The configuration we have considered is not so unlikely since it corresponds to the minimum number of broken bonds at the surface.[1] This is relevant to the surface energy, since the bonds are mainly covalent in character. Other ways of terminating the crystal would lead to other (smaller) values of the dipole moment and hence e_{pol}, but only very special ways of terminating the crystal would give a vanishing value of e_{pol}.

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High-Field Hall Factor of *n*-Ge at 200 °K

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The high-field Hall factor of *n*-type germanium at 200 °K has been theoretically calculated including the effect of carrier scattering into the ⟨100⟩ minima and that of the magnetic field dependence of the carrier temperature and population in the different valleys. The results calculated with the optical-phonon deformation-potential constant $D_0 = 0.4\times10^9$ eV cm^{-1} differ widely from the experimental values. Good agreement between theory and experiment is obtained for values of D_0 lying within 1×10^9 and 1.5×10^9 eV cm^{-1}.

I. INTRODUCTION

In a recent paper Heinrich *et al.*[1] reported experimental results on the hot-electron galvanomagnetic coefficients of *n*-type germanium at 200 °K. They have also shown that the results can be explained if the scattering of the electrons from the normally occupied ⟨111⟩ valleys to the ⟨100⟩ valleys at high fields is taken into account. The intervalley transfer into the ⟨100⟩ minima has been calculated on the basis of a model introduced by Omar.[2] In this model the electron temperature has been taken to be the same for all the valleys and has been obtained from experimental values of the average drift velocity and of the energy relaxation time. Further, Heinrich *et al.*[1] have neglected the effect of the magnetic field on the temperature and the carrier population in the different valleys. This effect, though negligible at low fields, is likely to be important at large values of the heating field.[3] Reasonable agreement between theory and experiment has been obtained by Heinrich *et al.*[1] for the ratio of the longitudinal and the transverse magnetoresistance. The agreement for the high-field Hall factor is, however, only qualitative.

It is of interest to determine if the agreement between theory and experiment is improved if the effect of the magnetic field on the carrier distribution function and that of the temperature inequality of the ⟨111⟩ and the ⟨100⟩ valleys are taken into account.

In this paper we have calculated the Hall factor on the basis of a model that has been found useful in explaining the negative differential conductivity of uniaxially strained *n*-type germanium at room temperature.[4] We have also included the carrier repopulation effect of the magnetic field. The model together with the method of analysis has been presented in Sec. II. The numerical results are compared with the experimental data in Sec. III.

II. MODEL AND THE ANALYTICAL DETAILS

We have assumed that the symmetrical part of the distribution function in each valley is Maxwellian with an electron temperature determined by the field and the prevalent scattering mechanisms. According to the revised estimate of Stratton[5] the carrier concentration required for establishing a Maxwellian distribution through predominant carrier-carrier scattering is 10^{15} cm^{-3} at 200 °K. The carrier concentration in the experimental sample (3×10^{14} cm^{-3}) is not much lower than this critical concentration. In the case of *n*-type germanium the adoption of the Maxwellian energy-distribution model may be justified even when carrier-carrier scattering does not predominate, as it has been shown that the predominance of optical-phonon scattering also makes the distribution function Maxwellian.[6,7] Further, the distribution function obtained by Dumke[8] at 300 °K, using the Levinson method, is not much different from the Maxwellian function except for low-energy electrons.

The important scattering processes for the electrons in n-type germanium at 200 °K are the intravalley acoustic, optic, and the intervalley phonon scattering among the equivalent and the nonequivalent valleys. However, the effects of the equivalent intervalley scattering among the $\langle 111 \rangle$ valleys in energy and momentum relaxation may be ignored.[7] It affects only the transfer of carriers among the $\langle 111 \rangle$ valleys.

