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Renormalization of Bloch electrons in coherent light

Ahmet Elçi
Center for Advanced Studies, University of New Mexico, Albuquerque, New Mexico 87131
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Separation of the coherent parts of the electromagnetic field modes from the vacuum-fluctuating parts leads to a new picture of single-particle excitations in crystals. We discuss the effects of this renormalization on simple band structures.

I. INTRODUCTION

In quantum theories which are concerned with the generation and propagation of light in crystals, one generally holds with few exceptions to the physical picture in which the elementary single-particle excitations are the Bloch electrons, whose fundamental characteristics are not directly influenced by the coherence of any electromagnetic field mode. Among the fundamental characteristics of a Bloch electron are its energy-momentum relations for different bands, its parameters such as band gaps and effective masses, which enter into the energy-momentum dispersion relations, and its charge and other coupling coefficients. In the conventional physical picture, these parameters are not affected by the coherence of any field mode. Of course, under intense light excitations, many mobile carriers can be created in crystals, which in turn affect, for example, band gaps, effective masses, and the screening of the electronic charge. However, such renormalizations are associated with incoherent processes involving nonequilibrium carrier populations and their relaxation via Coulomb scatterings, phonon emissions, and phonon absorptions. For such processes, the coherence or incoherence of field modes are not relevant. In fact, one need not use light to obtain a given reparametrization; any other excitation source which causes real transitions and creates the same degree of nonequilibrium in the crystal would do. In the description of real transitions and nonequilibrium processes, one still holds on to the basic Bloch-electron picture.

One exception is, of course, Hopfield's theory of polaritons.[1] Hopfield wrote the Hamiltonian of a classical dielectric in terms of the potential fields **A** and ϕ, and the polarization density **P**. He then quantized **A**, ϕ, and **P** by imposing appropriate commutation relations such that **P** is a transverse boson field. The quanta of the polarization field are of two types. One type is similar to photons of the radiation field **A** and was named polaritons by Hopfield. For optical or near-optical frequencies, polaritons correspond to excitons. The other type of the polarization field quanta is associated with the decaying electromagnetic waves above the plasma frequency. As elementary excitations, polaritons represent a significant improvement over Bloch electrons when excitonic lifetimes are long. The drawback of the polariton picture is that many essential processes of photoexcited solid-state plasmas (Auger processes in semiconductor lasers, phonon absorptions and emissions, impurity-defect scatterings, etc.) require description in terms of single-particle excitations rather than electron-hole pairs. One therefore rapidly reverts back to the Bloch-electron picture in many practical and theoretical problems.

However, the Bloch-electron picture is in fact incomplete for single-particle excitations in coherent light. If an electromagnetic field mode in a crystal is in a coherent state, the fundamental parameters of a Bloch electron are altered due to the coherence of the mode.[2,3] The modifications induced by the field coherence in the electronic band structure, coupling coefficients, etc., depend on both the intensity and the phase of the coherent mode. The effects of such coherent-field renormalizations are determined by means of two unitary transformations. If a set of coherent modes (the reference set) is postulated, a transformation T separates the coherent parts of the modes from the vacuum fluctuating parts. As this separation is carried out, the electronic part of the Hamiltonian becomes nondiagonal because of the coupling between electrons and the coherent-field modes. A second transformation U rediagonalizes the new Hamiltonian, yielding the renormalized electron operators. The new elementary excitations still refer to single-particle excitations unlike polaritons. However, they take into account the correlations induced by the coherent modes. The new picture is developed completely within the quantum theoretical framework. Therefore, the electronic states obtained from the coherent-field renormalization are distinct from the electronic states obtained from the usual classical treatment of the electromagnetic field.

In this paper we discuss the coherent-field renormalization and its effects on simple band structures. In Sec. II we discuss the renormalization transformations and the meaning of the physical picture they produce. We demonstrate that the renormalization transformations redefine the vacuum state. The transformation T maps the coherent state of the electromagnetic field, as represented by the modes in the reference set, onto the vacuum state. If a state of the field is partially coherent, T maps it onto a superposition of a finite set of number states for photons. There is a close analogy between this redefinition of the vacuum state and the redefinition of the vacuum state for the ideal electron gas. In the electron-gas problem the vacuum state is redefined to coincide with the ground state of the electron gas by introducing a hole field. In the present problem the transformation T redefines the vacuum state so that it contains a set of coherent modes. The energies produced by the transformation U are the actual single-particle energies. We demonstrate this by considering the coupling between the electron system and a field mode which is not included in the reference set. This result contrasts with the semiclassical theories, where one deals with time-dependent Hamiltonians and interprets certain frequencies as quasienergies.[4,5] We also give the formal expressions for the renormalized Bloch functions. If the photon momenta associated with the reference modes can be neglected relative to electronic crystal momenta, then the renormalized Bloch functions remain periodic in the reciprocal lattice space and one readily obtains the momentum Bloch functions. One also obtains an expression for the renormalized Fourier coefficients of the periodic crystal potential from the relation between this potential and the momentum Bloch functions. The renormalized crystal potential may be used as the starting point for investigations of crystal structure in the presence of coherent fields. In Sec. III we discuss the effects of the coherent-field renormalization on two-band and four-band models. For these simple band structures, the renormalized electron energies and the matrix elements of the transformation U are calculated exactly under the assumption that the photon wave vectors of the reference modes are negligible relative to electronic wave vectors. One finds that band gaps and effective masses are altered and that these alterations are experimentally observable. A band gap which is direct in the bare band structure may become indirect when the band structure is renormalized. The positions of the band edges shift in the Brillouin zone (BZ). If the bare bands are degenerate, this degeneracy may be removed partially or completely in the renormalized bands. The calculations carried out in this section are quite similar to those in Kane's band theory,[6] except that in the present problem various coupling parameters depend on the intensities and the phases of the reference modes. The renormalization effects appear mostly in terms of the ratio $(\hbar\Omega_E/E_G^0)$, where Ω_E is similar to a Rabi frequency and E_G^0 is a band gap. The coherent-field renormalization is enhanced either by increasing the intensities of the reference modes (Ω_E is proportional to the square root of essentially the sum of these intensities) or by decreasing the band gap. Thus, the renormalization effects are particularly significant in narrow-gap crystals. Section IV includes a few concluding remarks. The renormalization effects may be observed in excite-probe experiments[7,8] when excitation and probe pulses overlap. In such experimental configurations, coherent excitation modes constitute the reference set. Probe pulses then couple to coherent-field-renormalized electrons and holes. The picture of single-particle excitations presented in this paper should be especially useful in semiconductor lasers. When one or more intense coherent modes are obtained in these devices, couplings between different lasing modes and their fluctuations are most accurately analyzed in terms of the coherent-field-renormalized electrons and holes, or the excitons formed from the renormalized electrons and holes.

II. COHERENT FIELD RENORMALIZATION

This section presents a general discussion of the transformations which effect the separation of the coherent parts of the field modes from the vacuum fluctuating parts and the rediagonalization of the electronic Hamiltonian.

Let a set of complex numbers $\{\alpha_\mu\}$ represent the coherent state[9] of the electromagnetic field:

$$|\{\alpha_\mu\}\rangle=\prod_\mu\left[e^{-|\alpha_\mu|^2/2}\sum_{n_\mu=0}^{\infty}\frac{(\alpha_\mu)^{n_\mu}}{(n_\mu!)^{1/2}}|n_\mu\rangle\right], \tag{2.1a}$$

$$\alpha_\mu=(\bar{N}_\mu)^{1/2}e^{i\phi_\mu}. \tag{2.1b}$$

Here, $\bar{N}_\mu$ is the average number of the quanta in the mode μ. ϕ_μ is the phase of the mode. $|n_\mu\rangle$ is a number state. Let a_μ and $a_\mu^\dagger$ be the photon annihilation and creation operators. $|\{\alpha_\mu\}\rangle$ is an eigenstate of a_μ:

$$a_\mu|\{\alpha_\mu\}\rangle=\alpha_\mu|\{\alpha_\mu\}\rangle, \tag{2.2a}$$

$$\langle\{\alpha_\mu\}|a_\mu^\dagger=\alpha_\mu^*\langle\{\alpha_\mu\}|. \tag{2.2b}$$

One can separate the coherent parts of the modes from the vacuum fluctuating parts by means of the transformation

$$T=\exp\left[\sum_\mu(\alpha_\mu^*a_\mu-\alpha_\mu a_\mu^\dagger)\right]. \tag{2.3}$$

Using the fact that if $[A,B]$ is constant for two operators A and B, then

$$e^Ae^B=e^{A+B}e^{[A,B]/2}, \tag{2.4}$$

one readily sees that T is a unitary transformation:

$$TT^\dagger=1. \tag{2.5}$$

When T is applied on the photon operators, it yields

$$Ta_\mu T^\dagger=a_\mu+\alpha_\mu, \tag{2.6a}$$

$$Ta_\mu^\dagger T^\dagger=a_\mu^\dagger+\alpha_\mu^*. \tag{2.6b}$$

These equations follow from the fact that for two operators A and S,

$$e^SAe^{-S}=\sum_{n=0}^{\infty}\frac{1}{n!}C^n\{S,A\}, \tag{2.7a}$$

where the C^n's are the nested commutators

$$C^0\{S,A\}=A,$$
$$C^1\{S,A\}=[S,A], \tag{2.7b}$$
$$C^2\{S,A\}=[S,[S,A]],\ \text{etc.}$$

From Eq. (2.4),

$$Ta_\mu T^\dagger=e^{-\sum_\nu\alpha_\nu a_\nu^\dagger}a_\mu e^{\sum_\nu\alpha_\nu a_\nu^\dagger}. \tag{2.8}$$

When one sets $A=a_\mu$, $S=-\sum_\nu\alpha_\nu a_\nu^\dagger$, and uses (2.7), one sees that only $C^0=a_\mu$ and $C^1=\alpha_\mu$ are nonzero and obtains (2.6a).

