conductors (Dunod, Paris, 1964), p. 129. These authors reported measurements on the Franz-Keldysh effect in CdS and CdSe platelets for four orientations of applied field and light polarization. However, since these platelets contain the c axis and the light must propagate normal to the platelets, the important additional case in which the applied field is in a direction perpendicular to c and the optical electric field is in a direction perpendicular to both the applied field and to c could not be measured. A value for the Franz-Keldysh coefficient γ for this case would enable a direct comparison with the polarization asymmetry in our data for the orientation in Fig. 3(a). Fortunately, the magnitude of γ for this case can be inferred from those measurements the authors did make. Their data show that with the drift field parallel to the optical field, the coefficient $\gamma = -\Delta E/\mathcal{E}^2$ was 1.58 or 1.62 (10^{-12} eV cm^2/V^2) with the fields parallel to or perpendicular to the c axis, respectively. With the two electric fields perpendicular to each other, the coefficient was 1.08 or 1.23, depending on the orientation of the c axis. The last two numbers were even closer (1.84 and 1.88) for CdSe. Thus, it appears that while the orientations of the applied field and the optical field with respect to the c axis are relatively unimportant, the angle between the two fields is important, with the Franz-Keldysh effect about 40% larger when the fields are parallel. This agrees well with our measurements on acoustoelectric domains, where the shift with the fields parallel was about 30% larger.

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[21]Here, the drift field is in the "2" direction and the light propagates along the c axis, or "3" direction [Fig. 3(a)]. The strains that interact with fields in the 2 direction are S_{23}, here written as S_4. Thus d_{24} is the relevant piezoelectric tensor element.

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Phonon Conductivity of InSb and GaAs in the Temperature Range 2–300 °K

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(Received 24 May 1971)

The recent modification of Holland's model of two-mode conduction, as proposed by us, has been applied to explain the phonon conductivity of InSb and GaAs. This model, known as the Sharma-Dubey-Verma model, makes use of Guthrie's classification of three-phonon scattering events. In this model, the exponent m of the temperature, i.e., $T^{m(T)}$ is a continuous function of temperature and approaches unity in the high-temperature region for both the longitudinal phonons as well as transverse phonons. The dispersion of acoustic branches is taken into account in replacing v_g/v_p^2 in the conductivity integrals and this forms the basis of the division of the conductivity integrals for the different polarization branches. The present model gives excellent agreement between the theoretical and experimental values of phonon conductivity except near the maximum where the scattering of phonons by point defects dominates over phonon-phonon scattering as well as the boundary scattering of phonons.

INTRODUCTION

Recently we have proposed a modification[1] of Holland's[2] model of two-mode conduction in semiconductors. The most significant feature of the present model [hereafter called the Sharma-Dubey-Verma (SDV) model] is the use of Guthrie's[3] classification on three-phonon scattering events. In class-I events the carrier phonon is annihilated by combination and in class II the annihilation takes place by splitting. Thus τ_{3ph}^{-1} is expressed as

$$\tau_{3ph}^{-1} = \tau_{3ph}^{-1}(\text{class I}) + \tau_{3ph}^{-1}(\text{class II}) .$$