Let us assume that the heating field F_x is applied along the x direction of the chosen coordinate system, which in the present experiment coincides with the $\langle 100 \rangle$ direction of the crystal. The carrier temperature in the $\langle 111 \rangle$ valleys for this field can be obtained by solving the energy balance equation. However, for the range of field intensities ($0-3\ \mathrm{kV\,cm^{-1}}$) used in the experiment the carriers in the $\langle 100 \rangle$ minima will not be much heated in view of their low mobility. Further, the strong intervalley scattering among these valleys will tend to keep them at the same temperature.[9] The carrier temperature in the $\langle 100 \rangle$ valleys may, therefore, be conveniently set equal to the lattice temperature, and the population transfer to these valleys obtained by solving the particle balance equation.

Now let a magnetic field B_z be applied along the z direction. If the heating field F_x acts along a symmetry direction of the crystal, the Sasaki field will be zero and the field in the semiconductor will consist simply of the field F_x and the Hall field F_y. Since for a particular valley i, a current density J_{yi} flows in the y direction, the power absorbed by the carriers in the valley per unit volume of the sample is altered by $J_{yi}F_y$ when the magnetic field is imposed. As a result, the temperature and the carrier population in the valley are perturbed. Taking these perturbations into account and assuming that the magnetic field is so small that only the terms linear in B_z are important, the Hall mobility can be shown to be given by[3]

$$\mu_H = -\frac{e}{m_c}\,\frac{\sum_i n_{i0}\langle\tau_i^2\rangle_0\beta_{zzi}}{\sum_i n_{i0}\langle\tau_i\rangle_0 M_{yyi} + F_x^2\left(\sum_i C_{1i}M_{xyi}^2\langle\tau_i\rangle_0 + \sum_i C_{2i}n_{i0}M_{xyi}^2\, d\langle\tau_i\rangle/dT\right)}, \tag{1}$$

where e is the electronic charge; m_c, the conductivity effective mass; n_{i0} and $\langle\tau_i\rangle_0$ are, respectively, the carrier density and the average relaxation time for the ith valley in the absence of the magnetic field; M's are the components of the normalized reciprocal effective-mass tensor; and $\beta_{zzi} = M_{xxi}M_{yyi} - M_{xyi}^2$. The factors C_{1i} and C_{2i} depend on the unperturbed carrier number n_{i0} and temperature T_{i0}, and can be determined by solving the number and the energy conservation equations.[4,10]

The Hall coefficient can be obtained using the relation

$$R = \mu_H/\sigma , \tag{2}$$

where σ is the conductivity. Since for small magnetic fields only the first-order terms in B_z are important, the conductivity σ is not changed by the application of the magnetic field and is determined only by the heating field F_x. The Hall mobility, on the other hand, is affected by the presence of the two new terms in the denominator. These terms are independent of the magnetic field, but depend on the square of the heating field and, therefore, assume importance at high electric fields.

III. NUMERICAL RESULTS AND CONCLUSIONS

Numerical computations were made using the parameter values given in Table I.

The values of $\langle\tau\rangle$ and $\langle\tau^2\rangle$ for the $\langle 111 \rangle$ valleys were evaluated numerically with the aid of a computer. For the $\langle 100 \rangle$ valleys, the related parameters being not precisely known, the value of $\langle\tau\rangle$

TABLE I. Constants of germanium.

Parameters	Values
Density of the material	$5.33\times10^3\ \mathrm{kg\,m^{-3}}$
Velocity of sound in the material	$5.4\ \times10^3\ \mathrm{m\,sec^{-1}}$
Optical-phonon temperature	400 °K
Intervalley-phonon temperature	330 °K
Energy gap between the $\langle 100 \rangle$ and the $\langle 111 \rangle$ valleys (Ref. 1)	0.18 eV
Interaction constant for the acoustic-phonon scattering in	
(a) energy loss	25.6 eV ($=\Xi_0$)
(b) momentum loss	12.6 eV ($=\Xi_1$)
Deformation potential for	
(a) equivalent intervalley scattering among the $\langle 111 \rangle$ valleys (Refs. 9 and 11)	1 eV
(b) nonequivalent intervalley scattering (Ref. 11)	$1\times10^8 a_0$ eV
Longitudinal effective mass for	
(a) $\langle 111 \rangle$ valleys	$1.6m_0$
(b) $\langle 100 \rangle$ valleys (Ref. 11)	$0.9m_0$
Transverse effective mass for	
(a) $\langle 111 \rangle$ valleys	$0.082m_0$
(b) $\langle 100 \rangle$ valleys (Ref. 11)	$0.19m_0$

