity

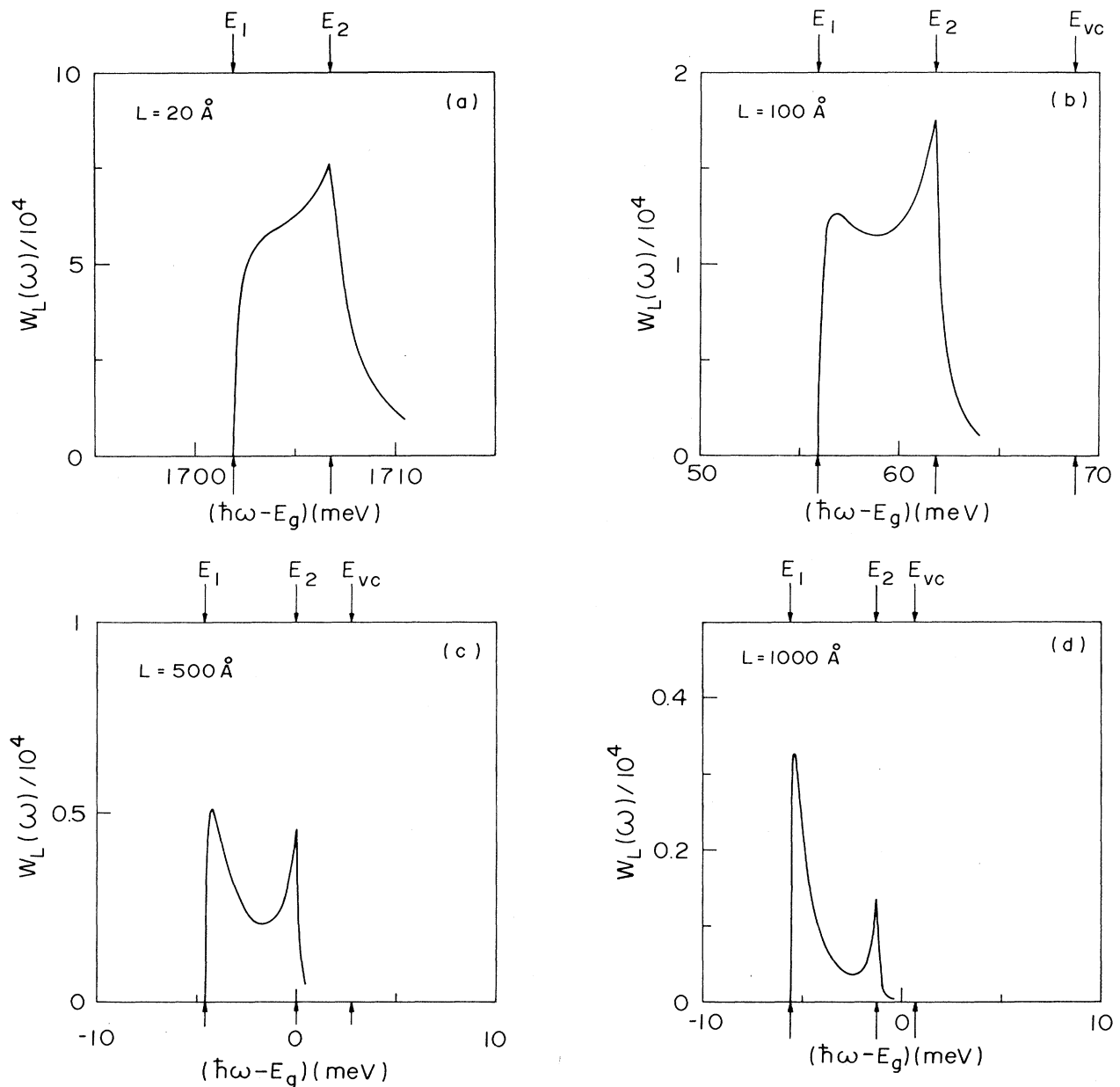


FIG. 4. Total absorption probability per unit time (in units of  $W_0$ ; see text) for valence-to-donor transitions for infinite GaAs quantum wells of width  $L = 20, 100, 500,$  and  $1000 \text{ \AA}$ . Notice that different scales are used.