The meaning of the separation in (2.6) can be discerned from the effects of T and $T^\dagger$ on the vacuum state. $T^\dagger$ operating on the vacuum produces the coherent state $|\{\alpha_\mu\}\rangle$:

$$T^\dagger|\text{vac}\rangle = e^{-\sum_\mu|\alpha_\mu|^2/2}\, e^{\sum_\mu \alpha_\mu a_\mu^\dagger}\, e^{-\sum_\mu \alpha_\mu^* a_\mu}|\text{vac}\rangle$$
$$= e^{-\sum_\mu|\alpha_\mu|^2/2}\, e^{\sum_\mu \alpha_\mu a_\mu^\dagger}|\text{vac}\rangle$$
$$= \prod_\mu \left[e^{-|\alpha_\mu|^2/2}\sum_{n_\mu}\frac{(\alpha_\mu)^{n_\mu}}{(n_\mu!)^{1/2}}|n_\mu\rangle\right]$$
$$= |\{\alpha_\mu\}\rangle \ . \tag{2.9}$$

From the unitarity of T it follows that T acting on the coherent state $|\{\alpha_\mu\}\rangle$ produces the vacuum state

$$T|\{\alpha_\mu\}\rangle = |\text{vac}\rangle \ . \tag{2.10}$$

This implies that the transformation T induces a redefinition of the vacuum state. If we view (2.6) as a canonical transformation into a new Hilbert space, the vacuum state in the new space is redefined in such a way that it incorporates a reference set of coherent modes given by $\{\alpha_\mu\}$. This can be seen more clearly by operating with T on (2.2a). Let the transformed operator be designated as A_μ:

$$A_\mu = T a_\mu T^\dagger \ . \tag{2.11a}$$

From (2.2a),

$$T a_\mu|\{\alpha_\mu\}\rangle = \alpha_\mu T|\{\alpha_\mu\}\rangle \ ,$$
$$A_\mu T|\{\alpha_\mu\}\rangle = \alpha_\mu T|\{\alpha_\mu\}\rangle \ , \tag{2.11b}$$
$$A_\mu|\text{vac}\rangle = \alpha_\mu|\text{vac}\rangle \ .$$

In other words, the vacuum state is an eigenstate of the annihilation operator A_μ, if μ is included in the reference set (that is $\alpha_\mu \neq 0$). This is a characteristic of a coherent state. In the new Hilbert space, the vacuum state itself becomes a coherent state of the electromagnetic field. If we choose to express the transformed field operators in terms of the old operators, that is, if we use $a_\mu + \alpha_\mu$ instead of a new label A_μ, then after the transformation T, a_μ and $a_\mu^\dagger$ operate either on the vacuum state or on pure number states in the Heisenberg picture, the coherent parts being separated by c numbers.

These statements are also demonstrated by operating with T on partially coherent states of the field. An example for a partially coherent state is

$$|\Psi_{\nu m}\rangle = \rho_{\nu m}(a_\nu^\dagger)^m|\{\alpha_\mu\}\rangle \ , \tag{2.12a}$$

where $\rho_{\nu m}$ is the normalization constant:

$$\rho_{\nu m} = \left[\sum_{l=0}^{m}\frac{[m!]^2}{[(m-l)!]^2 l!}|\alpha_\nu|^{2m-2l}\right]^{-1/2} \ . \tag{2.12b}$$

T transforms this state into a superposition of the vacuum state with a finite set of number states for the mode ν:

$$T|\Psi_{\nu m}\rangle = \rho_{\nu m} T(a_\nu^\dagger)^m T^\dagger T|\{\alpha_\mu\}\rangle$$
$$= \rho_{\nu m}(\alpha_\nu^* + a_\nu^\dagger)^m|\text{vac}\rangle$$
$$= \rho_{\nu m}\sum_{l=0}^{m}\frac{m!}{(m-l)!(l!)^{1/2}}(\alpha_\nu^*)^{m-l}|l_\nu\rangle \ . \tag{2.13a}$$

If the mode ν is not included in the reference set $\{\alpha_\mu\}$, that is if $\alpha_\nu = 0$, then T maps a number state onto itself as far as the mode ν is concerned:

$$T|\Psi_{\nu m}\rangle = (m!)^{-1/2}(a_\nu^\dagger)^m|\text{vac}\rangle = |m_\nu\rangle \ . \tag{2.13b}$$

Finally it should be noted that the original vacuum state is mapped by T onto a coherent state which has its mode phases shifted by π:

$$T|\text{vac}\rangle = |\{-\alpha_\mu\}\rangle = |\{\alpha_\mu e^{i\pi}\}\rangle \ . \tag{2.14}$$

It is worthwhile to point out the similarity between the redefinition of the vacuum state above and the redefinition of the vacuum state for an ideal electron gas. For an ideal electron gas at zero temperature, the single-particle states are filled up to a Fermi energy. The corresponding many-body state gives the ground state of the noninteracting electrons. Other states of the system are associated with an electron (or more) crossing the Fermi surface. One can go to a new field description of this system by redefining the vacuum state so that it coincides with the ground state of the system and by introducing a hole field. The transformation T produces a similar kind of redefinition of the vacuum state for the coherent states of photons.

Let us consider the Hamiltonian which represents Bloch electrons, photons, and their coupling:

$$H^B = H_e^0 + H_\gamma^0 + H_I^0 + H_{II}^0 \ , \tag{2.15a}$$

$$H_e^0 = \sum_\beta E_\beta^0 c_\beta^\dagger c_\beta \ , \tag{2.15b}$$

$$H_\gamma^0 = \sum_\mu \hbar\omega_\mu a_\mu^\dagger a_\mu \ , \tag{2.15c}$$

$$H_I^0 = \sum_{\beta\beta'} c_\beta^\dagger c_{\beta'}\left[\sum_\mu (g_{\beta\beta'}^\mu a_\mu + g_{\beta'\beta}^{\mu *} a_\mu^\dagger)\right] \ , \tag{2.15d}$$

$$H_{II}^0 = \sum_{\beta\beta'} c_\beta^\dagger c_{\beta'} \sum_{\mu\mu'}[d_{\beta\beta'}^{\mu\mu'}(\mathbf{q}_\mu + \mathbf{q}_{\mu'})a_\mu a_{\mu'}$$
$$+ d_{\beta\beta'}^{\mu\mu'}(\mathbf{q}_\mu - \mathbf{q}_{\mu'})a_\mu a_{\mu'}^\dagger$$
$$+ d_{\beta\beta'}^{\mu\mu'}(-\mathbf{q}_\mu + \mathbf{q}_{\mu'})a_\mu^\dagger a_{\mu'}$$
$$+ d_{\beta\beta'}^{\mu\mu'}(-\mathbf{q}_\mu - \mathbf{q}_{\mu'})a_\mu^\dagger a_{\mu'}^\dagger] \ . \tag{2.15e}$$

Here the β's designate the Bloch states: $\beta = (n\mathbf{k})$, where n is the band index and $\mathbf{k}$ is the electronic wave vector. $c_\beta, c_\beta^\dagger$ are the anticommuting electron operators. E_β^0 is the Bloch energy. $g_{\beta\beta'}^\mu$ represents the $\mathbf{p}\cdot\mathbf{A}$ coupling between electrons and photons. It is given by

$$g_{\beta\beta'}^\mu = (e/mc)\mathcal{A}_\mu\langle n\mathbf{k}|e^{i\mathbf{q}_\mu\cdot\mathbf{x}}\hat{\boldsymbol{\varepsilon}}_\mu\cdot\mathbf{p}|n'\mathbf{k}'\rangle \ , \tag{2.16a}$$

$$\mathcal{A}_\mu = \left[\frac{2\pi\hbar c^2}{V_{01} n_\mu^2 \omega_\mu}\right]^{1/2} \ . \tag{2.16b}$$

$\hat{\epsilon}_\mu$ and $\mathbf{q}_\mu$ are the polarization and propagation vectors for the mode μ, $\mathbf{p}$ is the momentum operator, n_μ is the index of refraction, $V_{\rm ol}$ is the quantization volume, ω_μ is the photon frequency, and $d^{\mu\mu'}_{\beta\beta'}$ represents the $\mathbf{A}^2$ coupling and is given by

$$d^{\mu\mu'}_{\beta\beta'}(\mathbf{q})=(e^2/2mc^2)\mathcal{A}_\mu\mathcal{A}_{\mu'}\langle n\mathbf{k}|e^{i\mathbf{q}\cdot\mathbf{x}}|n'\mathbf{k}'\rangle(\hat{\epsilon}_\mu\cdot\hat{\epsilon}_{\mu'}) . \tag{2.17}$$

T transforms H^B into

$$H=TH^BT^\dagger=H_e+H_\gamma+H_I+H^0_{II} , \tag{2.18a}$$

where

$$H_e=\sum_{\beta\beta'}h_{\beta\beta'}c^\dagger_\beta c_{\beta'} , \tag{2.18b}$$

$$h_{\beta\beta'}=E^0_\beta\delta_{\beta\beta'}+\sum_\mu(g^\mu_{\beta\beta'}\alpha_\mu+g^{\mu*}_{\beta\beta'}\alpha^*_\mu)+\sum_{\mu\mu'}[d^{\mu\mu'}_{\beta\beta'}(\mathbf{q}_\mu+\mathbf{q}_{\mu'})\alpha_\mu\alpha_{\mu'}+d^{\mu\mu'}_{\beta\beta'}(\mathbf{q}_\mu-\mathbf{q}_{\mu'})\alpha_\mu\alpha^*_{\mu'}$$
$$+d^{\mu\mu'}_{\beta\beta'}(-\mathbf{q}_\mu+\mathbf{q}_{\mu'})\alpha^*_\mu\alpha_{\mu'}+d^{\mu\mu'}_{\beta\beta'}(-\mathbf{q}_\mu-\mathbf{q}_{\mu'})\alpha^*_\mu\alpha^*_{\mu'}] , \tag{2.18c}$$

$$H_\gamma=\sum_\mu\hbar\omega_\mu(a^\dagger_\mu a_\mu+\alpha_\mu a^\dagger_\mu+\alpha^*_\mu a_\mu+|\alpha_\mu|^2) , \tag{2.18d}$$

$$H_I=\sum_{\beta\beta'}c^\dagger_\beta c_{\beta'}\sum_\mu[(g^\mu_{\beta\beta'}+\tilde{d}^{\,\mu}_{\beta\beta'})a_\mu+(g^{\mu*}_{\beta'\beta}+\tilde{d}^{\,\mu*}_{\beta'\beta})a^\dagger_\mu] , \tag{2.18e}$$

and

$$\tilde{d}^{\,\mu}_{\beta\beta'}=2\sum_{\mu'}[d^{\mu\mu'}_{\beta\beta'}(\mathbf{q}_\mu+\mathbf{q}_{\mu'})\alpha_{\mu'}+d^{\mu\mu'}_{\beta\beta'}(\mathbf{q}_\mu-\mathbf{q}_{\mu'})\alpha^*_{\mu'}] . \tag{2.18f}$$

