other ions in gold in this regard, but especially to examine the behavior of light ions in a variety of other target substances.

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Oscillation Frequencies of Protons in Planar Channels of Silicon*

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Measurements of the angular distributions and energy losses of 0.4-MeV protons transmitted through planar channels in thin silicon crystals have been used to relate the stopping power of the ions to the frequencies with which they oscillate in traversing the channels. In agreement with analogous experiments for several ions in gold, the stopping power is found to be accurately proportional to the channel oscillation frequency. The interaction between the proton and the silicon atoms is in reasonable agreement with expectations based on a Thomas-Fermi model.

A recent series of experiments on the energy-loss spectra of several ions transmitted through planar channels in thin gold crystals was interpreted in terms of a model in which the ions execute transverse anharmonic oscillations under the influence of the planes of atoms bordering the channel and in which an ion loses energy at a rate which is a function of its displacement from the atomic planes.[1-4] In these experiments, the incident beam and the small-acceptance-angle detector were colinear so that the well-resolved peaks in the energy spectra of the transmitted ions corresponded to particles which made an integral number of oscillations in passing through the crystal. An important result of the gold experiments[4] was that in every case the stopping power of the ion was accurately proportional to its channel oscillation frequency, an observation which could be made the basis of a quantitative application of the model.[2,3] It is therefore of considerable interest to know whether this linear relationship between stopping power and oscillation frequency applies to any target other than gold. Its general applicability would establish a remarkable connection between the stopping power of an ion and its interaction potential with the atoms of the target.

The energy spectra of protons transmitted through the planar channels of thin silicon crystals do not contain resolved peaks, so that the experimental procedure used for heavier ions in gold is inapplicable. Nevertheless, an alternative procedure, based on the same physical model, can circumvent this limitation. The basis of the experimental technique may be understood by reference to Fig. 1. Ions of initial energy E_0 are incident on the target at a small angle ψ_0 from the plane that defines the channel. In passing through the channel, the ions oscillate under the influence of the bordering planes with amplitude x_m and wavelength λ. After traveling a distance z, the ions leave the crystal with reduced energy E and at an angle ψ from the channel plane. As illustrated in Fig. 1, there are always two trajectories with the same amplitude, distinguished by having entered the crystal on opposite sides of the channel center and equidistant from it. According to the model,[2,3] these two trajectories will have the same wavelength

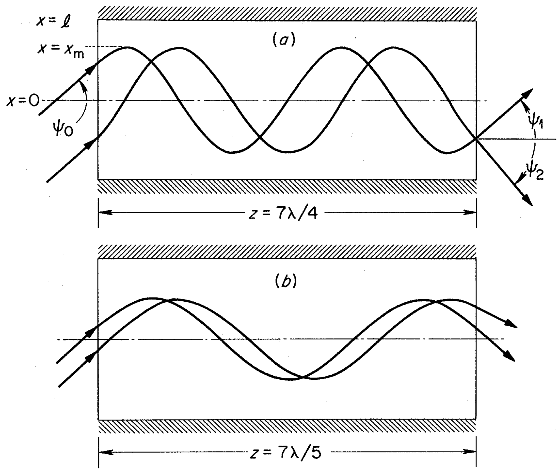

FIG. 1. Determination of the wavelengths (or periods) of oscillation of ions traversing planar channels by the observation of the angular distribution of monoenergetic transmitted particles.

and amplitude of oscillation, but the two emergent angles, ψ_1 and ψ_2, will generally bear no particular relationship to each other. However, in the special case illustrated in Fig. 1(a),

$$\text{when } z=(k\pm\tfrac{1}{4})\lambda \;\; (k \text{ an integer}), \text{ then } \psi_1=\psi_2. \tag{1}$$

When this condition is met, the two trajectories cross at the point of exit from the crystal. A case in which condition (1) is not met is illustrated in Fig. 1(b). In either case, a small-acceptance-angle detector is arranged to respond only to ions in a selected narrow energy interval. If this energy corresponds to a pair of trajectories meeting condition (1), as in Fig. 1(a), the angular distribution of emerging ions will consist of a pair of equally intense peaks, symmetrically disposed about the channel plane. If condition (1) is not met, there will generally still be two peaks in the angular distribution, but these will be of unequal intensity and not necessarily symmetrically disposed about the channel plane. Thus, condition (1) is used to estimate the wavelengths of ions of known energy loss by finding those final energies for which the emergent-ion angular distribution is symmetrical. It may in fact be applied even when the peaks in the distribution are not clearly resolved, although this would involve increased uncertainties.