band to the first conduction subband, respectively, for both the infinite- and finite- ( $x=0.30$ ) barrier GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum wells. It is clear from the Figs. that for  $L$  up to at least 100 Å there is a considerable shift of the  $W_L(\omega)$  curves to smaller values of  $\hbar\omega - E_g$  due essentially to changes in the energies of the first conduction and valence subbands when the finite character of the barrier potential is taken into account. Also, as a general feature, one notices a softening of the peak in  $W_L(\omega)$  associated with  $\hbar\omega_2$ , i.e., associated to transitions involving impurities at the edges of the quantum well. For a finite quantum well, the wave function associated with a donor (acceptor) at the border of the well penetrates into the Ga<sub>1-x</sub>Al<sub>x</sub>As barrier and therefore it

has a smaller overlap with the wave function of the valence (conduction) subband when compared to the case of the infinite barrier. In the situation of a very small value of  $L/\alpha_0^*$  [cf. Fig. 6(a)] and for a finite barrier, one eventually recovers the result for the Ga<sub>1-x</sub>Al<sub>x</sub>As bulk, i.e., a  $\delta$  function at  $\hbar\omega_1$ . Of course, for the bulk case, there is no inhomogeneous broadening of the impurity levels due to changes in the position of the impurity. On the other hand, for large values of  $L/\alpha_0^*$ , the density of impurity states associated with impurities at the edges of the well is very small, and therefore the influence of considering a finite barrier is practically negligible and one essentially recovers the results for an infinite quantum well [see Figs. 6(c), (6d), 7(c), and 7(d)].