In (2.18e) and (2.18f) we used the symmetry in the definition (2.17) that $d^{\mu\mu'}_{\beta\beta'}(\mathbf{q})=d^{\mu'\mu}_{\beta\beta'}(\mathbf{q})$ and $d^{\mu\mu'}_{\beta\beta'}(-\mathbf{q})=[d^{\mu\mu'}_{\beta'\beta}(\mathbf{q})]^*$. The last term in (2.18d) is a c number and may be omitted.

It is seen from (2.18b) and (2.18c) that the electronic Hamiltonian is now nondiagonal and depends on the coherent mode amplitudes in the reference set. Let U be a unitary transformation which diagonalizes $(h_{\beta\beta'})$:

$$(UhU^\dagger)_{\beta\beta'}=\delta_{\beta\beta'}E_\beta . \tag{2.19}$$

Let also

$$C_\beta=\sum_{\beta'}U_{\beta\beta'}c_{\beta'} , \tag{2.20a}$$

$$\{C_\beta,C^\dagger_{\beta'}\}=\delta_{\beta\beta'} . \tag{2.20b}$$

C_β and $C^\dagger_\beta$ are the renormalized electron operators. Then,

$$H_e=\sum_\beta E_\beta C^\dagger_\beta C_\beta , \tag{2.21}$$

where E_β is the renormalized energy. The transformation U also affects H_I and H^0_{II}. The renormalized electron-photon coupling which is linear in photon operators becomes

$$H_I=\sum_{\beta\beta'\mu}C^\dagger_\beta C_{\beta'}(G^\mu_{\beta\beta'}a_\mu+G^{\mu*}_{\beta'\beta}a^\dagger_\mu) , \tag{2.22a}$$

$$G^\mu_{\beta\beta'}=\sum_{\lambda\lambda'}U_{\beta\lambda}(g^\mu_{\lambda\lambda'}+\tilde{d}^{\,\mu}_{\lambda\lambda'})U^\dagger_{\lambda'\beta'} . \tag{2.22b}$$

The electron-photon coupling which is quadratic in photon operator becomes

$$H_{II}=H^0_{II}(d^{\mu\mu'}_{\beta\beta'}\to D^{\mu\mu'}_{\beta\beta'}) , \tag{2.23a}$$

$$D^{\mu\mu'}_{\beta\beta'}(\mathbf{q})=\sum_{\lambda\lambda'}U_{\beta\lambda}d^{\mu\mu'}_{\lambda\lambda'}(\mathbf{q})U^\dagger_{\lambda'\beta'} . \tag{2.23b}$$

One can express the eigenvalue equation for the renormalized energies compactly in terms of an infinite expansion matrix

$$Z_{\beta\beta'}(E)=h_{\beta\beta'}+\sum_\gamma\frac{\bar{h}_{\beta\gamma}\bar{h}_{\gamma\beta'}}{(E-h_{\gamma\gamma})}+\sum_{\gamma,\tau}\frac{\bar{h}_{\beta\gamma}\bar{h}_{\gamma\tau}\bar{h}_{\tau\beta'}}{(E-h_{\gamma\gamma})(E-h_{\tau\tau})}+\cdots , \tag{2.24a}$$

where

$$\bar{h}_{\beta\beta'}=(1-\delta_{\beta\beta'})h_{\beta\beta'} . \tag{2.24b}$$

E_β's are given by the solutions of the equations[10]

$$E=Z_{\beta\beta}(E) . \tag{2.25}$$

For a given β, this equation provides many solutions. The appropriate solution for a Bloch state can be picked out among the many solutions by requiring

$$\lim_{\{\alpha_\mu\}\to 0}E_\beta=E^0_\beta . \tag{2.26}$$

One can also express the matrix elements of U in terms of Z. Let the matrix $(h_{\beta\beta'})$ have normalized column eigenvectors $\underset{\sim}{\xi}^{(\beta)}$ so that

$$h\underset{\sim}{\xi}^{(\beta)}=E_\beta\underset{\sim}{\xi}^{(\beta)} . \tag{2.27}$$

If we form a matrix from these column eigenvectors,

$$U^\dagger=(\cdots,\underset{\sim}{\xi}^{(\beta)},\underset{\sim}{\xi}^{(\beta')},\cdots) , \tag{2.28a}$$

then

$$UhU^\dagger=h_D , \tag{2.28b}$$

where h_D is diagonal with matrix elements E_β. The matrix elements of U are therefore given by

$$U_{\alpha\beta}=\xi_\beta^{(\alpha)*} \; . \tag{2.28c}$$

From Lowdin's theorem,[10]

$$\xi_\beta^{(\alpha)}=\begin{cases}\eta_\alpha, & \text{if } \alpha=\beta\\ \dfrac{\eta_\alpha Z_{\beta\alpha}(E_\alpha)}{(E_\alpha-h_{\beta\beta})}, & \text{if } \alpha\neq\beta\end{cases} \tag{2.29a}$$

where

$$\eta_\alpha=\left[1+\sum_{\beta(\neq\alpha)}\frac{|Z_{\beta\alpha}(E_\alpha)|^2}{(E_\alpha-h_{\beta\beta})^2}\right]^{-1/2} \; . \tag{2.29b}$$

Using $Z^*_{\beta\alpha}(E)=Z_{\alpha\beta}(E)$, one obtains

$$U_{\beta\beta'}=\begin{cases}\eta_\beta, & \text{if } \beta=\beta'\\ \dfrac{\eta_\beta Z_{\beta\beta'}(E_\beta)}{(E_\beta-h_{\beta'\beta'})} & \text{if } \beta\neq\beta' \; .\end{cases} \tag{2.30}$$

In the next section we use the Z matrix to evaluate the renormalized energies and the matrix elements of U for simple band structures.

The transformation U renormalizes the Bloch functions. To obtain the renormalized Bloch functions, we note that a bare Bloch function is given by

$$\psi_\beta^B(\mathbf{x})=\langle \mathbf{x}|c_\beta^\dagger|\text{vac}\rangle=\langle \text{vac}|\psi(\mathbf{x})c_\beta^\dagger|\text{vac}\rangle \; , \tag{2.31}$$

where $\psi(\mathbf{x})$ is the field operator. Therefore, a renormalized Bloch function is given by

$$\begin{aligned}\psi_\beta^{\text{RB}}(\mathbf{x})&=\langle \text{vac}|\psi(\mathbf{x})C_\beta^\dagger|\text{vac}\rangle\\ &=\sum_{\beta'}U^*_{\beta\beta'}\psi^B_{\beta'}(\mathbf{x}) \; .\end{aligned} \tag{2.32}$$

$U_{\beta\beta'}$ depends on the electronic wave vectors through the matrix elements of $(h_{\beta\beta'})$. If one can neglect the photon momenta in (2.16a) and (2.17), as in the case of optical wavelengths, then $(h_{\beta\beta'})$ is periodic in the reciprocal lattice space, as are U and the renormalized Bloch functions.

The momentum Bloch functions[11] can be inferred from (2.32) under the periodicity assumption. The bare Bloch functions and the momentum Bloch functions are related by

$$\psi^B_{n\mathbf{k}}(\mathbf{x})=\sum_{\mathbf{G}}e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{x}}\phi_n^B(\mathbf{k}+\mathbf{G}) \; , \tag{2.33}$$

where the $\mathbf{G}$'s are the reciprocal lattice vectors. Expanding Eq. (2.32) as in (2.33), one infers that the renormalized momentum Bloch functions are given by

$$\begin{aligned}\phi_n^{\text{RB}}(\mathbf{k}+\mathbf{G})&=\sum_{n',\mathbf{k}'}U^*_{n\mathbf{k};n'\mathbf{k}'}\phi^B_{n'}(\mathbf{k}'+\mathbf{G})\\ &=\sum_{n'}U^*_{nn'}(\mathbf{k})\phi^B_{n'}(\mathbf{k}+\mathbf{G}) \; .\end{aligned} \tag{2.34}$$

The last step follows from the fact that $h_{\beta\beta'}\propto\delta_{\mathbf{kk}'}$ when photon wave vectors are neglected.