In describing this technique, it has been tacitly assumed that the two related trajectories correspond to exactly the same energy loss, so that the two particles emerge from the crystal with precisely equal energies. In fact, as may be seen by inspecting Fig. 1(a), between their entry into the crystal and their first crossing point, the two particles sample different (complementary) portions of the channel cross section and must experience different energy losses in this region. Beyond their first crossing, the two trajectories sample the channel identically, so that the error in the method described becomes less as the pathlength through the target is increased. In the experiments reported here, from 19 to 31 quarter-oscillations were observed. Thus, the error introduced by the differing energy losses in the first quarter-oscillation is not more than 3–5% of the observed energy loss; or, since these losses were about 5–10% of the emergent energies, the error is certainly less than 0.5%—the resolution of the detector. Thus, the assumption that the two related trajectories have the same energy loss does not introduce significant errors into the experimental results.

The basis of the technique has been presented from a viewpoint requiring that the ion have a constant velocity through the channel. Only in this case can one properly speak of wavelengths.[2] When slowing down of the ion is introduced, the lengths of successive waves will be different. In this case, however, condition (1) may be rewritten in terms of the time taken by the ion to traverse the crystal and the period of its oscillation. Because the trajectory always lies very nearly in the channel plane, the energy loss affects only the longitudinal motion and its influence on the transverse motion may be safely ignored.[2] That is to say, experiments performed in the manner sketched here can be used to determine the stopping power of an ion as a function of its oscillation frequency. The condition (1), whether written in spatial or temporal terms, is correct as long as the trajectory is a differentiable periodic function odd about its nodes and even about its antinodes, which will admit all possibilities of physical interest.

A portion of the experimental apparatus is shown schematically in Fig. 2. Thin silicon samples were mounted in a three-axis goniometer located in the center of the sample chamber. The electrostatic analyzer assembly could be moved in a horizontal plane about an axis through the center of the chamber. The angular distribution of transmitted protons having a chosen energy was measured by setting the deflection voltage corresponding to that energy on the analyzer plates and recording on an x-y recorder a signal proportional to the current deflected into the electron multiplier as a function of the angular position of the analyzer. The angular resolution of the analyzer was about 0.05 deg and its energy resolution was about 0.5%. The analyzer and goniometer have been described in more detail elsewhere.[5]

The thickness of the samples used in this work ranged from about 0.5 to 0.7 μm. They were made by etching well-polished slices of $\{100\}$-oriented

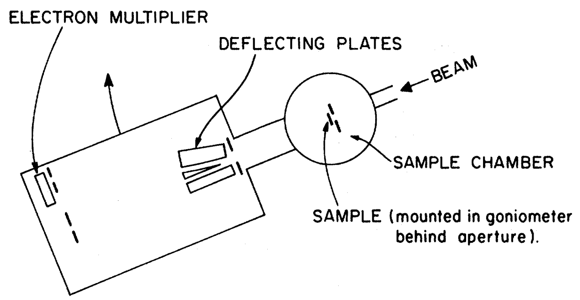

FIG. 2. Schematic diagram of the experimental arrangement.

silicon until a thin region could be detected by the transmission of light through it. They were mounted by a technique that has been described previously.[5] Examination of the interference pattern observed in the light from a sodium lamp indicated that the thickness variation was less than $\pm 0.035\ \mu m$.

The samples were oriented so that the atomic planes of interest were vertical with the angle ψ_0 set at 0.3 deg, that is, at about $\frac{2}{3}$ of the critical angle. The motion of the analyzer in the horizontal plane then made it possible to measure the angular distribution of protons of a given energy which resulted from their oscillatory motion between these atomic planes. The pathlength through the target was varied by tilting the sample about the horizontal axis of the goniometer. Care was taken to avoid working at orientations which could result in axial channeling effects. This was accomplished by observing the pattern of the transmitted protons on a phosphorescent screen a few inches behind the sample and avoiding those settings which gave an indication of axial channeling pattern. Preliminary measurements showed a deterioration in the ability to resolve the two peaks in the angular distribution with increasing exposure to the proton beam. A liquid-nitrogen-temperature shield was employed in front of the sample in an effort to decrease the deposition of hydrocarbons on the sample during the measurements. No change of the above type was observed while this was in use.

Typical experimental distributions for several values of the energy of the transmitted protons are shown in Fig. 3. In order to determine the oscillation frequency as a function of proton energy loss, it was necessary to find those values of the energy of the transmitted protons which yielded symmetric angular distribution with peaks of equal height, for various horizontal-tilt settings (crystal pathlengths). The measured angular distributions were affected by changes in the accelerator voltage and current. This necessitated repeating the measurements several times at the energy which yielded a symmetric pattern and at energies slightly above and below this value. With careful work the effects on the angular distribution of a change in transmitted energy of 0.5 keV could be observed. At the lowest observed transmitted-ion energies, a symmetric angular distribution was always found. It is believed that this was due at least partly to protons which had already been dechanneled and no longer represented ions whose motion was confined to an oscillatory path between two adjacent planes. For this reason only data points were used for which it was possible to observe an asymmetric distribution at an energy slightly lower than that which gave the symmetric distribution.