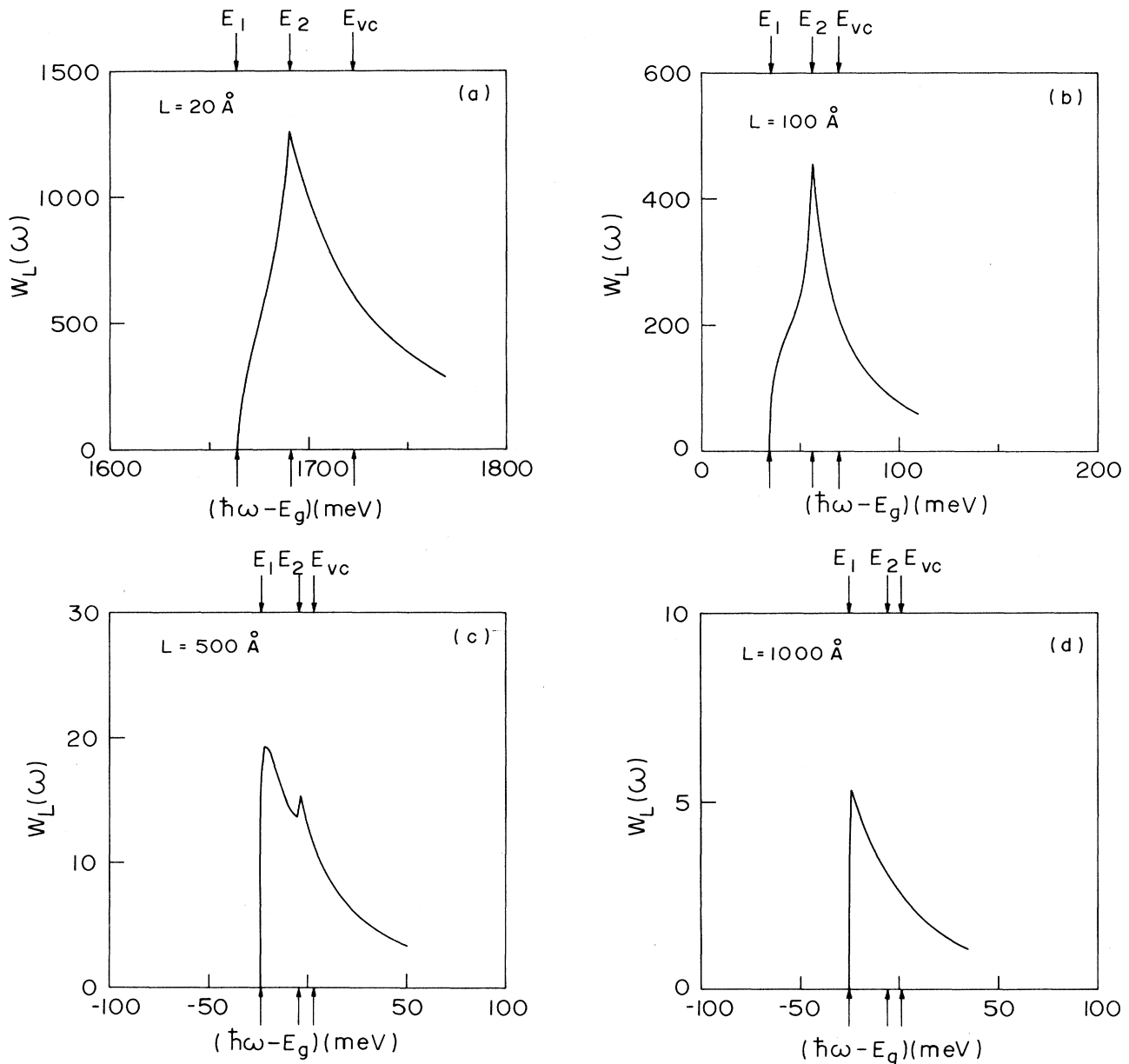


FIG. 5. Total absorption probability per unit time (in units of  $W_0$ ; see text) for acceptor-to-conduction transitions for infinite GaAs quantum wells of width  $L=20, 100, 500$ , and  $1000 \text{ \AA}$ . Notice that different scales are used.

## IV. CONCLUSIONS

Summing up, we have calculated the optical-absorption spectra associated with transitions between the  $n=1$  valence subband and the donor-impurity band and between the acceptor-impurity band and the  $n=1$  conduction subband for a GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As finite quantum well. We have shown that there are two special structures in the transition probability per unit time  $W_L(\omega)$ , i.e., an edge associated with transitions involving impurities at the center of the well and a peak associated with transitions related to impurities at the edges of the quantum well. We have unequivocally demonstrated that the total absorption probability  $W_L(\omega)$  goes continuously to zero as  $\omega$  approaches the absorption edge  $\omega_1$  from above, a behavior which contrasts with the previous calculation by Bastard.<sup>5</sup> Also, the results we obtained for the optical

absorption associated with band  $\leftrightarrow$  impurity transitions in the case of an infinite GaAs quantum well are rather different from those previously reported.<sup>5,21</sup> For the case of a finite GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum well, there is a softening of the  $E_2$  peak due to the smaller overlap between the wave functions of on-edge impurity and band states, and a considerable shift of the  $W_L(\omega)$  curves to smaller values of  $\hbar\omega - E_g$  due to the effect of the finite character of the barrier potential for  $L$  up to at least 100 Å.

Finally, although experimental results for the optical-absorption spectra associated with impurities in quantum wells are not yet available, we believe our results are of importance in the quantitative understanding of future experimental work in this field. Also, the study of luminescence properties associated with impurities in quantum wells is a subject of considerable experimental

