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Electromagnetic interactions between fluctuations near the superconducting phase transition

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We calculate the amplitude ratio C_+/C_- for specific-heat fluctuations near the superconducting-phase transition, taking into account the lowest-order fluctuations about mean-field theory. For a U(1) Ginzburg-Landau theory, minimally coupled to electromagnetism in d dimensions, $C_+/C_- = 2/(2^{d/2}+\kappa^{-d})$, where κ is the Ginzburg-Landau parameter. We discuss how this result arises from the microscopic theory of superconductivity.

I. INTRODUCTION

The short coherence length of the high-temperature superconductor $YBa_2Cu_3O_{7-\delta}$ has enabled fluctuation effects to be observed even in bulk samples near to the superconducting transition temperature T_c. Measurements have been reported for fluctuation contributions to the conductivity,[1–7] the magnetic susceptibility,[8–10] the thermopower,[11] and the specific heat.[12–16] In most of these cases, the temperature dependence of the data has been consistent with the interpretation that Gaussian fluctuations—the lowest-order fluctuations about mean-field theory—are responsible for the observations. The observation of a fluctuation contribution to the specific heat[12] in zero external magnetic field is of particular interest, because it can be used to test directly the hypothesis that the order parameter has only two real components. Indeed, in Ref. 12, the ratio of the amplitudes of the Gaussian fluctuation specific heat above and below the transition, C_+/C_-, was found to be significantly larger than expected for an s-wave superconductor, leading to the conclusion that the pairing involved a higher-angular-momentum state. It should be noted that this result does not require any special assumptions regarding the spatial anisotropy, the values of the parameters in the Ginzburg-Landau free energy, or the applicability of BCS theory.

The purpose of this Brief Report is to calculate C_+/C_- in the Gaussian approximation, where the direct interaction between fluctuations is ignored. Previously, this has been done for the $O(n)$ model,[17] but this calculation ignores the fact that in a superconductor, the fluctuations can interact indirectly through the electromagnetic field. In the present paper, we examine this effect: as might have been expected, the inclusion of the electromagnetic field only decreases C_+/C_-. Thus we may safely dismiss a possible explanation of the findings of Ref. 12 based solely on the previously neglected effects of electromagnetic interactions.

In Sec. II we calculate C_+/C_- in a U(1) Ginzburg-Landau theory minimally coupled to electromagnetism. Such a calculation only depends on the static properties of the fluctuations, but apparently not on the spectral density of excitations of the many-body system. We have found it instructive to investigate exactly why it is the dynamic properties of the fluctuations do not affect the result, and these considerations are explained in Sec. III.

II. GINZBURG-LANDAU THEORY

The calculation of C_+/C_- essentially amounts to counting the number of temperature-dependent, long-wavelength modes above and below T_c. Such modes are described by a propagator in momentum space $\mathbf{k}$ of the form $G(\mathbf{k}) \propto (k^2+m^2)^{-1}$. In the context of Ginzburg-Landau theory, the temperature T, enters through the propagator's "mass" m, which is simply proportional to $\sqrt{|T-T_c|}$. For an $O(n)$ model describing a neutral condensate, there are n massive modes above T_c, but there is only one massive mode below T_c. It corresponds to the amplitude of the order parameter, while the $n-1$ Goldstone modes correspond to the phases of the order parameter. Furthermore, for fixed $|t|$, where $t=(T-T_c)/T_c$, the mass of the longitudinal mode below T_c is $\sqrt{2}$ times as big as the mass above T_c. It is straightforward to find the most singular terms in the free energy and thence find the fluctuation specific heat[17] above (+) and below (−) T_c:

$$C(t) = \frac{C\pm}{t^{2-d/2}}\,, \tag{2.1}$$

$$\frac{C_+}{C_-} = \frac{n}{2^{d/2}}\,. \tag{2.2}$$

For a superconductor with a charged condensate, however, fluctuations in the order parameter generate currents and fields which can couple order-parameter fluctuations at different positions. This occurs when the longitudinal component of the vector potential becomes massive, through the Anderson-Higgs mechanism,[18] at the expense of one of the Golstone modes. Physically, this corresponds to the existence of a temperature-dependent electromagnetic penetration depth, $\lambda(T)$, below T_c.[19] As shown below, it is this temperature dependence which generates an additional term in the specific heat below T_c; the divergence of the electromagnetic penetration depth as $T \to T_c$ ensures that this term also contributes to the fluctuation specific heat below T_c and hence to the denominator of C_+/C_-.