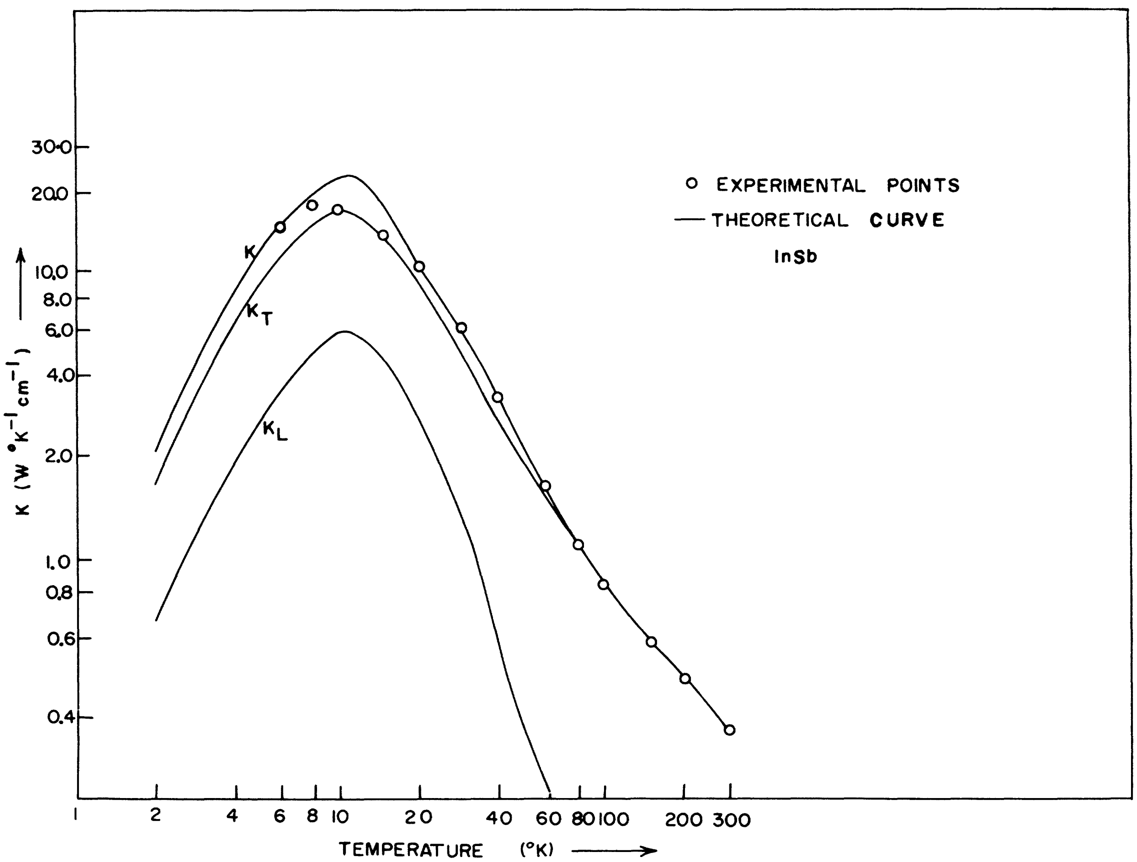

FIG. 1. Phonon conductivity of InSb. Solid line is the theoretical curve and circles represent the experimental points.

For transverse phonons this leads to

$$(\tau_{3\mathrm{ph}}^{-1})_{\mathrm{trans}} = B_T\,\omega T^{m_{T,\mathrm{I}}(T)}\,e^{-\Theta/\alpha T}$$

since only class-I events are possible. For longitudinal phonons one obtains

$$(\tau_{3\mathrm{ph}}^{-1})_{\mathrm{long}} = B_{L,\mathrm{I}}\,\omega^2 T^{m_{L,\mathrm{I}}(T)}\,e^{-\Theta/\alpha T} + B_{L,\mathrm{II}}\,\omega^2 T^{m_{L,\mathrm{II}}(T)}\,e^{-\Theta/\alpha T}\,.$$

Another significant feature of our present approach is the use of the dispersion relation[4] $\vec{q} = (\omega/\vec{v})(1+\gamma\omega^2)$ to replace v_g/v_p^2 in the conductivity integrals. This gives

$$\frac{v_g}{v_p^2} = \frac{1}{v}\,\frac{(1+\gamma\omega^2)^2}{1+3\gamma\omega^2}\,,$$

where γ is given by[4]

$$\gamma = \frac{1}{\omega^2}\left(\frac{\vec{q}\cdot\vec{v}}{\omega} - 1\right).$$

The conductivity integrals in the present model are divided on the basis of the nature of dispersion curves for transverse phonons and longitudinal phonons. Thus division of conductivity integrals both for transverse as well as longitudinal phonons into

TABLE I. Values of the different constants used in the analysis of phonon conductivity of InSb and GaAs.

	InSb	GaAs
	(10^{13} Hz)	(10^{13} Hz)
w_1	0.73	1.16
w_2	0.81	1.33
w_4	1.335	2.45
w_3	2.22	4.10
a	6.4788×10^{-8} cm	5.641907×10^{-8} cm
	(10^5 cm/sec)	(10^5 cm/sec)
$(v_T)_{0<\omega<\omega 1}$	2.28	2.48
$(v_T)_{\omega 1<\omega<\omega 2}$	0.82	0.90
$(v_L)_{0<\omega<\omega 3}$	3.77	4.73
	(°K)	(°K)
Θ_1	56	90
Θ_2	62	101.5
Θ_4	102	187
Θ_3	170	313
Θ_D	208	345
α	1.5	2
	(sec^2)	(sec^2)
γ_1	9.665×10^{-27}	2.084×10^{-27}
γ_2	2.618×10^{-26}	7.225×10^{-27}
γ_3	1.37×10^{-27}	1.163×10^{-28}

