# Impurity pinning of charge-density waves and spin-density waves

Kazumi Maki and Attila Virosztek*†
*Department of Physics, University of Southern California, Los Angeles, California 90089-0484*
(Received 15 August 1988)

We reexamine the threshold electric field of the charge-density wave (CDW) within the Fukuyama-Lee-Rice model, where the temperature dependence of the effective Hamiltonian for the phase $\phi$ of the CDW is explicitly included. Our theory describes very well the temperature dependence of the threshold field in $NbSe_3$ when the effects of the thermal fluctuations of $\phi$ are included. An extension of the model for the spin-density wave predicts that the threshold electric field in the spin-density wave does not exhibit a divergence at $T=T_c$ but is almost constant for $T \lesssim 0.5T_c$ and slightly increases ($\sim 33\%$) near $T=T_c$.

Usually the threshold electric field of the charge-density wave (CDW) in quasi-one-dimensional systems such as $NbSe_3$, etc., is described in terms of the Fukuyama-Lee-Rice (FLR) theory.[1,2] In this approach the spatial configuration of the phase $\phi(\mathbf{x})$ of the order parameter is described by the phase Hamiltonian

$$H(\phi)=\int d^D x \left[ \tfrac{1}{2} f_1 K |\nabla\phi|^2 - 2N_0\lambda^{-1}\Delta(T)V\sum_i \cos[\mathbf{Q}\cdot\mathbf{x}+\phi(\mathbf{x})]\delta(\mathbf{x}-\mathbf{x}_i) - enf_1 Q^{-1}\phi E \right], \tag{1}$$

where

$$f_1=\rho_s(T)/\rho, \quad K=\tfrac{1}{2}N_0 v_F^2, \quad Q=2k_F, \tag{2}$$

and $\lambda$, $V$, $\Delta(T)$, $\rho_s(T)$, and $n$ are the dimensionless electron-phonon coupling constant, the impurity potential, the temperature-dependent order parameter, the condensate density, which has the same temperature dependence as the superfluid density in a BCS superconductor, and the electron density, respectively. Here we include the temperature dependence of all the coefficients[3] explicitly but neglect the anisotropy of the elastic term, since this does not affect the temperature dependence of the threshold electric field. In particular, the coupling term to the electric field $E$ follows the chiral symmetry.[4] However, the temperature dependence of this term is different from that assumed in Refs. 1 and 2. In general, the coupling term is given by a limiting value[5] of $f(\omega,q)$ which has different limits depending on whether $\omega \gtrless v_F q$. Here $\omega$ and $q$ are the frequency and the wave vector associated with $\phi(\mathbf{x},t)$. In the static limit $f\rightarrow f_1$ [$\propto \Delta^2(T)$ in the vicinity of $T_c$] while in the dynamic limit $f\rightarrow f_0$ [$\propto \Delta(T)$ for $T\rightarrow T_c$]. In Refs. 1 and 2 the latter limit is used, which is appropriate to describe the microwave conductivity or the dc conductivity for $E \gg E_T$. However, in the analysis of the dc conductivity for $E \simeq E_T$, $f_1$ should be used since $q \sim L^{-1}$ the inverse of Fukuyama-Lee-Rice length[1,2] due to the spatial distortion of $\phi$, while $\omega=0$.

In this Brief Report we shall show that Eq. (1) predicts the unique temperature dependence of the threshold field though it depends on whether we are in the strong-pinning regime or in the weak-pinning regime.[1,2] Further in the latter case it depends on the dimensionality $D$ of the system.

First, in the strong-pinning regime the threshold field at $T=0$ K is given by[2]

$$E_T^S(0)=\frac{2Q}{e\lambda}\left(\frac{n_i}{n}\right)(N_0V)\Delta(0). \tag{3}$$

When the impurity concentration $n_i$ is the order of a few ppm, Eq. (3) will give $E_T^S \sim$ V/cm. Also, as already pointed out by FLR, $E_T^S(0)$ depends linearly on $n_i$, which has been verified in some experiments.[6,7] For $T \neq 0$ K, Eq. (1) predicts in the strong-pinning regime

$$E_T^S(T)/E_T^S(0)=[\Delta(T)/\Delta(0)][\rho/\rho_s(T)], \tag{4}$$

which increases monotonically with $T$. As $T$ approaches $T_c$, Eq. (4) diverges as

$$E_T^S(T)/E_T^S(0)\simeq 0.868(1-T/T_c)^{-1/2}. \tag{5}$$

Though such a divergence in $E_T(T)$ has been seen in a number of experiments,[8] the observed $E_T(T)$ exhibits a minimum slightly below $T=T_c$, which contradicts Eq. (4). The minimum in $E_T(T)$ is described if the effects of the thermal fluctuations[9] in $\phi$ is included. The thermal fluctuation in $\phi$ modifies Eq. (4) as[9]