One may use (2.34) to demonstrate the renormalization of the periodic crystal potential. In the case of normal momentum Bloch functions, the components of the periodic potential can be written as[11]

$$\mathcal{V}^0_{\mathbf{G}}=\sum_n E_n^0(\mathbf{k}=0)\phi_n^B(-\mathbf{G})\phi_n^B(0) \; . \tag{2.35}$$

Postulating the same relation for the renormalized quantities, for the component of the renormalized pseudopotential one obtains

$$\mathcal{V}^R_{\mathbf{G}}=\sum_n E_n(0)\phi_n^{\text{RB}}(-\mathbf{G})\phi_n^{\text{RB}}(0) \; . \tag{2.36}$$

Using (2.34) and

$$\begin{aligned}\sum_n U^*_{nn''}(0)E_n(0)U_{nn'}(0)&=(U^\dagger h_D U)_{n''n'}\\ &=[h(0)]_{n''n'} \; ,\end{aligned} \tag{2.37}$$

one finds

$$\mathcal{V}^R_{\mathbf{G}}=\mathcal{V}^0_{\mathbf{G}}+\sum_{n'n''}[\delta h(0)]_{n'n''}\phi^{B*}_{n'}(-\mathbf{G})\phi^B_{n''}(0) \; , \tag{2.38a}$$

where

$$[\delta h(0)]_{nn'}=h_{nn'}(0)-E_n^0(0)\delta_{nn'} \; . \tag{2.38b}$$

The transformation U affects all other electronic couplings such as the Coulomb coupling and the coupling to phonons. For example, writing the bare Coulomb coupling in the form

$$H_c^0=\sum_{\beta\beta'\beta''\beta'''}v^0(\beta\beta'\beta''\beta''')c^\dagger_\beta c^\dagger_{\beta'}c_{\beta''}c_{\beta'''} \; , \tag{2.39}$$

one finds from (2.20) that the renormalized Coulomb coupling becomes

$$v(\beta\beta'\beta''\beta''')=\sum_{\lambda\lambda'\lambda''\lambda'''}U_{\beta\lambda}U_{\beta'\lambda'}v^0(\lambda\lambda'\lambda''\lambda''')U^\dagger_{\lambda''\beta''}U^\dagger_{\lambda'''\beta'''} \tag{2.40a}$$

and

$$H_c=\sum_{\beta\beta'\beta''\beta'''}v(\beta\beta'\beta''\beta''')C^\dagger_\beta C^\dagger_{\beta'}C_{\beta''}C_{\beta'''} \; . \tag{2.40b}$$

Some consequences of (2.40) were investigated in Ref. 3. The main conclusion is that the charge of the Bloch electron which is concentrated at a point is transformed into a charge cloud that can couple to itself. The overall effectiveness of the Coulomb coupling is reduced, resulting for example in lower excitonic binding energies. Renormalized electron-phonon coupling will be discussed elsewhere.

The transformations T and U deal with the quantum kinematics of the electron-field coupling problem. The dynamics of the problem is determined by the Heisenberg equations of motion for the renormalized operators. These equations of motion determine the deviations of the system relative to a fixed set of coherent-field modes. The equations further demonstrate that the renormalized energies E_β are the actual single-particle excitation energies in coherent fields. For instance, taking just the coupling given by H_I, one finds

$$\left[i\hbar\frac{d}{dt}-\hbar\omega_\nu\right]a_\nu=\hbar\omega_\nu\alpha_\nu+\sum_{\beta\beta'}G^{\nu*}_{\beta'\beta}C^\dagger_\beta C_{\beta'} , \tag{2.41a}$$

$$\left[i\hbar\frac{d}{dt}-E_\beta\right]C_\beta=\sum_{\mu\beta'}(G^\mu_{\beta\beta'}a_\mu+G^{\mu*}_{\beta'\beta}a^\dagger_\mu)C_{\beta'} . \tag{2.41b}$$

In the interaction representation,

$$a_\nu(t)\to e^{-i\omega_\nu t}\tilde{a}_\nu(t) ,$$
$$C_\beta(t)\to e^{-iE_\beta t/\hbar}\tilde{C}_\beta(t) , \tag{2.42}$$

and the equations of motion become

$$i\hbar\frac{d}{dt}\tilde{a}_\nu=\omega_\nu\alpha_\nu e^{i\omega_\nu t}+\sum_{\beta\beta'}e^{i(\hbar\omega_\nu+E_\beta-E_{\beta'})t/\hbar}G^\nu_{\beta\beta'}\tilde{C}^\dagger_\beta\tilde{C}_{\beta'} , \tag{2.43a}$$

$$i\hbar\frac{d}{dt}\tilde{C}_\beta=\sum_{\mu\beta'}(G^\mu_{\beta\beta'}e^{i(E_\beta-E_{\beta'}-\hbar\omega_\mu)t/\hbar}\tilde{a}_\mu + G^{\mu*}_{\beta'\beta}e^{i(E_\beta-E_{\beta'}-\hbar\omega_\nu)t/\hbar}\tilde{a}^\dagger_\mu) . \tag{2.43b}$$

If the mode ν is not included in the reference set (or if it is not coherent at all), then $\alpha_\nu=0$ and (2.41a) and (2.43a) are identical in form to the equations of motion without the renormalization. Equation (2.43a) shows that, as far as a probe field is concerned, spectral resonances are at the renormalized energy differences. This is essentially an operational definition of the energy difference between two distinct states. E_β's are therefore the actual single-particle energies in the presence of a coherent field. They differ from quasienergies obtained in classical treatments of the electromagnetic field.[4,5] We emphasize again that the coherent-field renormalization differs fundamentally from the semiclassical theories of the electron-field coupling. The coherent-field renormalization transformations are carried out at a fixed moment in time. One deals with a Hamiltonian which is not explicitly time dependent. The electromagnetic field is treated completely within the quantum-theoretical framework. The formalism is capable of handling partial coherence. The results we obtain and their interpretations are different from those when the electromagnetic field is treated classically.

III. RENORMALIZATION OF SIMPLE BAND STRUCTURES

In this section we discuss the effects of the coherent-field renormalization on two-band and four-band models.

A particularly simple form of the transformation U results when we consider a two-band model with a conduction and a valence band. We assume that $|g^\mu_{\beta\beta'}|\gg|\tilde{a}^\mu_{\beta\beta'}|$ and neglect the $\tilde{a}$ coupling. We also assume that photon wave vectors are negligible compared to electronic wave vectors. One then has, for interband transitions,

$$g^\mu_{c\mathbf{k};v\mathbf{k}'}\simeq\delta_{\mathbf{k}\mathbf{k}'}(e\mathcal{A}_\mu/mc)\hat{\varepsilon}_\mu\cdot\mathbf{P}_{cv}(\mathbf{k}) , \tag{3.1a}$$

$$h_{c\mathbf{k};v\mathbf{k}'}\simeq\delta_{\mathbf{k}\mathbf{k}'}(\hbar/m)\mathbf{K}_f\cdot\mathbf{P}_{cv}(\mathbf{k}) , \tag{3.1b}$$

where

$$\mathbf{K}_f=(2e/\hbar c)\sum_\mu\hat{\varepsilon}_\mu\mathcal{A}_\mu\mathrm{Re}\alpha_\mu . \tag{3.1c}$$

For intraband transitions,

$$g^\mu_{n\mathbf{k};n\mathbf{k}'}\simeq\delta_{\mathbf{k}\mathbf{k}'}(e\mathcal{A}_\mu/c\hbar)\hat{\varepsilon}_\mu\cdot\nabla_\mathbf{k}E^0_{n\mathbf{k}},\quad n=c,v \tag{3.2a}$$

$$h_{n\mathbf{k};n\mathbf{k}'}\simeq\delta_{\mathbf{k}\mathbf{k}'}(E^0_{n\mathbf{k}}+\mathbf{K}_f\cdot\nabla_\mathbf{k}E^0_{n\mathbf{k}}) . \tag{3.2b}$$

The matrix elements (3.1) and (3.2) do not couple different electronic wave vectors. Therefore in the following, we will sometimes omit explicit references to **k**'s. The form of Eq. (3.2a) follows from the fact that[12]

$$\langle\mathbf{v}\rangle=m^{-1}\langle n\mathbf{k}|\mathbf{p}|n\mathbf{k}\rangle=\hbar^{-1}\nabla_\mathbf{k}E^0_{n\mathbf{k}} . \tag{3.3}$$

Consider now $Z_{cc}(E)$. It is clear from (3.1), (3.2), and (2.24b) that only even powers of h_{cv} appear in the sum in (2.24a):

$$Z_{cc}(E)=h_{cc}+\frac{|h_{cv}|^2}{(E-h_{vv})}+\frac{|h_{cv}|^4}{(E-h_{vv})^2(E-h_{cc})}+\frac{|h_{cv}|^6}{(E-h_{vv})^3(E-h_{cc})^2}+\cdots . \tag{3.4a}$$

Regrouping,

$$Z_{cc}(E)=h_{cc}+\frac{|h_{cv}|^2}{(E-h_{vv})}\left[1+\frac{|h_{cv}|^2}{(E-h_{cc})(E-h_{vv})}+\frac{|h_{cv}|^4}{(E-h_{cc})^2(E-h_{vv})^2}+\cdots\right]$$
$$=h_{cc}+\frac{|h_{cv}|^2(E-h_{cc})}{[(E-h_{cc})(E-h_{vv})-|h_{cv}|^2]} . \tag{3.4b}$$

Replacing h_{cc} by h_{vv} in (3.4b) yields $Z_{vv}(E)$. Using (2.25) we obtain the eigenvalue equation

$$(E-h_{cc})(E-h_{vv})-2|h_{cv}|^2=0 , \tag{3.5}$$

which has solutions

$$E=\tfrac{1}{2}(h_{cc}+h_{vv})\pm\tfrac{1}{2}[(h_{cc}-h_{vv})^2+8|h_{cv}|^2]^{1/2} . \tag{3.6}$$

Define

$$e_\mathbf{k}=-(h_{cc}-h_{vv})+[(h_{cc}-h_{vv})^2+8|h_{cv}|^2]^{1/2} . \tag{3.7}$$

The renormalized conduction- and valence-band energies are given by

$$E_c = h_{cc} + e_{\mathbf{k}}/2 \ , \tag{3.8a}$$

$$E_v = h_{vv} - e_{\mathbf{k}}/2 \ . \tag{3.8b}$$

The same solutions are obtained from $E = Z_{vv}(E)$.