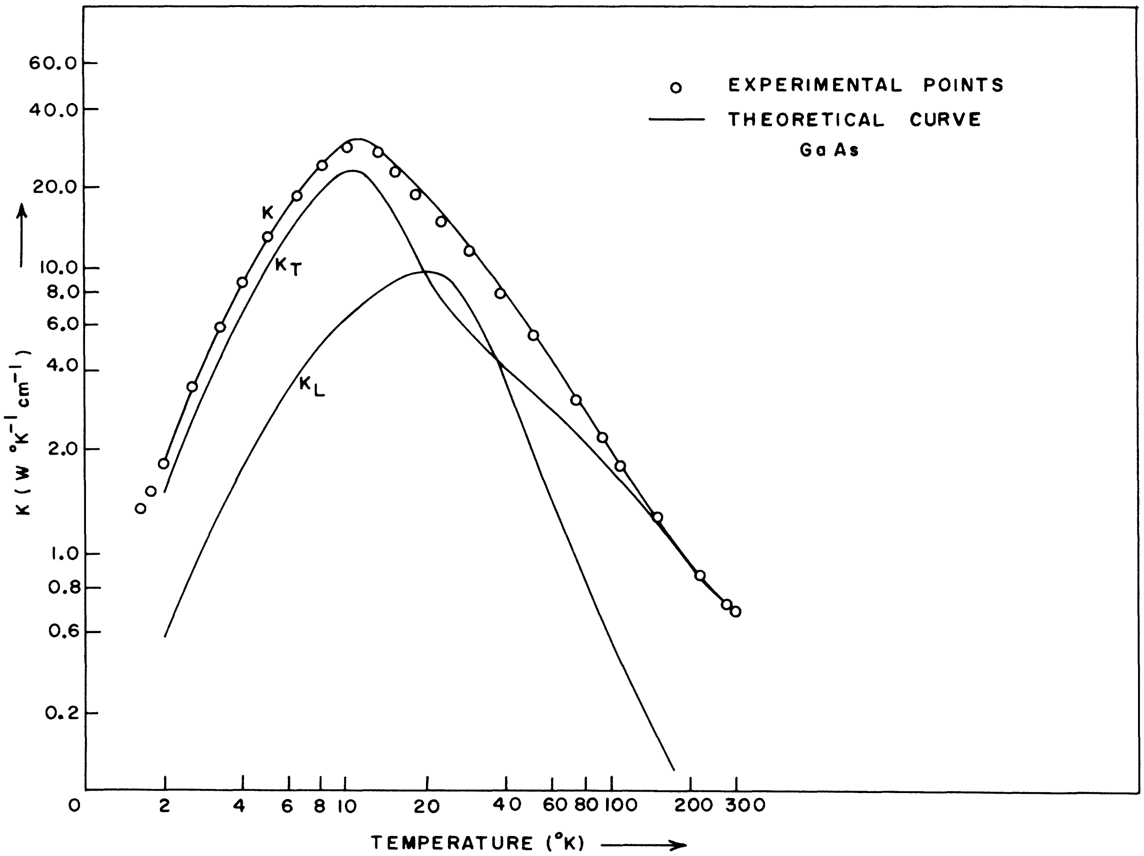

FIG. 2. Phonon conductivity of GaAs. Solid line is the theoretical curve and circles represent the experimental points.

two integrals is based upon the different values $v_g/v_p{}^2$ in the two regions 0 to $\frac{1}{2}$ q_{max} and $\frac{1}{2}$ q_{max} to q_{max}.

There is another very important aspect of the present calculations. This will be more evident if we reproduce Guthrie's[3] table for InSb and GaAs

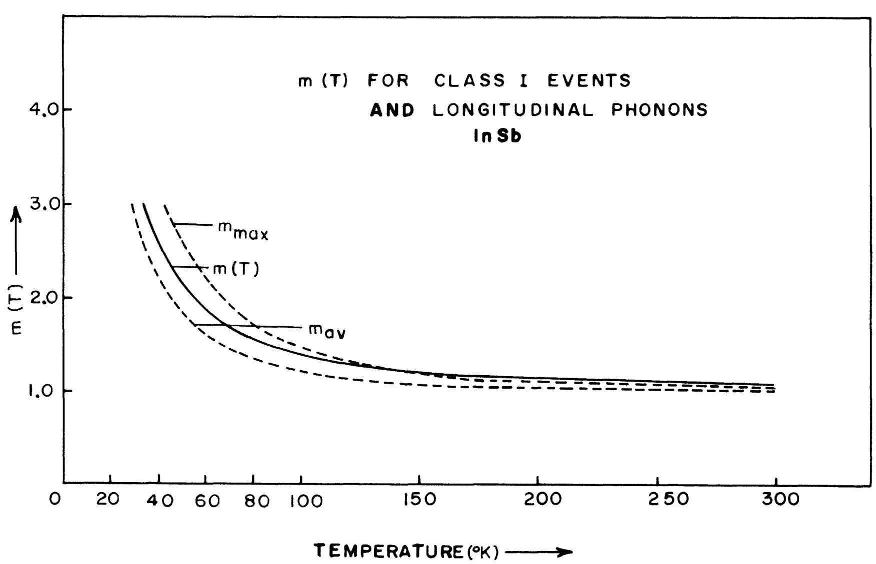

FIG. 3. $m(T)$ for class-I events for transverse phonons in InSb. m_{max} is the upper bound given by Guthrie, m_{av} is average value of upper and lower bounds of Guthrie.