The peaks in the angular distributions are broadened by the mosaic spread of the target and by multiple-scattering effects, both nuclear and electronic. The finite energy and angular resolution of the detector and the divergence of the incident beam also contribute to the width of the observed peaks. Perhaps more important than any of these, however, is the effect of the variations in the target thickness. Ions transmitted through regions of the target of different thickness cannot simultaneously satisfy condition (1) at a particular emergent energy. Particles emerging from regions of different thickness must do so at different angles, thus contributing to the width of the observed peaks. Since (presumably) variations in the thickness of the targets are symmetrically disposed about the mean value, the observed distributions will be less asymmetric than one might anticipate from Fig. 1. This may explain the fact that the channel plane always lies near the centroid of the observed angular distribution (compare Fig. 3).

The experimental data consist of observations of the energies E of ions of incident energy E_0 transmitted through a pathlength z in such a way as to satisfy condition (1). From these, the oscillation frequency and the stopping power, averaged over the oscillation period and corrected to the incident-ion energy, are obtained as described in the preceding paper on gold.[4] In the present case, the number of oscillations n is of the form $k \pm \frac{1}{4}$, k an integer. The values of n (or k) were estimated from the difference in pathlength between successive appearances of particular values of the stopping power.[1,2] In Fig. 4, the initial stopping power is plot-

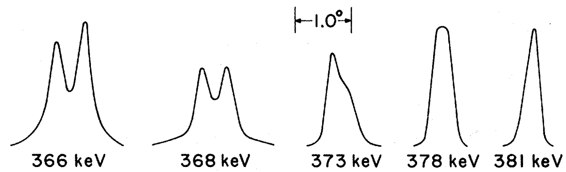

FIG. 3. Angular distributions observed for 0.4-MeV protons transmitted through the {111} channels of a 0.53-μm Si crystal. The {111} plane lies near the centroid of each distribution.

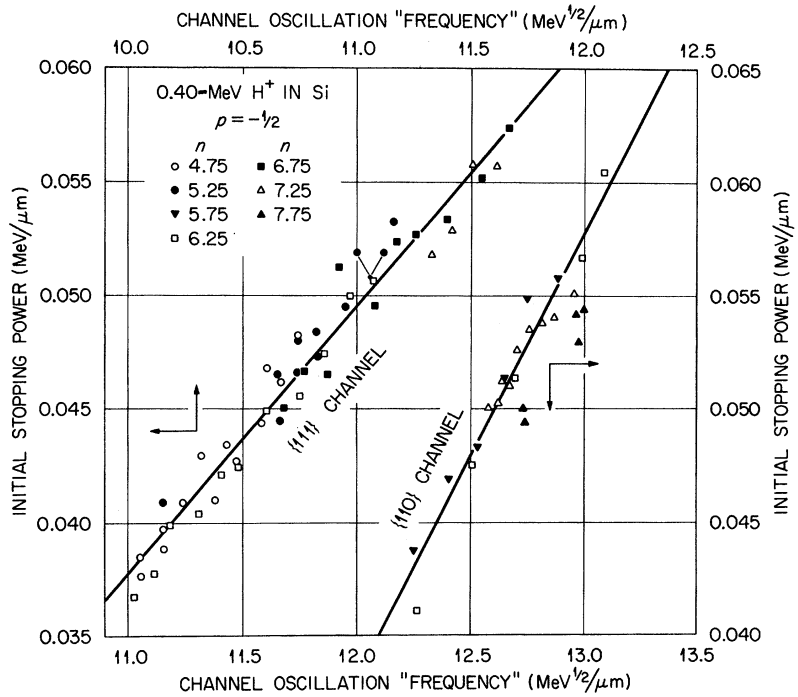

FIG. 4. Dependence of the initial stopping power of 0.4-MeV protons on their oscillation frequencies in the {111} and {110} channels of thin silicon crystals.