Consider the usual (i.e., $n=2$) expression for the Ginzburg-Landau free-energy density of a superconductor of volume V in d dimensions:

$$F\{\psi,\mathbf{A}\} = \int d^d\mathbf{x} \left[\alpha |\psi|^2 + \tfrac{1}{2}\beta|\psi|^4 + |D\psi|^2 + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2 \right], \tag{2.3}$$

$$|D\psi|^2 \equiv \sum_{i=1}^{2}\sum_{\mu=1}^{d} \left| \left(\frac{\partial}{\partial x_\mu} - iqA_\mu \right) \psi_i \right|^2. \tag{2.4}$$

Here, $\psi \equiv \psi_1 + i\psi_2$, $q=2e/\hbar c$, e is the electronic charge, c is the speed of light in vacuum, β is assumed to be a temperature-independent constant, and α is related to the order-parameter correlation length ξ by

$$\alpha(T) = \xi(T)^{-2} = \alpha_0 |T - T_c| \,. \tag{2.5}$$

We work in the Coulomb gauge $\nabla\cdot\mathbf{A}=0$. The thermodynamics is obtained from the partition function

$$Z = \int D\mathbf{A}\, D\psi\, e^{-F\{\psi,\mathbf{A}\}/T}. \tag{2.6}$$

In order to examine the Gaussian fluctuations about mean-field theory, we minimize $F\{\psi,\mathbf{A}\}$ and expand to quadratic order in the fields; after a gauge transformation, we obtain for the fluctuation contribution to the free energy

$$\tilde{F} = \int d^d\mathbf{x} \left[(\nabla\tilde{\psi}_1)^2 - 2\alpha\tilde{\psi}_1^2 + m_A^2\tilde{A}^2 + \frac{1}{8\pi}(\nabla\times\tilde{\mathbf{A}})^2 \right], \tag{2.7}$$

where the tilde denotes the fluctuation in a quantity about its mean-field value. The "photon mass" m_A is given by

$$m_A^2 = \frac{1}{8\pi\lambda(T)^2}\,. \tag{2.8}$$

Performing the functional integral in Eq. (2.6), and identifying the most singular terms in $\tilde{F}$ near T_c, F_s, we find that

$$F_s = -\tfrac{1}{2}T\frac{1}{V}2\sum_{\mathbf{k}}\ln\left[\frac{\pi}{\alpha+k^2}\right], \quad T>T_c\,. \tag{2.9}$$

$$F_s = -\tfrac{1}{2}T\frac{1}{V}\sum_{\mathbf{k}}\ln\left[\frac{\pi}{2\alpha+k^2}\right] - \tfrac{1}{2}T\frac{1}{V}\sum_{\mathbf{k}}\ln\left[\frac{\pi}{k^2+\lambda^{-2}}\right], \quad T<T_c\,. \tag{2.10}$$

The most singular terms in the specific heat, C_s, are obtained from $C_s = -T\partial^2F_s/\partial T^2$:

$$C_s = T^2\alpha_0^2\int\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{1}{(\alpha+k^2)^2}, \quad T>T_c\,, \tag{2.11}$$

$$C_s = 2^{d/2-1}T^2\alpha_0^2\int\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{1}{(\alpha+k^2)^2} + \frac{1}{2\lambda_0^2}T^2\int\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{1}{[\lambda(T)^{-2}+k^2]^2}, \quad T<T_c\,, \tag{2.12}$$

where the temperature-dependent penetration depth is given by $\lambda = \lambda_0 t^{-1/2}$. After some algebra, we obtain

$$\frac{C_+}{C_-} = \frac{2}{2^{d/2}+\kappa^{-d}}\,, \tag{2.13}$$

where the Ginzburg-Landau parameter $\kappa \equiv \lambda/\xi$. This result is valid for both type-I and type-II superconductors in the presence of zero external magnetic field.