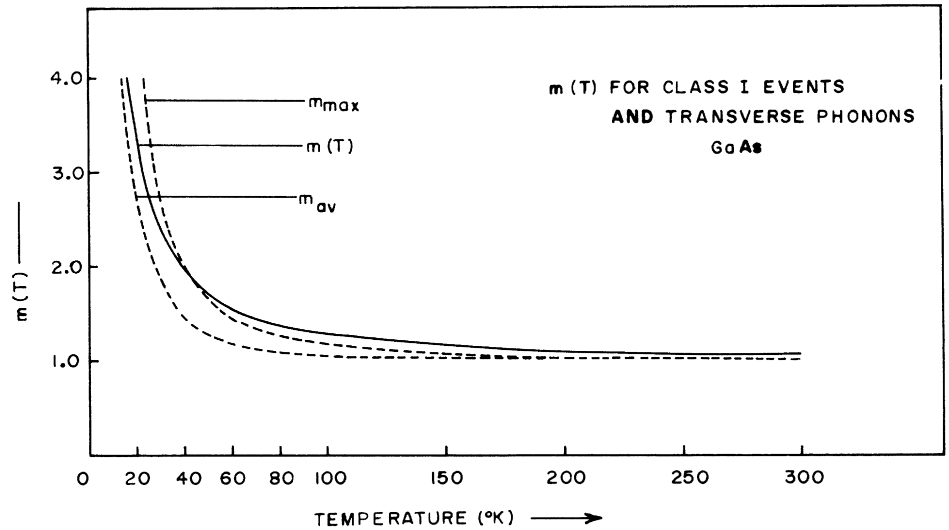

FIG. 4. $m(T)$ for class-I events for transverse phonons in GaAs. m_{max} is the upper bound given by Guthrie while m_{av} is the average value of upper and lower bounds of Guthrie.

(all values in °K):

Material	Assumption of T^4 relation invalid for $T>$	Assumption of T^3 relation invalid for $T>$	$m<4$ if $T>$	$m<3$ if $T>$	$m<2$ if $T>$
InSb	13	16.5	54	69	103
GaAs	20	26	85	108	159

This shows that the temperature dependence of the three-phonon relaxation rate has not been taken into account properly in the earlier calculations of phonon conductivity of InSb and GaAs. However, in the present calculations m is a continuous function of temperature and is either below or close to the upper limit at different temperatures. Thus present calculations are the only calculations in which the temperature dependence of the three-phonon relaxation rate has been taken into account properly. In view of the continucus nature of the function $m=m(T)$, the interpretation of the high-temperature data is consistent with low-temperature results, and there is no need to invoke different temperature dependences in the different regions.

THEORY

According to the SDV model,[1] the contributions of transverse phonons and longitudinal phonons towards thermal conductivity are expressed as

$$K_T=\frac{2}{3}\left(\frac{k_B}{2\pi^2}\right)\left(\frac{k_B}{\hbar}\right)^3 T^3\Bigg[\int_0^{\Theta_1/T}\left((v_T)^{-1}_{0<\omega<\omega_1}\frac{x^4e^x(e^x-1)^{-2}(1+R_1x^2T^2)^2(1+3R_1x^2T^2)^{-1}\,dx}{\tau_B^{-1}+Dx^4T^4+\beta_T xT^{m_{\mathrm{I}}(T)+1}e^{-\Theta/\alpha T}}\right)$$

$$+\int_{\Theta_1/T}^{\Theta_2/T}\left((v_T^{-1})_{\omega_1<\omega<\omega_2}\frac{x^4e^x(e^x-1)^{-2}(1+R_2x^2T^2)^2(1+3R_2x^2T^2)^{-1}\,dx}{\tau_B^{-1}+Dx^4T^4+\beta_T xT^{m_{\mathrm{I}}(T)+1}e^{-\Theta/\alpha T}}\right)\Bigg] \qquad (1)$$

and

$$K_L=\frac{1}{3}\left(\frac{k_B}{2\pi^2}\right)\left(\frac{k_B}{\hbar}\right)^3 T^3\Bigg[\int_0^{\Theta_4/T}\left((v_L^{-1})_{0<\omega<\omega_4}\frac{x^4e^x(e^x-1)^{-2}\,dx}{\tau_B^{-1}+Dx^4T^4+\beta_{L,\mathrm{II}}x^2T^{m_{\mathrm{II}}(T)+2}e^{-\Theta/\alpha T}}\right)$$

$$+\int_{\Theta_4/T}^{\Theta_3/T}\left((v_L^{-1})_{\omega_4<\omega<\omega_3}\frac{x^4e^x(e^x-1)^{-2}(1+R_3x^2T^2)^2(1+3R_3x^2T^2)^{-1}\,dx}{\tau_B^{-1}+Dx^4T^4+\beta_{L,\mathrm{II}}x^2T^{m_{\mathrm{II}}(T)+2}e^{-\Theta/\alpha T}}\right)\Bigg], \qquad (2)$$