$$E_T^S(T)/E_T^S(0)=e^{-T/T_0}[\Delta(T)/\Delta(0)][\rho/\rho_s(T)], \tag{6}$$

where $T_0$ is a parameter proportional to $\xi=v_F/T_c$. Now Eq. (6) predicts a minimum in $E_T(T)$ at $T=T_c-\frac{1}{2}T_0$ if $T_0 \ll T_c$ consistent with the experiments.[8] In Fig. 1 we compare Eq. (6) with $E_T(T)$ measured for $NbSe_3$ by Fleming.[10] In this comparison we adjusted both $E_T(0)$ and $T_0$. As seen from Fig. 1 the present theory gives an excellent description of $E_T(T)$ of the second CDW with $T_{c2}=59$ K. On the other hand, for the first CDW, the measured $E_T(T)$ diverges stronger than Eq. (6) predicts

 

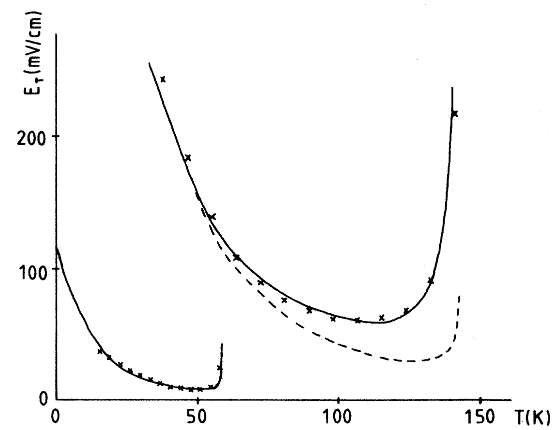


FIG. 1. The threshold electric field of $NbSe_3$ as a function of temperature. The crosses are the experimental data taken from Ref. 10, while solid curves are theoretical results; for the first CDW with $T_c = 144$ K, we used the $D=2$ weak-pinning model with $E_T(0) = 765$ mV cm$^{-1}$ and $T_0 = 60.6$ K. For the second CDW with $T_c = 59$ K, we used the strong-pinning model with $E_T(0) = 117$ mV cm$^{-1}$ and $T_0 = 14.6$ K. The dashed curve is a tentative comparison of the strong-pinning model with $E_T$ for the first CDW.

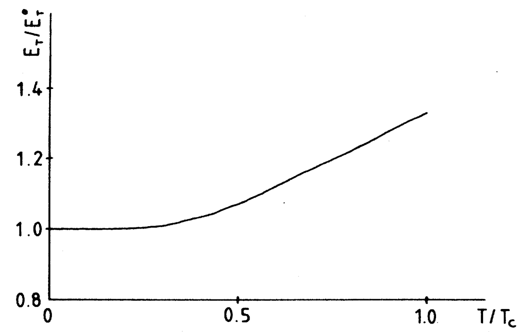


FIG. 2. The predicted temperature dependence of $E_T(T)$ of a spin-density wave in the strong-pinning limit is shown as function of the reduced temperature $T/T_c$.

at $T = T_{c1}$. Here we treat two CDW's below $T_{c1}$ and $T_{c2}$ independently as is commonly done.

We shall see shortly that $E_T(T)$ of the first CDW is described by the weak-pinning expression with $D=2$. We note also that $T_0 = 14.6$ K deduced from the above fitting is very close to $T_0 = 15.2$ K determined from the threshold field of $NbSe_3$ in a strong transverse magnetic field ($\sim 23$ T) by Coleman *et al.*[11] This also indicates that $T_0$ is not only insensitive to the impurity concentration[9] but also to the transverse magnetic field.

Repeating FLR, we obtain in the weak-pinning regime

$$E_T^W(0) \propto n_i^{2/4-D} . \qquad (7)$$

Furthermore, we obtain

$$E_T^W(T)/E_T^W(0) = [E_T^S(T)/E_T^S(0)]^{4/4-D} , \qquad (8)$$

where $D$ is the dimensionality of the CDW. In general the divergence of $E_T(T)$ near $T \simeq T_c$ becomes stronger in the weak-pinning regime. As is seen in Fig. 1, Eq. (8) with $D=2$ gives an excellent description of $E_T(T)$ of the first CDW in $NbSe_3$, though we do not know why $D=2$ should be used. Perhaps this implies that the elastic constant $K$ for the first CDW is strongly anisotropic so that we have $K_1 > K_2 \gg K_3$. Further, $D=2$ predicts also $E_T^W \propto n_i$, which appears to be consistent with a recent study[12] of the impurity effect on $NbSe_3$. In summary, we have shown that the effective Hamiltonian (1) predicts quantitatively the temperature dependence of the threshold electric field of the CDW.