For convenience, let us also define the quantities

$$\hbar\Omega_E = 2\sqrt{2}|h_{cv}| = 4\sqrt{2}(e/mc)\left|\sum_\mu \mathcal{A}_\mu \hat{\varepsilon}_\mu \cdot \mathbf{P}_{cv} \mathrm{Re}\alpha_\mu\right| \ , \tag{3.9a}$$

$$\delta W_{n\mathbf{k}} = \mathbf{K}_f \cdot \nabla_{\mathbf{k}} E^0_{n\mathbf{k}} = (2e/c\hbar)\sum_\mu \mathcal{A}_\mu (\mathrm{Re}\alpha_\mu)\hat{\varepsilon}_\mu \cdot \nabla_{\mathbf{k}} E^0_{n\mathbf{k}} \ . \tag{3.9b}$$

Ω_E is similar to a Rabi frequency.[13] It is slightly different from the usual definition of a Rabi frequency in that $\mathbf{P}_{cv}$ appears in (3.9a) instead of a genuine dipole moment, and there is a phase factor $\cos\phi_\mu$ coming from $\mathrm{Re}\alpha_\mu$ for each coherent mode in the reference set. With the definitions (3.9a) and (3.9b), $e_{\mathbf{k}}$, E_c, and E_v can be written in more explicit forms:

$$e_{\mathbf{k}} = -(E^0_{c\mathbf{k}} - E^0_{v\mathbf{k}} + \delta W_{c\mathbf{k}} - \delta W_{v\mathbf{k}}) + [(E^0_{c\mathbf{k}} - E^0_{v\mathbf{k}} + \delta W_{c\mathbf{k}} - \delta W_{v\mathbf{k}})^2 + \hbar^2\Omega_E^2]^{1/2} \ , \tag{3.10a}$$

$$E_{c\mathbf{k}} = \tfrac{1}{2}(E^0_{c\mathbf{k}} + E^0_{v\mathbf{k}} + \delta W_{c\mathbf{k}} + \delta W_{v\mathbf{k}}) + \tfrac{1}{2}[(E^0_{c\mathbf{k}} - E^0_{v\mathbf{k}} + \delta W_{c\mathbf{k}} - \delta W_{v\mathbf{k}})^2 + \hbar^2\Omega_E^2]^{1/2} \ , \tag{3.10b}$$

$$E_{v\mathbf{k}} = \tfrac{1}{2}(E^0_{c\mathbf{k}} + E^0_{v\mathbf{k}} + \delta W_{c\mathbf{k}} + \delta W_{v\mathbf{k}}) - \tfrac{1}{2}[(E^0_{c\mathbf{k}} - E^0_{v\mathbf{k}} + \delta W_{c\mathbf{k}} - \delta W_{v\mathbf{k}})^2 + \hbar^2\Omega_E^2]^{1/2} \ . \tag{3.10c}$$

Next, consider $Z_{cv}(E)$ in order to determine the matrix elements of U. From (2.24a), (2.24b), and (3.1), one has (for a given electronic wave vector)

$$Z_{cv} = h_{cv} + \frac{h_{cv}h_{vc}h_{cv}}{(E-h_{vv})(E-h_{cc})} + \frac{h_{cv}h_{vc}h_{cv}h_{vc}h_{cv}}{(E-h_{vv})^2(E-h_{cc})^2} + \cdots = h_{cv}\sum_{n=0}^{\infty}\frac{|h_{cv}|^{2n}}{(E-h_{cc})^n(E-h_{vv})^n} \ . \tag{3.11}$$

Thus,

$$Z_{cv}(E) = \frac{h_{cv}(E-h_{cc})(E-h_{vv})}{[(E-h_{cc})(E-h_{vv}) - |h_{cv}|^2]} \ . \tag{3.12}$$

$Z_{vc}(E)$ is obtained by replacing h_{cv} with $h_{vc} = h^*_{cv}$ in (3.12). Because E_c and E_v satisfy (3.5),

$$Z_{cv}(E_c) = \frac{h_{cv}}{|h_{cv}|^2}(E_c - h_{cc})(E_c - h_{vv}) \ , \tag{3.13a}$$

$$Z_{vc}(E_v) = \frac{h^*_{cv}}{|h_{cv}|^2}(E_v - h_{cc})(E_v - h_{vv}) \ . \tag{3.13b}$$

Using (3.8), these equations can be rewritten as

$$Z_{cv}(E_c) = \frac{4h_{cv}e_{\mathbf{k}}}{\hbar^2\Omega_E^2}(E_c - h_{vv}) \ , \tag{3.14a}$$

$$Z_{vc}(E_v) = -\frac{4h^*_{cv}e_{\mathbf{k}}}{\hbar^2\Omega_E^2}(E_v - h_{cc}) \ . \tag{3.14b}$$

One sees from (3.14) and (2.29b) that the normalization constants for the conduction and valence bands are identical:

$$\eta_{\mathbf{k}} = \frac{\hbar\Omega_E}{(2e^2_{\mathbf{k}} + \hbar^2\Omega_E^2)^{1/2}} \ . \tag{3.15}$$

Define

$$\lambda_{\mathbf{k}} = \frac{4h_{cv}e_{\mathbf{k}}}{\hbar^2\Omega_E^2} \ . \tag{3.16}$$

From (2.30), the matrix elements of U are given by

$$U_{cc} = \eta_{\mathbf{k}}, \quad U_{cv} = \eta_{\mathbf{k}}\lambda_{\mathbf{k}} \ , \qquad U_{vc} = -\eta_{\mathbf{k}}\lambda^*_{\mathbf{k}}, \quad U_{vv} = \eta_{\mathbf{k}} \ . \tag{3.17}$$

Note that

$$\eta^2_{\mathbf{k}}(1 + |\lambda_{\mathbf{k}}|^2) = 1 \tag{3.18}$$

and U is properly unitary. We may now write the transformation (2.20) as

$$C_{c\mathbf{k}} = \eta_{\mathbf{k}}(c_{c\mathbf{k}} + \lambda_{\mathbf{k}}c_{v\mathbf{k}}) \ , \qquad C_{v\mathbf{k}} = \eta_{\mathbf{k}}(-\lambda^*_{\mathbf{k}}c_{c\mathbf{k}} + c_{v\mathbf{k}}) \ . \tag{3.19a}$$

Conversely,

$$c_{c\mathbf{k}} = \eta_{\mathbf{k}}(C_{c\mathbf{k}} - \lambda_{\mathbf{k}}C_{v\mathbf{k}}) \ , \qquad C_{v\mathbf{k}} = \eta_{\mathbf{k}}(\lambda^*_{\mathbf{k}}C_{c\mathbf{k}} + C_{v\mathbf{k}}) \ . \tag{3.19b}$$

The renormalized Bloch functions are given by

$$\psi^{\mathrm{RB}}_{c\mathbf{k}}(\mathbf{x}) = \eta_{\mathbf{k}}[\psi^B_{c\mathbf{k}}(\mathbf{x}) + \lambda^*_{\mathbf{k}}\psi^B_{v\mathbf{k}}(\mathbf{x})] \ , \qquad \psi^{\mathrm{RB}}_{v\mathbf{k}}(\mathbf{x}) = \eta_{\mathbf{k}}[-\lambda_{\mathbf{k}}\psi^B_{c\mathbf{k}}(\mathbf{x}) + \psi^B_{v\mathbf{k}}(\mathbf{x})] \ . \tag{3.20}$$

Let us go back to the renormalized energies. It is clear from (3.10) that the effects of the renormalization vary as one moves to different regions of the BZ. Let $E^0_{c,v}$ near the center of the BZ be given by

$$E^0_{c\mathbf{k}} = \frac{E^0_G}{2} + \frac{\hbar^2\mathbf{k}^2}{2m^0_c} \ , \qquad E^0_{v\mathbf{k}} = -\frac{E^0_G}{2} - \frac{\hbar^2\mathbf{k}^2}{2m^0_v} \ . \tag{3.21}$$

Define also

$$m_{cv} = \frac{m^0_v m^0_c}{m^0_v - m^0_c} \ , \tag{3.22a}$$

$$m_r = \frac{m_v^0 m_c^0}{m_v^0 + m_c^0} . \quad (3.22b)$$

From (3.10), the renormalized energies in the vicinity of $\mathbf{k}=0$ are given by[14]

$$E_{c,v}(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{4m_{cv}} + \frac{\hbar^2 \mathbf{k}\cdot\mathbf{K}_f}{2m_{cv}} \pm \frac{1}{2}\left[\hbar^2\Omega_E^2 + \left[E_G^0 + \frac{\hbar^2\mathbf{k}^2}{2m_r} + \frac{\hbar^2\mathbf{k}\cdot\mathbf{K}_f}{m_r}\right]^2\right]^{1/2} . \quad (3.23)$$

Let us estimate the size of the various terms in (3.23). For this purpose let us assume that only one mode designated by $\mu=0$ is coherent. For practical semiconductors like the III-V compounds, typical parametric values are

$$|\mathbf{P}_{cv}|/\hbar \sim 2\times 10^8 \text{ cm}^{-1}, \quad E_G^0 \sim 1 \text{ eV}, \quad m_c^0 \sim 0.1m . \quad (3.24a)$$

These numbers are chosen so that they are consistent with the approximate f-sum rule for the two-band model:

$$\frac{\hbar^2|\mathbf{P}_{cv}|^2}{m^2 E_G^0} \simeq \sum_i \frac{\partial^2}{\partial k_i^2} E_c . \quad (3.25)$$

Let

$$\hbar\omega_0 \sim 1 \text{ eV}, \quad n_0 \sim 4, \quad \bar{N}_0/V_{\text{ol}} \sim 10^{18} \text{ cm}^{-3} . \quad (3.24')$$

This photon density corresponds to an intensity

$$I_0 = \hbar\omega_0 c (n_0)^{-1} \bar{N}_0 V_{\text{ol}}^{-1} \sim 10^9 \text{ W/cm}^2 . \quad (3.24'')$$

For these numbers, one finds from (3.1c) and (3.9a) that

$$\hbar\Omega_E \sim \frac{8e|\mathbf{P}_{cv}|}{m}\left[\frac{\pi\hbar\bar{N}_0}{n_0^2\omega_0 V_{\text{ol}}}\right]^{1/2} \sim 10^{-1} \text{ eV} \quad (3.24''')$$

and

$$|\mathbf{K}_f| \sim 2\left[\frac{2\pi e^2 \bar{N}_0}{\hbar n_0^2 \omega_0 V_{\text{ol}}}\right]^{1/2} \sim 5\times 10^5 \text{ cm}^{-1} . \quad (3.24'''')$$

Equation (3.24''') shows that the energy shifts and the changes in the density of states brought about by the renormalization are significant and experimentally observable. The value of $|\mathbf{K}_f|$ is comparable to Fermi wave vectors. Consider the Fermi wave vector for an electron density $N_e \sim 10^{18}$ cm^{-3} at zero temperature:

$$k_F = (3\pi^2 N_e)^{1/3} \sim 3\times 10^6 \text{ cm}^{-1} . \quad (3.26)$$

This is only six times larger than $|\mathbf{K}_f|$.