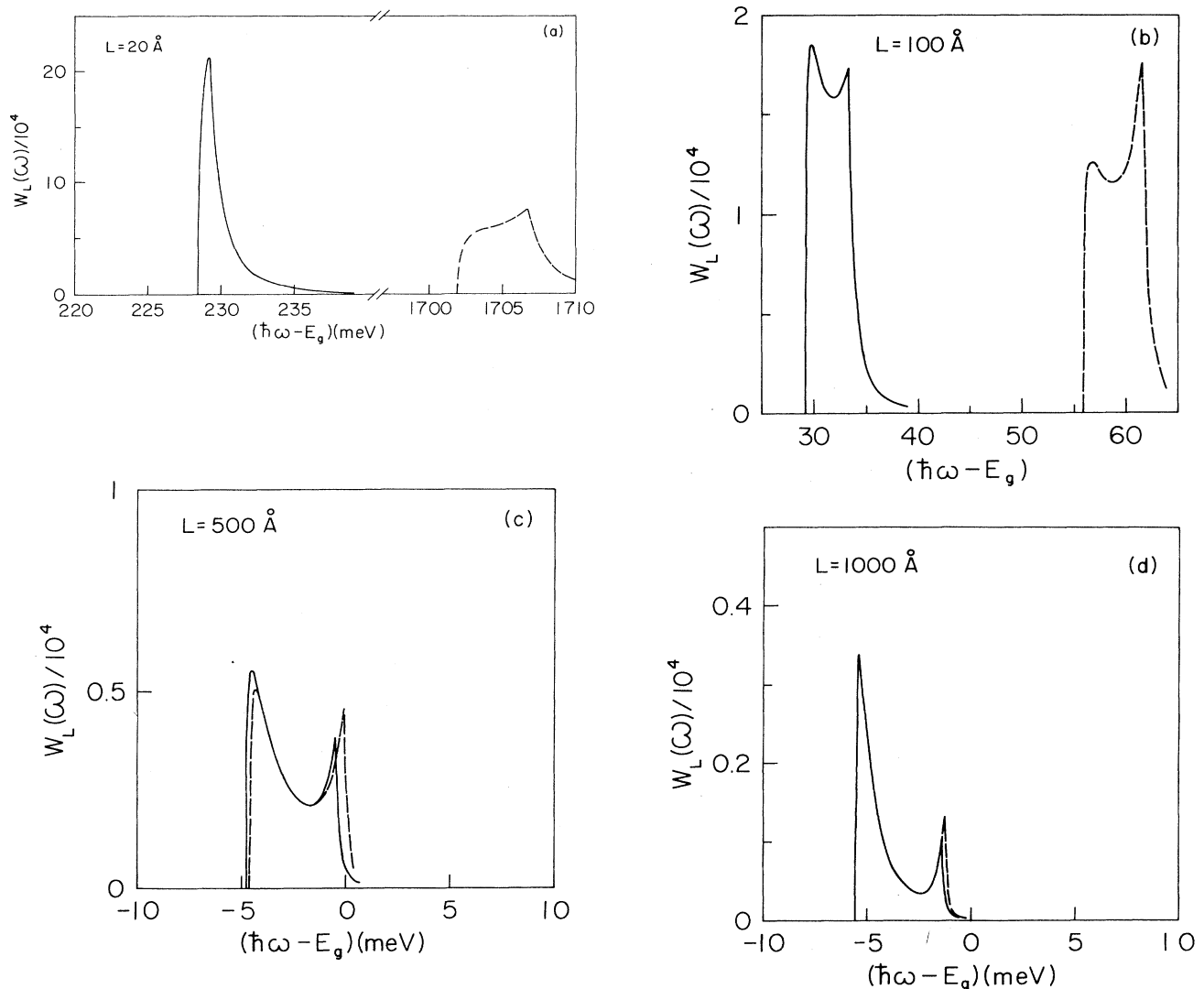


FIG. 6. Total absorption probability per unit time (in units of  $W_0$ ; see text) for valence-to-donor transitions for GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum wells of width  $L=20, 100, 500,$  and  $1000$  Å. The dashed lines correspond to GaAs quantum wells of infinite depth, whereas the solid curves are for an  $x=0.30$  Al concentration (finite-barrier potential). Notice that different scales are used.

interest<sup>20-23</sup> and a theoretical analysis along the same lines of the present work is in progress.

#### ACKNOWLEDGMENTS

We are grateful to Gerald Weber and Peter A. B. Schulz for helpful discussions in the early stages of this work. R.P.A. is grateful to the Institute of Physics at UNICAMP for their hospitality, and to the Third World Academy of Science (Trieste, Italy) and the Brazilian Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for financial support.

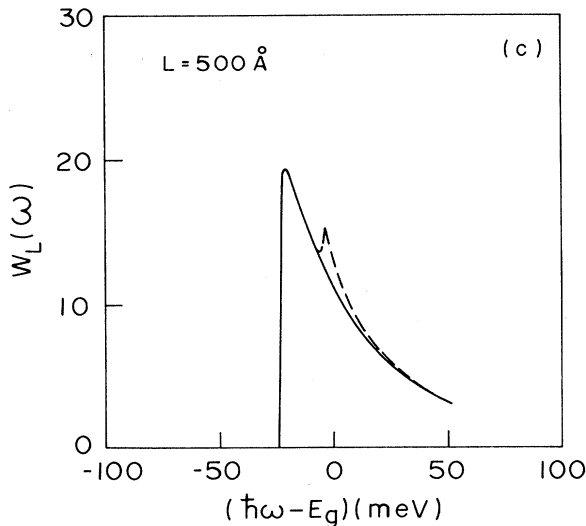
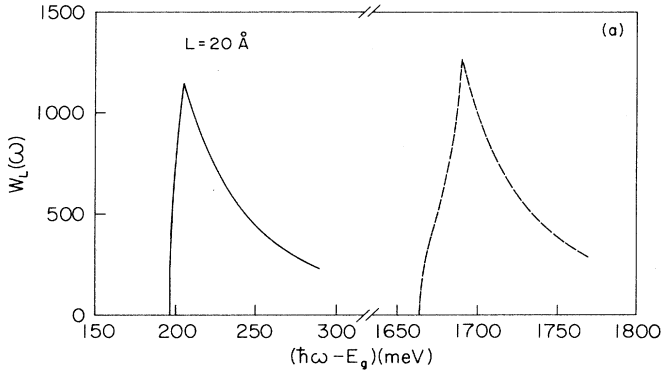
#### APPENDIX

