PHYSICAL REVIEW B VOLUME 39, NUMBER 13 1 MAY 1989

Dissipative quantum mechanics of a particle in the washboard potential: Application to the Josephson junction

Yu. N. Ovchinnikov*
Istituto di Cibernetica, CNR, 80072 Arco Felice, Napoli, Italy

B. I. Ivlev
Institute for Theoretical Physics, Academy of Sciences of U.S.S.R., Moscow, U.S.S.R.
(Received 10 August 1988)

The dependence of the mean value of the voltage of a Josephson junction on temperature and current is obtained for low viscosity. No resonant response to high-frequency-driven force exists in the mean value of the voltage when the frequency is close to the distances between levels at quasilocalized states.

I. INTRODUCTION

In the last years many authors have actively investigated the dissipative quantum mechanics. This interest is partially connected with the large progress in the preparation of Josephson junctions of very small size. Such junctions are in fact particularly suitable to the study of dissipative phenomena in quantum mechanics.

The essential theoretical progress in this topic was made by Caldeira and Leggett.[1] They proposed an effective action, which allows the description of the influence of the environment on the motion of a quantum particle. Later the effective action was obtained microscopically by Schmidt.[2] For the Josephson junctions the effective action was discussed in Refs. 3 and 4. In principle, the method of functional integration enables to find the density matrix and hence it gives a full description of the system. The validity of such a quantum-mechanical description of a Josephson tunnel junction was demonstrated by Martinis, Devoret, and Clarke[5,6] in experiments on the enhancement of the decay rate of metastable states by high-frequency field. There is good agreement between experimental and theoretical results.[7,8]

In many cases the knowledge of the transition probability from one well to another is sufficient. If the energy difference between two wells is not too small, this transition probability is known in all regions of values of both temperature and viscosity.[1,9–11] Here we are dealing with small values of the slope of the washboard potential, that is, small currents and small values of temperature and viscosity. In this region the quantum-mechanical interference phenomena are essential. This problem was studied by Likharev and Zorin.[1] Our results differ markedly from those of Ref. 12.

II. THE EQUATION FOR THE DENSITY MATRIX IN THE REPRESENTATION OF QUASILOCALIZED STATES

The dynamics of a particle, interacting with the environment, can be described with the aid of the density matrix $\hat{\rho}(t,\phi,\tilde{\phi})$.[13] The value of the density matrix at a time t_f can be found from its value at time t_i by resorting to the functional integral,[14]

$$\hat{\rho}(t_f,\tilde{\phi},\tilde{\phi}) = \int \mathcal{D}\phi\, \mathcal{D}\tilde{\phi} \exp[iA(\phi,\tilde{\phi})]\hat{\rho}(t_i,\phi,\tilde{\phi}) \; . \qquad (1)$$

The functional integral is taken over all values of coordinates $\phi,\tilde{\phi}$ at times between t_i and t_f. The effective action $A(\phi,\tilde{\phi})$ is equal to

$$A(\phi,\tilde{\phi}) = A_0(\phi) - A_0(\tilde{\phi}) + A_2(\pi,\tilde{\phi}) \; , \qquad (2)$$

where

$$A_0(\phi) \int_{t_i}^{t_f} dt \left[\frac{m}{2} \left(\frac{\partial\phi}{\partial t} \right)^2 - V(\phi) \right] \; . \qquad (3)$$

The potential $V(\phi)$ and the "mass" m for the tunnel junction are

$$V(\phi) = -E_J\cos(2\phi) - I\phi/e \; ,$$
$$E_J = I_c/2e, \quad m = C/e^2, \quad C = C_0 + 3E_J e^2/8\Delta^2 \; . \qquad (4)$$

Here I is the driven current through the junction. The mass m depends on the capacitance C_0 of the junction and has some additional contributions arising from virtual transitions of the quasiparticles.[3,4] Δ is the order parameter of the superconductor, and I_c is the critical current of the junction. The voltage V across the junction is equal to $eV = \langle \partial\phi/\partial t \rangle$. The functional $A_2(\phi,\tilde{\phi})$ is given by Eq. (4),

$$iA_2(\phi,\tilde{\phi}) = -\left[\frac{\pi}{R_s e^2} \right] \int_{t_i}^{t_f} dt \int_{t_i}^{t} dt_1 \{ \mathcal{K}(t-t_1)\cos[\phi(t)-\phi(t_1)] + \mathcal{K}(t_1-t)\cos[\tilde{\phi}(t)-\tilde{\phi}(t_1)] \}$$
$$+ \left[\frac{\pi}{R_s e^2} \right] \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt_1\, \mathcal{K}(t_1-t)\cos[\phi(t)-\tilde{\phi}(t_1)] \; . \qquad (5)$$