where

$$\Theta_i=\frac{\hbar\omega_i}{k_BT},\quad i=1,2,3,4;\quad R_i=\gamma_i\left(\frac{k_B}{\hbar}\right)^2,\quad i=1,2,3;\quad \beta_T=B_T\left(\frac{k_B}{\hbar}\right),\quad \beta_{L\,\mathrm{II}}=B_{L\,\mathrm{II}}\left(\frac{k_B}{\hbar}\right)^2,\quad D=A\left(\frac{k_B}{\hbar}\right)^4.$$

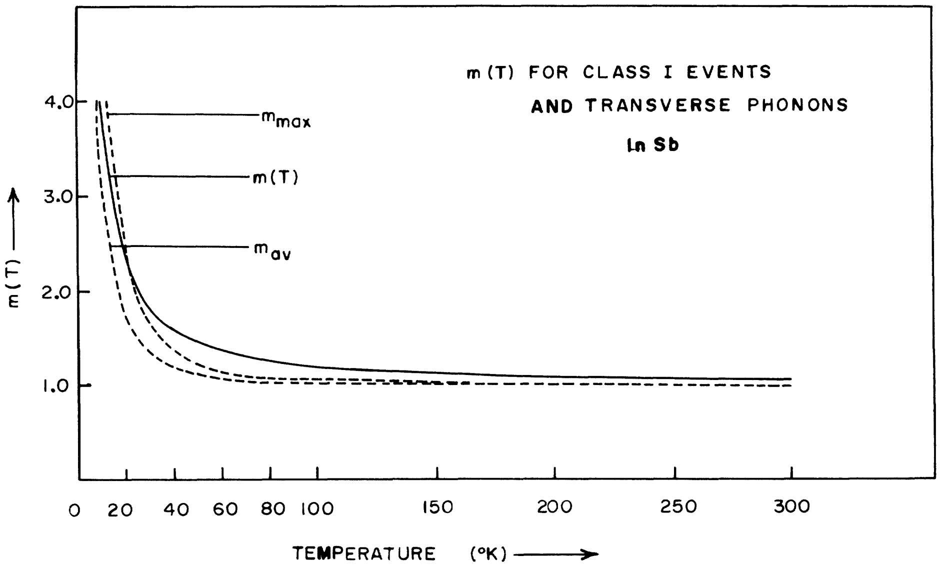

FIG. 5. $m(T)$ for class-I events for longitudinal phonons in InSb. m_{max} is the upper bound given by Guthrie while m_{av} is the average value of upper and lower bounds of Guthrie.

The three-phonon relaxation rate for longitudinal phonons is given by

$$(\tau_{3\text{ph}}^{-1})_L = (B_{L,\text{I}}\, T^{m_\text{I}(T)} + B_{L,\text{II}}\, T^{m_\text{II}(T)})\, \omega^2 e^{-\Theta/\alpha T} . \tag{3}$$

Similarly, the three-phonon relaxation rate for transverse phonons is given by

$$(\tau_{3\text{ph}}^{-1})_T = B_{T,\text{I}}\, \omega\, T^{m_\text{I}(T)} e^{-\Theta/\alpha T} . \tag{4}$$

Here

$$m_\text{I}(T) = m_{\text{av},\text{I}} + \frac{\ln(1+\Theta/\alpha T)}{\ln T}$$
$$= x_{\max}(e^{x_{\max}}-1)^{-1} + 0.5\, x_{\max} + \frac{\ln(1+\Theta/\alpha T)}{\ln T} ,$$

$$m_\text{II}(T) = m_{\text{av},\text{II}} + \frac{\ln(1+\Theta/\alpha T)}{\ln T}$$
$$= 0.5\, x_{\max}(e^{x_{\max}}-1)^{-1} e^{0.5 x_{\max}} + 0.5 + \frac{\ln(1+\Theta/\alpha T)}{\ln T} ,$$

where $x_{\max} = \hbar\omega_{\max}/k_B T$ and the last term is known as the correction term, which is needed to express $m(T)$ in terms of the average value of $m(T)$, i.e.,