One can see that  $W_L(\omega)$  should go to zero as  $\omega - \omega_1 \rightarrow 0^+$  by considering (2.6),

$$W_L(z_i, \omega) = F(z_i, \omega) Y(\hbar\omega - \varepsilon_g + E(L, z_i)), \quad (\text{A1})$$

with  $\omega \simeq \omega_1$ , i.e.,

$$W_L(z_i, \omega) \simeq F(z_i, \omega_1) Y(\hbar\omega - \varepsilon_g + E(L, 0) - \gamma z_i^2), \quad (\text{A2})$$



where we have used the fact that  $F(z_i, \omega)$  is a well-behaved function and

$$E(L, z_i) \simeq E(L, 0) - \gamma z_i^2, \quad (\text{A3})$$

as the impurity binding energy at the neighborhood of  $z_i = 0$  [note that  $E(L, z_i)$  is an even function of the impurity position].

Using (2.14), one obtains

$$W_L(\omega) \simeq \frac{2}{L} \int_0^{(\Delta_0/\gamma)^{1/2}} F(z_i, \omega_1) Y(\Delta_0 - \gamma z_i^2) dz_i, \quad (\text{A4})$$

with  $\Delta_0 = \hbar\omega - \hbar\omega_1 = \hbar\omega - \varepsilon_g + E(L, 0)$ . Finally,

$$W_L(\omega) \simeq \frac{2F(0, \omega_1)}{L\gamma^{1/2}} (\hbar\omega - \hbar\omega_1)^{1/2}, \quad (\text{A5})$$

for  $\omega - \omega_1 \rightarrow 0^+$ , which proves that the total absorption goes continuously to zero, in contrast to the result by Bastard.<sup>5</sup>

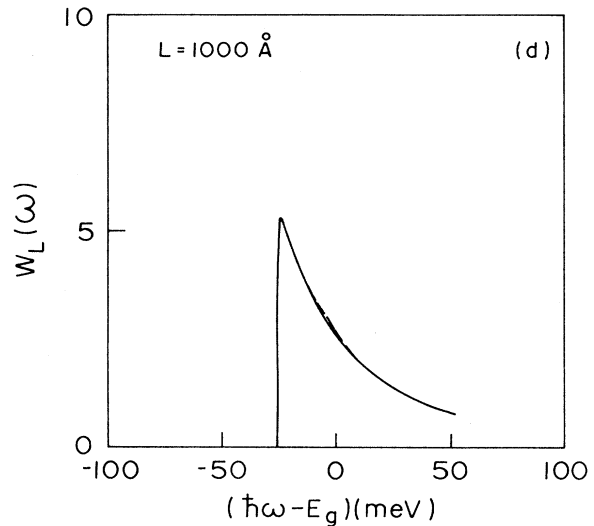
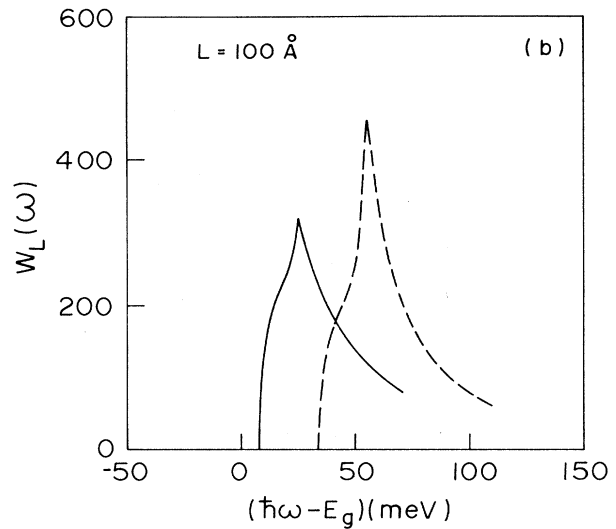


FIG. 7. Total absorption probability per unit time (in units of  $W_0$ ; see text) for acceptor-to-conduction transitions for GaAs-Ga<sub>1-x</sub>Al<sub>x</sub>As quantum wells of width  $L=20, 100, 500,$  and  $1000 \text{ \AA}$ . The dashed lines correspond to GaAs quantum wells of infinite depth, whereas the solid curves are for an  $x=0.30$  Al concentration (finite-barrier potential). Notice that different scales are used.



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