One can obtain the renormalized effective masses from (3.23) by taking appropriate derivatives. Some care must be shown in the definition of the renormalized effective masses, since the renormalized bands are not parabolic. Near the BZ center we define the effective mass in the ith direction as

$$m_{ni}^* = \hbar \lim_{\mathbf{k}\to 0}\left[\frac{\partial^2 E_n}{\partial k_i^2}\right]^{-1} . \quad (3.27)$$

Consequently the sign of the renormalized effective mass can be positive or negative according to this definition, although the unrenormalized masses m_c^0 and m_v^0 are positive quantities. From (3.23) one obtains

$$\frac{1}{m_{ni}^*} = \frac{1}{2m_{cv}} + \frac{\sigma_n}{2m_r\gamma_E}\left[1 + \frac{\hbar^2 K_{fi}^2}{2m_r E_G^0}(2 - 1/\gamma_E^2)\right] , \quad (3.28a)$$

where

$$\sigma_n = \begin{cases} 1 & \text{if } n=c \\ -1 & \text{if } n=v \end{cases} \quad (3.28b)$$

and

$$\gamma_E = [1 + (\hbar\Omega_E/E_G^0)^2]^{1/2} . \quad (3.28c)$$

If $\hbar^2 K_{fi}^2/2m_r \sim E_G^0$, for example for extremely narrow band gaps, then the renormalized effective masses become directional. The increased inertia induced by γ_E is partially compensated by the K_{fi} term. If $\hbar^2 K_{fi}^2/2m_r \ll E_G^0$, then the effective masses are isotropic and one can omit the subscript i:

$$m_c^* = \frac{2m_v^0 m_c^0 \gamma_E}{m_v^0 + m_c^0 + (m_v^0 - m_c^0)\gamma_E} , \quad (3.29a)$$

$$m_v^* = \frac{2m_v^0 m_c^0 \gamma_E}{[-(m_v^0 + m_c^0) + (m_v^0 - m_c^0)\gamma_E]} . \quad (3.29b)$$

If $m_c^0 \ll m_v^0$, then

$$m_c^* \simeq 2m_c^0\left[\frac{\gamma_E}{\gamma_E + 1}\right] , \quad (3.30a)$$

$$m_v^* \simeq 2m_c^0\left[\frac{\gamma_E}{\gamma_E - 1}\right] . \quad (3.30b)$$

If $m_c^0 \gg m_v^0$, then

$$m_c^0 \simeq -2m_v^0\left[\frac{\gamma_E}{\gamma_E - 1}\right] , \quad (3.31a)$$

$$m_v^0 \simeq -2m_v^0\left[\frac{\gamma_E}{\gamma_E + 1}\right] . \quad (3.31b)$$

If $m_c^0 = m_v^0 = m^0$, then

$$m_c^* = -m_v^* = m^0\gamma_E . \quad (3.32)$$

Consider the band separation at $\mathbf{k}=0$. This separation need not be the actual band gap for the renormalized bands, since the maximum and the minimum of the conduction and valence bands, respectively, need not be at $\mathbf{k}=0$. From (3.23),

$$E_c(0) - E_v(0) = E_G^0\gamma_E . \quad (3.33)$$

Thus, the band separation at the center of the BZ is increased by the coherent fields.

Equations (3.30) and (3.31) show that the signs of the curvatures of the renormalized bands may be changed relative to their original signs if the two bands have vastly different effective masses. The unrenormalized band curvatures are, from (3.21), $m_c^0>0$ and $-m_v^0<0$. According to (3.30), if $m_c^0 \ll m_v^0$, then the renormalized band curvatures are both positive near $\mathbf{k}=0$ (in the plane perpendicular to $\mathbf{K}_f$). This implies a coherent-field-induced depression in the band around $\mathbf{k}=0$, as illustrated in Fig. 1(a). Conversely, if $m_c^0 \gg m_v^0$, then according to (3.31) a hill forms in the renormalized conduction band near $\mathbf{k}=0$ in the plane perpendicular to $\mathbf{K}_f$, as shown in Fig. 1(b). If the bare bands are symmetric, the renormalized bands remain symmetric, although the renormalized effective mass increases as indicated in (3.33).

More quantitatively, the extrema of $E_{c,v}$ can be obtained from (3.23) by setting $\partial E_{c,v}/\partial k_i=0$:

$$(k_i+K_{fi})\left\{\frac{\left[E_G^0+\dfrac{\hbar^2k^2}{2m_r}+\dfrac{\hbar^2\mathbf{K}_f\cdot\mathbf{k}}{m_r}\right]}{\left[\hbar^2\Omega_E^2+\left[E_G^0+\dfrac{\hbar^2k^2}{2m_r}+\dfrac{\hbar^2\mathbf{K}_f\cdot\mathbf{k}}{m_r}\right]^2\right]^{1/2}}+\sigma_n\frac{(m_v^0-m_c^0)}{(m_v^0+m_c^0)}\right\}=0\ . \tag{3.34}$$

The first factor yields the solution

$$\mathbf{k}=-\mathbf{K}_f\ . \tag{3.35}$$

The second factor in (3.34) yields a solution only if

$$\frac{\hbar^2K_f^2\cos^2\theta}{2m_rE_G^0}>1+\sigma_n\frac{(m_v^0-m_c^0)}{2(m_v^0m_c^0)^{1/2}}\left[\frac{\hbar\Omega_E}{E_G^0}\right], \tag{3.36}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{K}_f$. If (3.36) is satisfied, then there are two more extrema for each band:

$$k_{1,2}^{(n)}=-K_f\cos\theta\pm\left[K_f^2\cos^2\theta+\sigma_n\frac{(m_v^0-m_c^0)}{2(m_v^0m_c^0)^{1/2}}\left[\frac{\hbar\Omega_E}{E_G^0}\right]\right]^{1/2}. \tag{3.37}$$

If (3.36) is not satisfied, there is only one solution given by (3.35), where E_c is minimum and E_v is maximum. If (3.36) is satisfied, E_c has a local maximum at $\mathbf{k}=-\mathbf{K}_f$, and E_v a local minimum. E_c is minimum at $k_{1,2}^{(c)}$; E_v is maximum at $k_{1,2}^{(v)}$. Thus, whenever (3.36) is satisfied, the renormalized band structure has an indirect band gap.

If $m_c^0=m_v^0=m^0$, then

$$E_n(\mathbf{k})=\frac{\sigma_n}{2}\left[\hbar^2\Omega_E^2+\left[E_G^0+\frac{\hbar^2\mathbf{k}^2}{m^0}+\frac{2\hbar^2\mathbf{k}\cdot\mathbf{K}_f}{m^0}\right]^2\right]^{1/2}. \tag{3.38}$$

The gap is direct, but the band edge has moved from $\mathbf{k}=0$ to $\mathbf{k}=-\mathbf{K}_f$, where the gap is given by

$$E_c(-\mathbf{K}_f)-E_v(-\mathbf{K}_f)=\left[\hbar^2\Omega_E^2+\left[E_G^0-\frac{\hbar^2\mathbf{K}_f^2}{m^0}\right]^2\right]^{1/2}. \tag{3.39}$$

We can express K_f in terms of $\hbar\Omega_E$ when there is just one coherent mode:

$$K_f=\frac{m\,\Omega_E}{2\sqrt{2}|\mathbf{P}_{cv}|}\ , \tag{3.40a}$$

which implies

$$\frac{\hbar^2K_f^2}{m^0E_G^0}=\frac{1}{8}\left[\frac{\hbar\Omega_E}{E_G^0}\right]^2\left[\frac{m^2E_G}{m^0|P_{cv}|^2}\right]=\frac{1}{24}\left[\frac{\hbar\Omega_E}{E_G^0}\right]^2. \tag{3.40b}$$

The second step of (3.40b) follows from the f-sum rule (3.25). One can thus rewrite (3.39) in the form

$$E_c(-\mathbf{K}_f)-E_v(-\mathbf{K}_f)=E_G^0\left[1+\frac{11}{12}\left[\frac{\hbar\Omega_E}{E_G^0}\right]^2+\frac{1}{576}\left[\frac{\hbar\Omega_E}{E_G^0}\right]^4\right]^{1/2}. \tag{3.41}$$

Near the boundaries of the BZ, typically $k\sim10^8\ \text{cm}^{-1}$. Comparing this with (3.24''''), one sees that $K_f \ll k$ and

$$\mathbf{K}_f\cdot\nabla_{\mathbf{k}}E_n^0(\mathbf{k})\simeq E_n^0(\mathbf{k}+\mathbf{K}_f)-E_n^0(\mathbf{k})\ . \tag{3.42a}$$

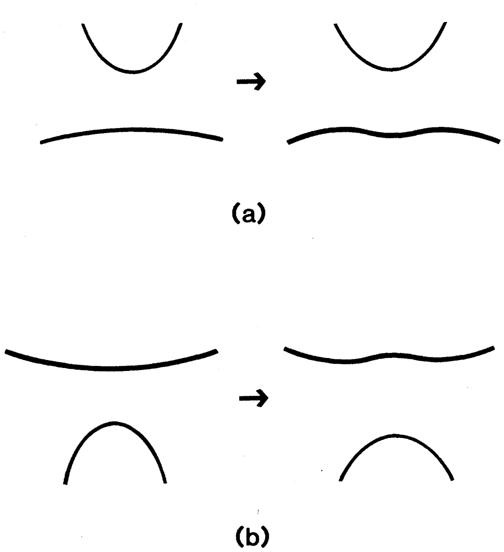

FIG. 1. Indirect-gap formations.