ted as a function of the channel oscillation frequency for 0.4-MeV protons traversing the {111} and {110} planar channels of thin silicon crystals. Approximate values of the wavelengths of the ion trajectories are given by $\lambda = 2E_0^{1/2}/\omega$; in the present experiments they ranged from 0.096 to about 0.126 μm. In constructing Fig. 4, the stopping parameter p was assumed to be $-\frac{1}{2}$, in agreement with observations on the random stopping power of 0.4-MeV protons in silicon.[6] As was observed also for several ions in gold,[4] the data are well represented by a straight line with intercept α and slope β. These parameters are employed, together with the stopping power at the center of the channel, s_0, to construct the curvature parameter y. The values of α and β, though not the observation of linearity, depend on the value chosen for p. In the experiments on gold,[4] it was found that y was independent of p as long as a single value was chosen to describe the energy dependence of α, β, and s_0. Guided by this result, p was chosen to be -1 and $-\frac{3}{4}$, respectively, for {111} and {110} planes, using experimental values obtained separately for minimum stopping powers in these channels.[6] Unfortunately, the data necessary to evaluate y for a range of values of p are not available in the present experiments. Its precise choice should not influence the results significantly, however. The resulting curvature parameters are shown in Table I. The two values shown for the {111} plane will be discussed below.

According to the planar-channeling model,[3] if the interaction potential is assumed to be of the screened Coulomb type with a screening function given by the long-range part of the Molière potential,[7] a functional form which has been justified before,[2] the curvature parameter is

$$y = 0.35\,(8\pi\rho\kappa Z_1 Z_2 e^2 b)\,e^{-bl} \quad , \tag{2}$$

where ρ is the atomic density of the target, Z_1e and Z_2e are the nuclear charges of the ion and the lattice atom, respectively, b is a screening constant, and κ allows for the possibility that all interplanar spacings may not be the same. This situation occurs in silicon for the {111} planes, where a long and a short spacing alternate. If the channel is regarded as defined by the longer spacing and only the nearer bounding planes are included, then $\kappa_{111} = \frac{2}{3}$ is required in Eq. (2); if, on the other hand, the channel is regarded as defined by a pair of effective planes at the average {111} spacing, then $\kappa_{111} = 1$. Note that the product κl is constant. The two {111} entries in Table I correspond to these two possi-

TABLE I. Curvature parameters observed for 0.4-MeV protons in planar channels of silicon.

Orientation	Half-width l (Å)	Curvature parameter y (eV/Å^3)
{111}	1.1757	14.5 ± 1.4
	1.5675	10.9 ± 1.0
{110}	0.9599	30.1 ± 4.7

TABLE II. Parameters for an exponentially screened Coulomb potential in silicon.

	Screening constant b ($Å^{-1}$)	Charge product Z_1Z_2	Proton charge $Z_1Z_2/14$
Experimental,			
$l_{111}=1.1757$ Å	1.51 ± 0.85	13.4 ± 5.3	0.96 ± 0.38
$l_{111}=1.5675$ Å	1.68 ± 0.30	14.1 ± 3.6	1.01 ± 0.26
$l_{111}=1.1757$, 1.9594 Å	2.24 ± 0.55	18.2 ± 5.6	1.30 ± 0.40
Thomas-Fermi theory	1.543	14	1

bilities. In Table II, the potential parameters deduced from the data using Eq. (2) are shown. The two choices of {111} spacing make little difference in the results, the greater spacing in the second case being largely offset by the greater density of atoms in the effective defining planes. A larger effect is produced if the {111} channel is defined using both sets of bounding planes, located at their correct positions. In this case the exponential in Eq. (2) is replaced by the sum of two, each with its own value of l. In all cases, the calculated proton charge is in satisfactory agreement with the expected value. The screening constant is also reasonably close to that expected theoretically if the electron density in silicon is given by the Thomas-Fermi model[8] and the Molière approximation is used for the screening function. This agreement, which is similar to that achieved for gold,[4] lends considerable confidence to the underlying model.

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Deduction of Interaction Potentials from Planar-Channeling Experiments*

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A model is presented which allows planar-channeling data to be used to deduce the parameters of the interaction potential between the channeled ions and the atoms of the solid and which relates the stopping power of channeled ions to that of randomly directed ones. The analysis is based on the experimental observation of the proportionality between the stopping power and the transverse oscillation frequency of ions traversing the planar channels of thin crystal targets. It is applied to data on He, O, and I ions transmitted through the {111} and {100} channels of Au crystals and on H ions transmitted through the {111} and {110} channels of Si crystals. For both targets, the screening constants of the interaction potentials are in excellent agreement with recent Hartree calculations. The same potentials allow the calculation of random stopping powers from the channeling data which are in good agreement with observation, especially when account is taken of thermal vibrations of the lattice atoms.

INTRODUCTION

In an earlier paper,[1] a model was described for the interpretation of the energy-loss spectra observed[2–6] in beams of energetic ions transmitted through planar channels in thin single-crystal targets. The ions were regarded as executing transverse anharmonic oscillations under the influence of the planes of lattice atoms bordering the channel and as losing energy at a rate which depended upon their position within the channel. After choosing a particular plausible form for the interaction