$$m(T) = m_{\text{av}}(T) + \frac{\ln(1+\Theta/\alpha T)}{\ln T} .$$

The factor

$$\frac{(1+Rx^2T^2)^2}{1+3Rx^2T^2}$$

in the different conductivity integrals is due to the

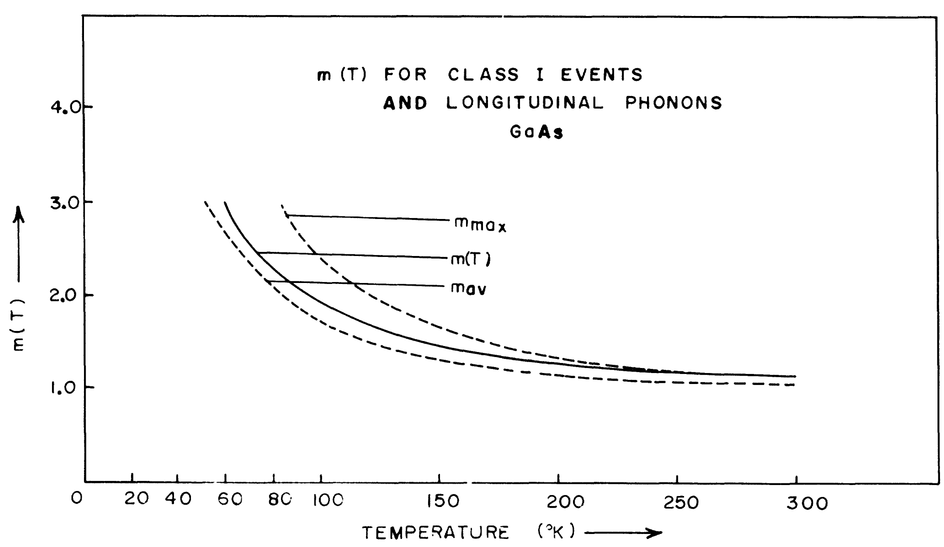

FIG. 6. m(T) for class-I events for longitudinal phonons in GaAs. m_{max} is the upper bound given by Guthrie while m_{av} is the average value of upper and lower bounds of Guthrie.

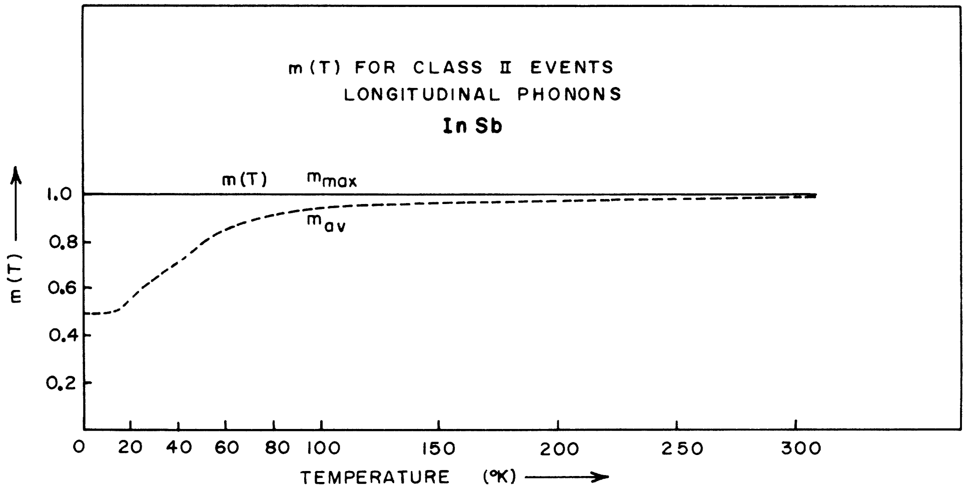

FIG. 7. $m(T)$ for class-II events for longitudinal phonons in InSb. m_{max} is the upper bound given by Guthrie which is same as $m(T)$. m_{av} is the average value of upper and lower bounds of Guthrie.

replacement of v_g/v_p^2 by

$$\frac{1}{v}\,\frac{(1+\gamma\omega^2)^2}{1+3\gamma\omega^2},$$

where one makes use of the dispersion relation $\vec{q} = (\omega/\vec{v})(1+\gamma\omega^2)$. Since the contribution of longitudinal phonons in class-I events is very small, the term $\beta_{L,I}\, x^2 e^{-\Theta/\alpha T} T^{m_I(T)+2}$ has been neglected in the conductivity integrals of longitudinal phonons.

Using the above integral expressions we have calculated phonon conductivity of GaAs and InSb in the entire temperature range 2–300 °K. For this we have used the values of zone boundary frequencies as given by Bhandari and Verma[6] in the case of InSb. For GaAs we have taken the limits of the different conductivity integrals from the dispersion curve given by Waugh and Dolling.[7] In the absence of a dispersion curve for InSb we have assumed that the nature of the dispersion curve of InSb is the same as that of GaAs. Thus it is assumed that the ratios ω_1/ω_2 and ω_4/ω_3 for InSb are the same as in the case of GaAs. Similarly, it has been assumed that

$$\frac{(v_T)_{0<\omega<\omega_1}}{(v_T)_{\omega_1<\omega<\omega_2}} \quad \text{and} \quad \frac{(v_{L1})_{0<\omega<\omega_1}}{(v_{L2})_{\omega_1<\omega<\omega_2}}$$

are the same for InSb as for GaAs. For GaAs we have calculated γ_i at $\frac{1}{2}q_{max}$ and q_{max} in the [100] and [111] directions, and taken the average value of γ_i for these two directions.