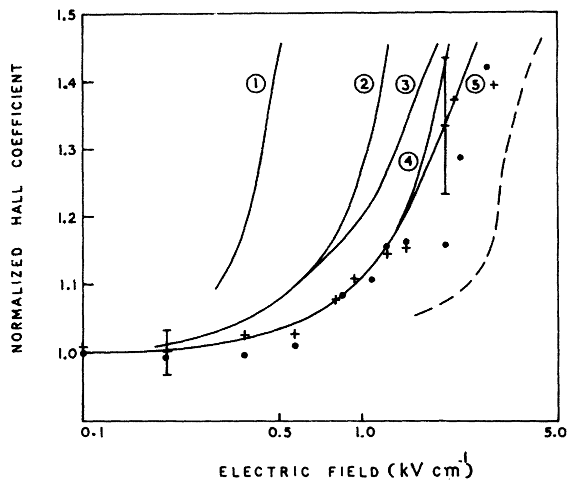

FIG. 1. Plots of normalized Hall coefficient against electric field. + and • represent experimental points for $B_z = 0.6$ and 3.2 kG (Ref. 1). Dashed curve indicates results obtained by Heinrich *et al.* (see Ref. 1). Curves ①, ②, and ④, are calculated for $D_0 = (0.4, 1.0,$ and $1.5) \times 10^9$ eV cm^{-1}, respectively, considering carrier transfer to the ⟨100⟩ minima. Curves ③ and ⑤ are for $D_0 = (1.0$ and $1.5) \times 10^9$ eV cm^{-1}, respectively, excluding electron scattering to the ⟨100⟩ minima.

was estimated by assuming the mobility in these valleys to be one-sixth of the low-field mobility in the ⟨111⟩ valleys.[4] The value of $\langle\tau^2\rangle$ for the ⟨100⟩ minima was taken to be equal to $\langle\tau\rangle^2$. This last approximation is not expected to introduce much error since the ratio of $\langle\tau^2\rangle/\langle\tau\rangle^2$ is usually very close to unity.

The normalized Hall factor was calculated from Eq. (2) taking the optical-phonon deformation-potential constant D_0 equal to 0.4×10^9 eV cm^{-1}, which is the same as that used by Heinrich *et al.*[1] in their computations. The calculated results are plotted in Fig. 1 together with the experimental points of Heinrich *et al.*[1] The dashed curve represents the calculated results of Heinrich *et al.*[1] It is found that there is a remarkable difference between this curve and the curve ① which is obtained from the present analysis, using the same value of D_0. The disagreement arises partly from the difference in the models, but primarily from the inclusion of the magnetic field dependence of the carrier temperature and population in the present calculations.

We also find that the curve ① differs widely from the experimental results and one may conclude that the high-field Hall mobility results at 200 °K cannot be explained from theory if D_0 is taken to be 0.4×10^9 eV cm^{-1} and the effect of the magnetic field on the carrier distribution is taken into account.