Thus, near the BZ boundaries (or more generally, whenever $k \gg K_f$), one can set

$$E^0_{n\mathbf{k}} + \delta W_{n\mathbf{k}} \simeq E^0_n(\mathbf{k}+\mathbf{K}_f) \ . \tag{3.42b}$$

The renormalized energies become

$$E_{c,v}(\mathbf{k}) = \tfrac{1}{2}[E^0_c(\mathbf{k}+\mathbf{K}_f) + E^0_v(\mathbf{k}+\mathbf{K}_f)] \pm \tfrac{1}{2}\{[E^0_c(\mathbf{k}+\mathbf{K}_f) - E^0_v(\mathbf{k}+\mathbf{K}_f)]^2 + \hbar^2\Omega_E^2\}^{1/2} \ . \tag{3.43}$$

Equation (3.23) and the equations following it are valid if $|g^{\mu}_{cv}| \gg |\tilde{d}_{nn}|$. For a single coherent mode, this implies that

$$\frac{\bar{N}_0}{V_{\text{ol}}} \ll \frac{n_0^2\omega_0^2|\mathbf{P}_{cv}|^2}{8\pi e^2\hbar} \ , \tag{3.44a}$$

which for the example of (3.24) becomes

$$\frac{\bar{N}_0}{V_{\text{ol}}} \ll 2\times 10^{23}\ \text{cm}^{-3}, \quad I_0 \ll 2\times 10^{14}\ \text{W/cm}^2 \ . \tag{3.44b}$$

Thus, for most practical intensities, the approximation stated at the beginning of the section is quite reasonable.

In useful semiconductors like the III-V compounds, there is more than one valence band. Typically there are three degenerate valence bands, if the spin-orbit couplings are neglected. The spin-orbit coupling separates and lowers one of these bands. Therefore a more realistic band structure is the following four-band model. Suppose that there are three valence bands and one conduction band which are coupled by the interband momentum matrix elements $\mathbf{P}_{cv_i}$, $v_i = 1,2,3$. We allow these matrix elements to be distinct. We also let $\mathbf{P}_{v_iv_j} = 0$. The matrix elements $h_{\beta\beta'}$ take the form

$$h_{c\mathbf{k};v_i\mathbf{k}'} = \delta_{\mathbf{k}\mathbf{k}'}\frac{\hbar}{m}\mathbf{K}_f\cdot\mathbf{P}_{cv_i}(\mathbf{k}) \ , \tag{3.45a}$$

$$h_{n\mathbf{k};n\mathbf{k}'} = \delta_{\mathbf{k}\mathbf{k}'}(E^0_{n\mathbf{k}} + \mathbf{K}_f\cdot\nabla_{\mathbf{k}}E^0_{n\mathbf{k}}), \quad n = c, v_1, v_2, v_3 \ . \tag{3.45b}$$

Consider $Z_{cc}(E)$. From (2.24),

$$Z_{cc} = h_{cc} + \sum_i w_i + \sum_{i,j}\frac{w_iw_j}{(E-h_{cc})} + \sum_{i,j,l}\frac{w_iw_jw_l}{(E-h_{cc})^2} + \cdots \ , \tag{3.46a}$$

where

$$w_i = \frac{|h_{cv_i}|^2}{E-h_{v_iv_i}}, \quad i = 1,2,3 \ . \tag{3.46b}$$

Thus,

$$Z_{cc}(E) = h_{cc} + \frac{(E-h_{cc})\left[\sum w_i\right]}{\left[E-h_{cc}-\sum w_i\right]} \ . \tag{3.47}$$

The eigenvalue equation becomes

$$E - h_{cc} = 2\sum w_i \ . \tag{3.48a}$$

More explicitly,

$$(E-h_{cc})\prod_i(E-h_{v_iv_i}) = 2\sum_j\left[|h_{cv_j}|^2\prod_{i\neq j}(E-h_{v_iv_i})\right] \ . \tag{3.48b}$$

This is a biquadratic equation and can be solved exactly.[15] Rather than dealing with the complicated exact solutions, let us consider a few special cases.

Case 1. Let $h_{v_iv_i} = 0$ (that is, the three valence bands are degenerate with infinite effective mass). Equation (3.48b) yields

$$E^2\left[E(E-h_{cc}) - 2\sum_i|h_{cv_i}|^2\right] = 0 \ . \tag{3.49}$$

Two of the renormalized bands remain degenerate:

$$E_{v_1,v_2} = 0 \ . \tag{3.50a}$$

Setting

$$E^2 - h_{cc}E - 2\sum_i|h_{cv_i}|^2 = 0 \ , \tag{3.51}$$

one finds

$$E_{v_3} = \tfrac{1}{2}h_{cc} - \tfrac{1}{2}\left[|h_{cc}|^2 + 8\sum_i|h_{cv_i}|^2\right]^{1/2} \ , \tag{3.50$'$}$$

$$E_c = \tfrac{1}{2}h_{cc} + \tfrac{1}{2}\left[|h_{cc}|^2 + 8\sum_i|h_{cv_i}|^2\right]^{1/2} \ . \tag{3.50$''$}$$

Thus, one of the valence bands is no longer degenerate. It now has a finite effective mass. This is illustrated in Fig. 2(a). Clearly, the total density of states per unit en-

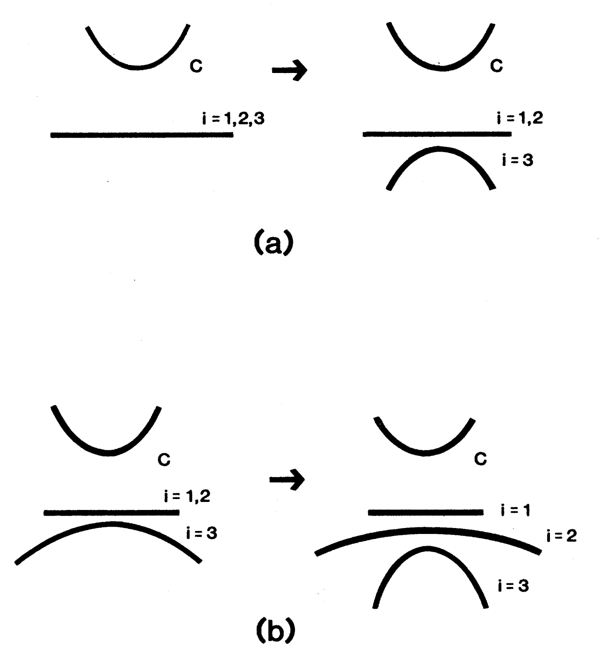

FIG. 2. Removal of degeneracies by the coherent-field renormalization.

ergy for the valence bands has been decreased.

Case 2. Let $h_{v_1v_1}=h_{v_2v_2}=0$, $h_{v_3v_3}\neq 0$. Equation (3.48b) yields

$$E[E(E-h_{cc})(E-h_{v_3v_3})-2|h_{cv_1}|^2(E-h_{v_3v_3})-2|h_{cv_2}|^2(E-h_{v_3v_3})-2|h_{cv_3}|^2E]=0 \; . \tag{3.52}$$

Suppose $|h_{v_3v_3}|<h_{cc}$ and $h_{v_3v_3}<0$. We solve the second factor of (3.52) approximately. One solution for the valence bands is

$$E_{v_1}=0 \; . \tag{3.53a}$$

Let $E_c=h_{cc}+\Delta$, where $\Delta \ll h_{cc}$. Then,

$$E_c \simeq h_{cc}+\frac{2}{h_{cc}}(|h_{cv_1}|^2+|h_{cv_2}|^2)+\frac{2|h_{cv_3}|^2}{(h_{cc}-h_{v_3v_3})} \; . \tag{3.53b}$$

Let $E_{v_3}=h_{v_3v_3}+\Delta'$, $|\Delta'| \ll |h_{v_3v_3}|$. Then,

$$E_{v_3}\simeq h_{v_3v_3}-\frac{2h_{v_3v_3}|h_{cv_3}|^2}{[h_{cc}h_{v_3v_3}-h_{v_3v_3}^2+2(|h_{cv_1}|^2+|h_{cv_2}|^2)]} \; . \tag{3.53c}$$

Finally, let $E_{v_2}=\Delta''$, where $|\Delta''| \ll h_{cc}$, $|h_{v_3v_3}|$. Then,

$$E_{v_2}\simeq \frac{-2h_{v_3v_3}(|h_{cv_1}|^2+|h_{cv_2}|^2)}{(h_{cc}h_{v_3v_3}-2|h_{cv_3}|^2)} \; . \tag{3.53d}$$

Thus, the degeneracy is completely removed. This is illustrated in Fig. 2(b).