Now we shall apply a similar model to the spin-density wave (SDW) in the quasi-two-dimensional systems like Bechgaard salts. From the analysis of the fluctuation propagator and the pinning potential in the SDW,[5] the only necessary change in Eq. (1) is the coupling to the random impurities (the second term). This is now given[5]

$$H_{\mathrm{imp}}(\phi) = -[(\pi/2)N_0V]^2\Delta(T)\tanh[\Delta(T)/2T] \times \sum_i \cos 2[\mathbf{Q}\cdot\mathbf{x}_i + \phi(\mathbf{x}_i)] . \qquad (9)$$

Also in the small $V$ limit ($N_0V \ll 1$) a more general result of Tüttö and Zawadowski[13] reduces to Eq. (9). Again in the strong-pinning limit Eq. (1) with Eq. (9) predicts

$$E_T^S(0) = \frac{Q}{e}\left(\frac{n_i}{n}\right)(\pi N_0 V)^2\Delta(0) \qquad (10)$$

which is now of the order of $10^{-2}$ V/cm for $n_i/n \sim 10^{-6}$. The temperature dependence of $E_T^S(T)$ is given by

$$E_T^S(T)/E_T^S(0) = \frac{\Delta(T)}{\Delta(0)}\tanh[\Delta(T)/2T][\rho/\rho_s(T)] , \qquad (11)$$

which increases monotonically with temperature from unity to 1.33 at $T = T_c$. There is no divergence in $E_T^S(T)$ near $T = T_c$. Equation (11) is numerically calculated and shown in Fig. 2. In particular for $T < 0.5T_c$, $E_T^S(T)$ is practically independent of $T$, which is consistent with a recent experiment on $E_T(T)$ of the SDW in $(TMTSF)_2NO_3$ by Tomić *et al.*[14]

Note that unlike the case of CDW, the thermal fluctuation is unimportant here, since $T_c$ is rather small ($\sim 10$ K), and $T_0 \propto \xi = v_F/T_c$ is rather large ($T_0 \sim 10^2$ K).

In the weak-pinning regime we obtain again Eqs. (7) and (8), where Eq. (11) has to be used for $E_T^S(T)/E_T^S(0)$ in Eq. (8) now. Therefore, unlike the case of the CDW, we do not expect any divergence in $E_T(T)$ in the vicinity of the transition temperature.

We have benefitted from a clarifying discussion with S. Tomić on her recent experiments on $(TMTSF)_2NO_3$. This work is in part supported by the National Science Foundation under Grant No. DM86-11829.

*Present address: Department of Physics, University of Virginia, Charlottesville, VA 22901.

†Permanent address: Central Research Institute for Physics, H-1525, Budapest 114, Hungary.

[1]H. Fukuyama and P. A. Lee, Phys. Rev. B **17**, 535 (1978).

[2]P. A. Lee and T. M. Rice, Phys. Rev. B **19**, 3970 (1979).

[3]See, for example, H. Fukuyama and H. Takayama, in *Electronic Properties of Inorganic Quasi-One-Dimensional Materials,* edited by P. Monceau (Reidel, Boston, 1985), p. 41.

[4]Z. B. Su and B. Sakita, Phys. Rev. Lett. **56**, 780 (1986).

[5]A. Virosztek and K. Maki, Phys. Rev. B **37**, 2028 (1988); K. Maki and A. Virosztek (unpublished).

[6]J. W. Brill, N. P. Ong, J. C. Eckert, J. Savage, S. K. Khanna, and R. B. Somoano, Phys. Rev. B **23**, 1517 (1981).

[7]W. W. Fuller, G. Grüner, P. M. Chaikin, and N. P. Ong, Phys. Rev. B **23**, 6259 (1981).

[8]P. Monceau, in Ref. 3, p. 139.

[9]K. Maki, Phys. Rev. B **33**, 2852 (1986).

[10]R. M. Fleming, Phys. Rev. B **22**, 5606 (1980).

[11]R. V. Coleman, M. P. Everson, G. Eiserman, and A. Johnson, Phys. Rev. B **32**, 537 (1985).

[12]M. Underweiser, M. Maki, B. Alavi, and G. Grüner, Solid State Commun. **64**, 181 (1987).

[13]I. Tüttö and A. Zawadowski, Phys. Rev. Lett. **60**, 1442 (1988).

[14]S. Tomić, J. R. Cooper, D. Jérome, and K. Bechgaard, Phys. Rev. Lett. **62**, 2466 (1989); S. Tomić, D. Jérome, J. R. Cooper, and K. Bechgaard, Synth. Met. (to be published).