III. MICROSCOPIC THEORY

An interesting feature of the calculation presented above is that the result is obtained solely from the static properties of the fluctuations. This might appear to be in conflict with the results of standard many-body theory calculations of fluctuations in quantum systems such as almost ferromagnetic Fermi liquids[20] or superconductors near T_c.[21] In these calculations, fluctuation contributions are expressed in terms of frequency integrals of the spectral density of excitations, and accordingly, one might expect that dynamical properties of the fluctuations would enter into the calculation. In other words, the fact that the plasma frequency, $\omega_p \gg T$ might lead one to expect that charge fluctuations are suppressed, thus freezing out the phase as a dynamical variable.

To investigate this point in greater detail, and to show why this is, in fact, not the case, we start from the usual result for the fluctuation contribution to the thermodynamic potential

$$\Delta\Omega = -T\sum_{\omega_n,\mathbf{q}}\ln[G^{-1}(\mathbf{q},i\omega_n)G^0(\mathbf{q},i\omega_n)]\,. \tag{3.1}$$

Here, G is the propagator for the fluctuation in question, G^0 is its bare value in the absence of interactions, and the frequencies are given by $\omega_n = 2\pi nT$, where n is an integer. Converting the summation into an integration, we find

$$\Delta\Omega = -\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\operatorname{Im}\ln[G^{-1}(\mathbf{q},\omega)G^0(\mathbf{q},\omega)]}{e^{\omega/T}-1}\,. \tag{3.2}$$

This expression gives the fluctuation contribution to the thermodynamic potential in terms of finite frequency properties of the fluctuations. In the case of transverse electromagnetic fluctuations in a superconductor, the fluctuation propagator at long wavelengths has a pole at the plasma frequency of the metal, $\omega_p = (4\pi n_e e^2/m)^{1/2}$, where n_e is the total electron density, and m is the mass of the electron. ω_p remains finite as $T \to T_c$, and therefore one might expect that there would be no singular contribution to thermodynamic properties in this limit. Since ω_p is typically much greater than T_c, electromagnetic effects would effectively be frozen-out.

This argument neglects the fact that the critical behavior arises from frequencies $\omega \ll T$. Thus, we may take the classical limit of Eq. (3.2),

$$(\Delta\Omega)_{\text{critical}} = -T\sum_{\mathbf{q}} P\int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\operatorname{Im}\ln[G^{-1}(\mathbf{q},\omega)G^{0}(\mathbf{q},\omega)]}{\omega} . \tag{3.3}$$

Since at high frequencies $G^{-1}(\mathbf{q},\omega)G^{0}(\mathbf{q},\omega)$ tends to unity, it follows that $\ln G^{-1}(\mathbf{q},\omega)G^{0}(\mathbf{q},\omega)$ obeys a Kramers-Kronig relation, and therefore, one may write

$$(\Delta\Omega)_{\text{critical}} = -T\sum_{\mathbf{q}} \ln[G^{-1}(\mathbf{q},0)G^{0}(\mathbf{q},0)] . \tag{3.4}$$

This is just the $n=0$ term in the summation of Eq. (3.1), namely the only term which arises in the purely classical treatment of Sec. I.

The fallacy in the physical argument given at the beginning of this section, which indicated that electromagnetic effects would suppress fluctuations, is the incorrect assumption that all of the spectral weight of G lies at the plasmon pole. This is not the case, because there is a cut in G which arises from the low-lying excitations such as electron-hole pairs associated with the normal fluid in a two-fluid model. These local charge fluctuations do allow the phase to relax. If the normal fluid is neglected, then $G^{-1}(\mathbf{q},0) \propto (q^2+\omega_p^2/c^2)/T$, while if the response of the normal fluid is taken into account, the corresponding result is $G^{-1}(\mathbf{q},0) \propto (q^2+\lambda^{-2})/T$. As before, it is the variation of λ as $T \to T_c$ which gives a singular contribution to the thermodynamic functions. In conclusion, we see that there is no conflict between the two ways of calculating the fluctuation contribution to thermodynamic properties, provided that one properly takes into account the low-frequency behavior of the spectral density in Eq. (3.3).

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