Consider the matrix elements of U. From (2.34),

$$\begin{aligned} Z_{v_iv_j}(E) &= \frac{h_{v_ic}h_{cv_j}}{(E-h_{cc})}+\sum_l \frac{h_{v_ic}h_{cv_l}h_{v_lc}h_{cv_j}}{(E-h_{cc})(E-h_{v_lv_l})(E-h_{cc})}+\cdots \\ &= \frac{h_{v_ic}h_{cv_j}}{(E-h_{cc})}\left\{1+\frac{\left[\sum_l w_l\right]}{(E-h_{cc})}+\frac{\left[\sum_l w_l\right]^2}{(E-h_{cc})^2}+\cdots\right\} \\ &= \frac{h_{v_ic}h_{cv_j}}{\left[E-h_{cc}-\sum_l w_l\right]} \; . \end{aligned} \tag{3.54}$$

More explicitly,

$$Z_{v_iv_j}(E)=\frac{h_{v_ic}h_{cv_j}\left[\prod_l (E-h_{v_lv_l})\right]}{\left[(E-h_{cc})\prod_l (E-h_{v_lv_l})-\sum_l \left[|h_{cv_l}|^2\prod_{k\neq l}(E-h_{v_kv_k})\right]\right]} \; . \tag{3.55}$$

Using the eigenvalue equation (3.48b), one finds

$$Z_{v_iv_j}(E_{v_i})=\frac{h_{v_ic}h_{cv_j}\prod_l (E_{v_i}-h_{v_lv_l})}{\sum_l \left[|h_{cv_l}|^2\prod_{k\neq l}(E_{v_i}-h_{v_kv_k})\right]} \; . \tag{3.56}$$

The normalization constant η_{v_i} is given by

$$\eta_{v_i}=\left\{1+\frac{|h_{cv_i}|^2\sum_{j\neq i}|h_{cv_j}|^2\left[\prod_{l\neq j}(E_{v_i}-h_{v_lv_l})\right]^2}{\left[\sum_l \left[|h_{cv_l}|^2\prod_{k\neq l}(E_{v_i}-h_{v_kv_k})\right]\right]^2}\right\}^{-1/2} \; . \tag{3.57}$$

Thus, for $i=j$,

$$U_{v_iv_i}=\eta_{v_i} \; . \tag{3.58a}$$

For $i\neq j$,

$$U_{v_iv_j}=\frac{\eta_{v_i}h_{v_ic}h_{cv_j}\prod_{l\neq j}(E_{v_i}-h_{v_lv_l})}{\left[\sum_l \left[|h_{cv_l}|^2\prod_{k\neq l}(E_{v_i}-h_{v_kv_k})\right]\right]} \; . \tag{3.58b}$$

Similarly,

$$Z_{cv_i}(E)=\frac{h_{cv_i}(E-h_{cc})\prod_j (E-h_{v_jv_j})}{\left[(E-h_{cc})\prod_j (E-h_{v_jv_j})-\sum_j \left[|h_{cv_j}|^2 \prod_{k\neq j} (E-h_{v_kv_k})\right]\right]}, \tag{3.59a}$$

$$\eta_c=\left[1+\frac{\sum_i |h_{cv_i}|^2 \left[\prod_j (E_c-h_{v_jv_j})\right]^2}{\left[\sum_j \left[|h_{cv_j}|^2 \prod_{k\neq j}(E_c-h_{v_kv_k})\right]\right]^2}\right]^{-1/2}, \tag{3.59b}$$

$$U_{cc}=\eta_c , \tag{3.59c}$$

and

$$U_{cv_i}=\frac{\eta_c h_{cv_i}\prod_j (E_c-h_{v_jv_j})}{\left[\sum_j \left[|h_{cv_j}|^2 \prod_{k\neq j}(E_c-h_{v_kv_k})\right]\right]} . \tag{3.59d}$$

The degenerate states are significantly mixed. For example, set $h_{v_iv_i}=0$. Then, for $i\neq j$,

$$U_{v_iv_j}=\frac{\eta_{v_i}h_{v_ic}h_{cv_j}}{(\sum_l |h_{cv_l}|^2)} , \tag{3.60a}$$

where

$$\eta_{v_i}=\frac{\left[\sum_l |h_{cv_l}|^2\right]}{\left[\left[\sum_l |h_{cv_l}|^2\right]^2+|h_{cv_i}|^2\sum_{j\neq i}|h_{cv_j}|^2\right]^{1/2}} . \tag{3.60b}$$

Interestingly, if $|h_{cv_i}|$ is the same for all i, then the above quantities in (3.60a) and (3.60b) are independent of $|h_{cv_i}|$:

$$\eta_{v_i}=\frac{3}{(11)^{1/2}},\quad |U_{v_iv_j}|=\frac{1}{(11)^{1/2}} . \tag{3.61}$$

On the other hand,

$$\eta_c=\left[\frac{3|h_{cv_i}|^2}{E_c^2+3|h_{cv_i}|^2}\right]^{1/2}, \quad |U_{cv_i}|=\left[\frac{E_c^2}{3E_c^2+9|h_{cv_i}|^2}\right]^{1/2} . \tag{3.62}$$

Recalling that $h_{cv_i}\propto \mathbf{K}_f\cdot\mathbf{P}_{cv_i}$, one sees from the results above that the amount of the degeneracy breaking, as well as the separations induced between two distinct bands, depends on the polarization directions of the coherent modes. h_{cv_i} can be made quite small or be made to vanish by altering the polarization directions relative to the crystal axes. This in turn affects the mixing of the valence bands (via their coupling to the conduction band), hence the coherent-field-induced separations of the bands.

The preceding calculations and results are completely analogous to those in Kane's band theory.[6] The only difference is in the coupling coefficients. In Kane's theory the coupling coefficients between different bands arise from $\mathbf{k}\cdot\mathbf{p}$ and spin-orbit coupling. In our problem the coupling coefficients depend on the coherent mode amplitudes and polarization directions. One can therefore adapt many of the results of the $\mathbf{k}\cdot\mathbf{p}$ method to the coherent-field renormalization.

An important conclusion of the preceding calculation is that the renormalization effects depend on the ratio $(\hbar\Omega_E/E_G^0)$; they do not depend on the detuning between the band gap and the frequency of a reference mode. This is completely different from the result which would be obtained from the classical treatment of the electromagnetic field. In semiclassical theories, the quasienergies depend on the detunings between the mode frequencies and the Bloch-electron transition frequencies.[5] We may say that the coherent-field renormalization is an infinitely-many-photon-transition process, since by summing the Z matrices to all orders, we take into account all possible virtual transitions. Therefore the renormalization effects do not depend on the resonance properties of a crystal.

IV. CONCLUDING REMARKS

Our estimates with the parameter values given in (3.24) indicate that the effects of the coherent-field renormalization on simple band structures can be substantial and observable in the excite-probe experiments of the type performed by Mysyrowicz *et al.*[8,3] When an intense coherent excitation pulse overlaps with a weak probe pulse, the probe pulse detects the electronic energies which are renormalized by the excitation pulse. Also, as we saw above, the renormalization effects involve the ratio $(\hbar\Omega_E/E_G^0)$. Thus, the renormalization effects can be enhanced either by increasing coherent-field intensities, or by decreasing band gaps. Consequently, one would expect the renormalization effects to be much more readily observable in narrow-gap crystals for a given coherent-field intensity.

As we mentioned earlier, the coherent-field renormalization significantly affects the electronic coupling coefficients with photons and phonons, which may also be observable. These effects will be discussed in another paper.

We expect the concept of the coherent-field-renormalized Bloch electron to be particularly suited to the analysis of the dynamics and statistics of semiconductor lasers in which one or more intense coherent modes are built up. If the dynamics of an electron population in a semiconductor laser is formulated in terms of the renormalized electrons, various coefficients such as the rates for Auger processes, exciton formation, diffusion, etc., become dependent on the intensities and the phases of the coherent modes in the reference set. Such a formulation would be particularly useful for the study of fluctuations. Of course, the numerical accuracy of the new picture depends on the accuracy of the matrix elements of U. As we showed in the preceding discussion, with reasonable assumptions, one can evaluate these matrix elements exactly for simple band structures. For more complicated bands, one can adopt various numerical methods developed in standard band-structure calculations.

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[1]J. J. Hopfield, Phys. Rev. **112**, 1555 (1958).
[2]A. Elçi, Phys. Lett. A **136**, 145 (1989).
[3]A. Elçi and D. Depatie, Phys. Lett. A **135**, 471 (1989).
[4]K. Henneberger and H. Haug, Phys. Rev. B **38**, 9759 (1988).
[5]C. Comte and G. Mahler, Phys. Rev. B **34**, 7164 (1986).
[6]E. O. Kane, in *Semiconductors and Semimetals I: Physics of III-V Compounds,* edited by R. K. Willardson and A. C. Beer (Academic, New York, 1966).
[7]A. Elçi, M. O. Scully, A. L. Smirl, and J. C. Matter, Phys. Rev. B **16**, 191 (1977).
[8]A. Mysyrowicz, D. Hulin, A. Antonetti, A. Migus, W. T. Masselink, and H. Morkoç, Phys. Rev. Lett. **56**, 2748 (1986).
[9]J. R. Klauder and E. C. G. Sudarshan, *Fundamentals of Quantum Optics* (Benjamin, New York, 1968), Chap. 7.
[10]A. Elçi, Phys. Lett. A **111**, 448 (1985).
[11]A. Elçi and E. D. Jones, Phys. Rev. B **34**, 8611 (1986).
[12]C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1967), Chap. 9.
[13]P. L. Knight and P. W. Milloni, Phys. Rep. **66**, 21 (1980).
[14]There is an error in Eqs. (7a) and (7b) of Ref. 2. $\overleftrightarrow{M}_c^{-1}$ and $\overleftrightarrow{M}_v^{-1}$ should be replaced by $\overleftrightarrow{M}_{cv}^{-1}=\overleftrightarrow{M}_c^{-1}-\overleftrightarrow{M}_v^{-1}$. Also, indirect gap occurs only if m_c^0 and m_v^0 differ. Note that in Ref. 3, the contributions of $\delta W_{n\mathbf{k}}$ to $e_{\mathbf{k}}$ are neglected.
[15]J. V. Uspenski, *Theory of Equations* (McGraw-Hill, New York, 1948).