The values of the various parameters used in the analysis of phonon conductivity of GaAs and InSb in the entire temperature range 2–300 °K are given in Tables I and II. The approximate values of the various adjustable parameters can be obtained as follows. At high temperatures one can obtain analytical expression of the integral Eq. (1) by as-

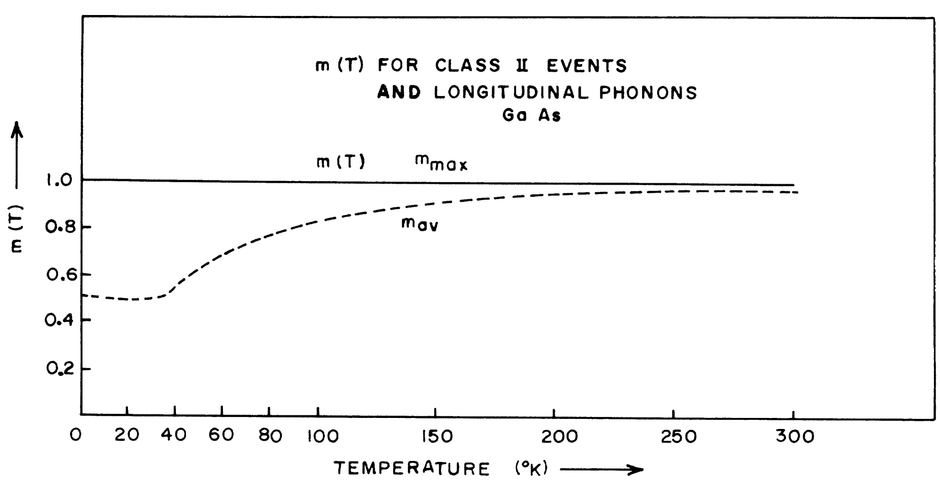

FIG. 8. $m(T)$ for class-II events for longitudinal phonons in GaAs. m_{max} is the upper bound given by Guthrie which is same as $m(T)$. m_{av} is average value of upper and lower bounds of Guthrie.

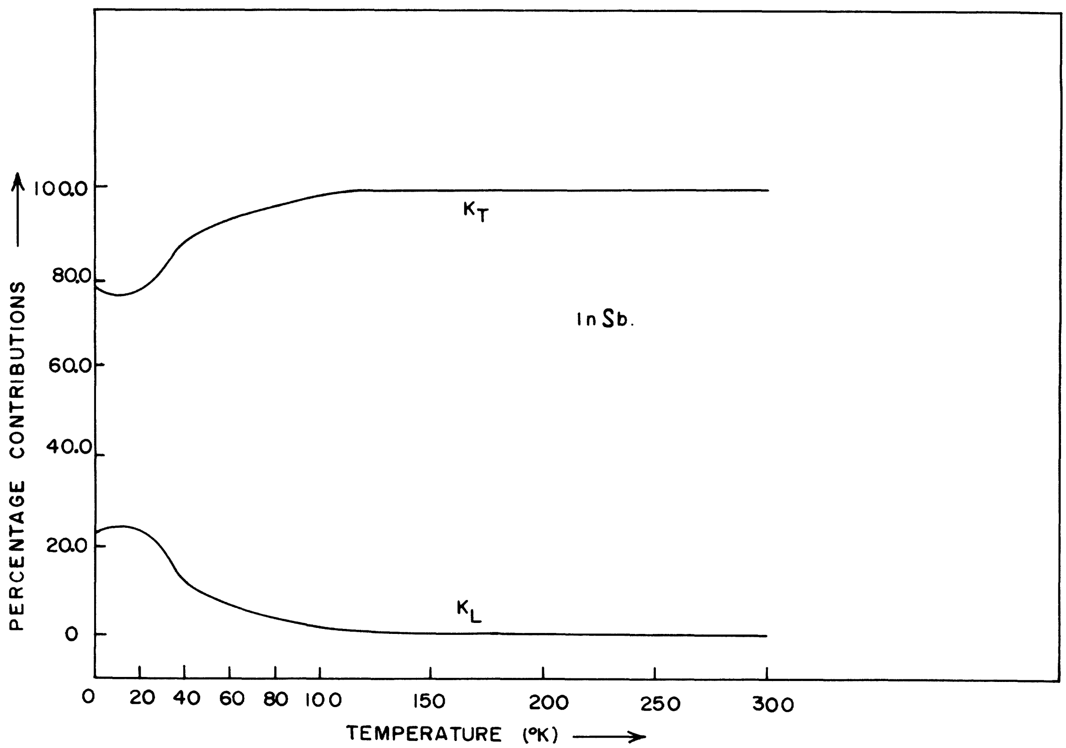

FIG. 9. Percentage contribution of transverse and longitudinal phonons in InSb. K_T is the percentage contribution of transverse phonons while K_L is the percentage contribution of longitudinal phonons.