The Hall-factor curves are largely affected by the value of D_0. In order to investigate whether the agreement between theory and experiment could be improved by altering the value of D_0, we have performed calculations for $D_0 = 1 \times 10^9$ eV cm^{-1} (curve ②) and $D_0 = 1.5 \times 10^9$ eV cm^{-1} (curve ④). Calculations were also made for these values of D_0 neglecting the effect of carrier scattering to the ⟨100⟩ minima (curves ③ and ⑤). It is seen that the agreement improves as the value of D_0 is increased and fairly good agreement is obtained for values of D_0 lying between 1.0×10^9 and 1.5×10^9 eV cm^{-1}, particularly if scattering to the ⟨100⟩ valleys is neglected. The values of D_0 reported in the literature lie between 0.4×10^9 and 1.15×10^9 eV cm^{-1}. A value of 1.15×10^9 eV cm^{-1} was obtained by Meyer[12] and by Rosenberg and Lax[13] from the analysis of infrared absorption results. This value was revised to 0.4×10^9 eV cm^{-1} by deVeer and Meyer.[14] However, the value of D_0 is required to be 1×10^9 eV cm^{-1} to explain the temperature dependence of low-field mobility.[15] This value is reduced by a factor of $(0.125/0.195)^{1/2} \approx 0.8$ when the nonparabolicity of the band structure is taken into account.[8] It is also seen that the field dependence of mobility may be explained with a value of 0.8×10^9 eV cm^{-1} in the simple theory[16] and 0.7×10^9 eV cm^{-1} in a more detailed theory.[8] A value of 0.5×10^9 eV cm^{-1} is obtained from the analysis of the saturation region of the hot-electron characteristics,[7] while a value of 0.7×10^9 eV cm^{-2} is required to explain the temperature dependence of the saturation velocity.[17] But in such analyses the effect of scattering into the ⟨100⟩ valleys has not been included. The majority of the experimental evidence is thus in favor of a value of D_0 equal to $0.7 \times 10^9 - 0.8 \times 10^9$ eV cm^{-1}. We may therefore conclude that the value of D_0 required for explaining the experimental results on the Hall factor at 200 °K is higher than what is indicated from the analyses of other experiments and the experimental results should be considered to remain unexplained by theory even if scattering to the ⟨100⟩ valleys is included.

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Magnetic Freezeout and Impact Ionization in GaAs†

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Low-temperature measurements of impurity effects have been conducted in n-type GaAs with impurity concentrations of the order of $(1-3)\times 10^{15}$ cm^{-3}. The behavior of these materials in strong magnetic fields is well described by a recently developed theory which characterizes the impurity level with a broadened energy distribution and the conduction band with a Gaussian-like tail. The reduced impurity ionization energies in strongly doped materials are explained in terms of this model. Electric field ionization of these impurity levels in strong magnetic fields was found to be consistent with the model and recent theories of impact-ionization phenomena.

INTRODUCTION

Recent work on epitaxial growth of n-type GaAs films has yielded samples with electron concentrations as low as 1×10^{12} cm^{-3} and a well-defined impurity level separated from the conduction-band edge by approximately 0.005 eV.[1-3] In studies of far-infrared photoconductivity and cyclotron resonance in this material, samples with donor impurity concentrations on the order of 1×10^{15} cm^{-3} have proven to be widely used. For GaAs with impurity concentrations in this range, the electron concentration in the conduction band depends on the temperature and magnetic field strength in a manner similar to that previously described for InSb[4] and InAs.[5]

At low temperature, some electrons in GaAs are frozen out of the conduction band into impurity bound states. However, in the absence of a magnetic field, an appreciable fraction of the carriers remain free in the conduction band. These free carriers are the result of a reduced donor activation energy in strongly doped materials caused by tails on the densities of states of the conduction band and impurity level. Application of an intense magnetic field H shrinks the volume of the electronic wave function to a region less than that occupied by an impurity ion. Charge carriers are frozen out of the lowest-order conduction-band Landau level onto localized states with a binding energy $\mathcal{E}_b$ which increases with magnetic field. This magnetic freezeout is characterized by an increase in the Hall coefficient with magnetic field at a fixed temperature T.

This paper reports on measurements of the temperature and magnetic field dependence of electron concentration of n-type GaAs in the freezeout regime. The binding energy $\mathcal{E}_b(H)$ has a magnetic field dependence of $\mathcal{E}_b \propto H^{1/3}$ as has been previously observed in n-InSb[4] and n-InAs.[5] The magnitude of the binding energy is less than that measured in experiments on lightly doped samples or predicted by theoretical calculations.[6,7] The results can be explained by resorting to the recently developed model of a strongly doped semiconductor in a magnetic field.[8] In this model, which is based on spatial fluctuations in a random impurity potential, there is a tail on the conduction-band density of states and a broadened impurity level leading to a temperature- and field-dependent ionization energy. A reasonably accurate picture of the band tailing and impurity