suming $x^2 e^x(e^x - 1)^{-2}$ to be unity and that three-phonon scattering dominates over other phonon scattering processes. This gives an approximate value of B_T and the exact value of B_T is obtained by numerical integration of Eq. (1) for the best fit between theory and experiment at 300 °K. Similarly we have adjusted $B_{L,\mathrm{II}}$ at 150 °K by numerical integration of Eq. (2), in GaAs and InSb, respectively. For τ_B we have used the average phonon velocity as $1/v_s = \frac{2}{3}(v_T)^{-1} + \frac{1}{3}(v_L)^{-1}$ and adjusted its value

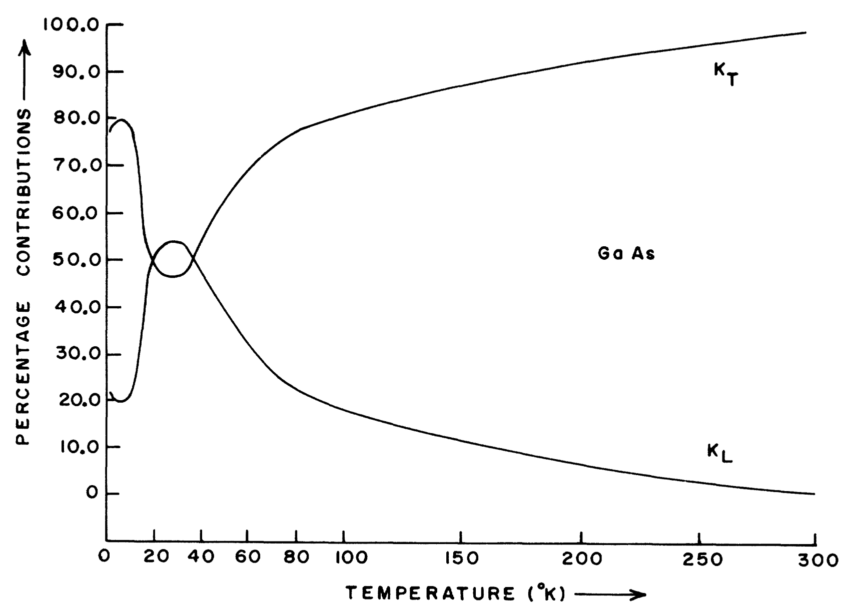

FIG. 10. Percentage contribution of transverse and longitudinal phonons in GaAs. K_T is the percentage contribution of transverse phonons while K_L is the percentage contribution of longitudinal phonons.

TABLE II. Values of the adjusted parameters used in the analysis of the phonon conductivity of Insb and GaAs.

	InSb	GaAs
B_T	1.85×10^{-6} sec^{-m}	1.57×10^{-6} (°K)$^{-m}$
$B_{L,\mathrm{II}}$	1.70×10^{-17} sec deg^{-m}	2.07×10^{-18} sec (°K)$^{-m}$
τ_B^{-1}	$5.0\ \times10^{5}$ sec^{-1}	5.94×10^{5} sec^{-1}
A	2.80×10^{-44} sec^3	1.30×10^{-44} sec^3

at 2 °K. This value is found to be the same as given by Holland[2] for GaAs and InSb.

RESULTS

Using the parameters given in Tables I and II and with the temperature dependences of the three-phonon relaxation rate discussed previously, we have calculated phonon conductivity of GaAs and InSb in the entire temperature range 2–300 °K. The results of calculation are shown in Figs. 1 and 2. Except near the maximum, it is found that the agreement between theory and experiment is excellent. The temperature dependence of the parameter $m = m(T)$ for the best fit between theory and experiment is shown in Figs. 3–8 for class-I and class-II events as well as for longitudinal and transverse phonons. It may be observed from these figures that m lies either in between the upper and lower bounds of $m(T)$ obtained on the basis of Guthrie's expressions or close to the upper bound.

The contribution of transverse phonons towards thermal conductivity is in general greater than that of longitudinal phonons. At high temperatures the contribution of longitudinal phonons is approximately (1–5)% of the total conductivity. At low temperatures the contributions of longitudinal and transverse phonons are in the ratio 1 : 3. These results are shown in Figs. 9 and 10 and are in agreement with the similar findings in Ge by Hamilton and Parrott.[8]

ACKNOWLEDGMENTS

The authors wish to express their thanks to Professor B. Dayal and Professor K. S. Singwi for their interest in the present work. One of us (K. S. D.) is grateful to the Council of Scientific and Industrial Research, India for the financial assistance.

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