For a tunnel junction shunted by a normal resistance R_s the kernel $\mathcal{K}(t)$ is equal to

$$\mathcal{K}(t)=\int_{-\infty}^{\infty} d\varepsilon \frac{1}{2\pi}\mathcal{K}(\varepsilon)\exp(-i\varepsilon t) ,$$
$$\mathcal{K}(\varepsilon)=\frac{\varepsilon}{\pi}\left[1+\coth\left[\frac{\varepsilon}{2T}\right]\right] . \tag{6}$$

In the quasiclassical approximation[2] the action (2) leads to the Langevin equation for ϕ

$$m\frac{\partial^2\phi}{\partial t^2}+\frac{\partial V}{\partial\phi}+\eta\frac{\partial\phi}{\partial t}=\xi ,$$
$$\eta=\frac{1}{R_s e^2} . \tag{7}$$

The random quantity ξ has a correlation function

$$\langle \xi(t)\xi(t')\rangle=2T\eta\delta(t-t') . \tag{8}$$

In the potential (4) quasilocalized states exist which are solutions of the Schrödinger equation

$$\left[-\frac{1}{2m}\frac{\partial^2}{\partial\phi^2}+V(\phi)\right]\psi_N(\phi)=E_N\psi_N(\phi) . \tag{9}$$

The function ψ_N has the form

$$\psi_N(\phi)=\sum_k A_k\mathcal{D}_0((2m\Omega)^{1/2}(\phi-\phi_k-\pi N)) , \tag{10}$$

where

$$\phi_k=\pi k+\tfrac{1}{2}\arcsin\left[\frac{I}{I_c}\right] ,$$
$$\Omega^2=\frac{4E_J}{m}\left[1-\left[\frac{I}{I_c}\right]^2\right]^{1/2} , \tag{11}$$
$$E_N=E_0-\frac{\pi I}{e}N,\quad \mathcal{D}_0(x)=\exp(-x^2/4) .$$

To find the coefficients A_k we use the quasiclassical solutions of Eq. (9) under the barrier and exact solutions of Eq. (9) near the bottom of the potential $V(\phi)$. Matching these solutions we find the equation for the coefficients A_k,

$$A_{k+1}+\tilde{z}(k-\beta)A_k+A_{k-1}=0,\quad k=0,\pm 1,\dots , \tag{12}$$

where

$$\beta=\frac{e\Omega}{\pi I}\left[\frac{1}{2}-\frac{E_0-V(\phi_0)}{\Omega}\right] ,$$
$$\tilde{z}=\left[\frac{\pi I}{e\Omega}\right]2\pi^{1/2}\exp(S+\tfrac{1}{2}) . \tag{13}$$

Here S is the action under the barrier for the transition from well 0 to 1. The solution of Eq. (12) that vanishes for $k>\pm\infty$ is

$$A_k=C_0(-1)^kJ_k\left[\frac{2}{\tilde{z}}\right],\quad \beta=0 , \tag{13a}$$

where J_k is a Bessel function. The constant C_0 can be found from the normalization condition and is equal to

$$C_0=\left[\frac{m\Omega}{\pi}\right]^{1/4} . \tag{14}$$

Now we shall write the density matrix $\hat{\rho}(t,\phi,\tilde{\phi})$ in the form

$$\hat{\rho}(t,\phi,\tilde{\phi})=\sum_{n,k}\rho_k^n(t)\psi_n(\phi)\psi_k^*(\tilde{\phi})\exp\left[-i(\varepsilon_n-\varepsilon_k)t\right] , \tag{15}$$

where

$$\varepsilon_n=-\frac{\pi I}{e}n .$$

To get the equation for the density matrix $\rho_k^n(t)$ we can use the standard method[14] consisting in the expansion of the exponent in formula (1) in powers of $A_2(\phi,\tilde{\phi})$. As a result we get to the first approximation on the parameter η the equation

$$\frac{\partial\rho_f^j}{\partial t}=\left[\frac{\pi}{R_se^2}\right]\left[-\rho_f^j\sum_{k=-\infty}^{\infty}\mathcal{K}(-\varepsilon_k)J_k^2(4/\tilde{z})+(-1)^{j-f}\sum_{k=-\infty}^{\infty}\mathcal{K}(\varepsilon_k)J_k^2(4/\tilde{z})\rho_{k+f}^{k+j}\right] . \tag{16}$$

To obtain Eq. (16) we use the following formula for the transition matrix elements:

$$\int d\phi\, d\phi_1\, \psi_n(\phi)\psi_k(\phi)\psi_k(\phi_1)\psi_j(\phi_1)\cos(\phi-\phi_1)=(-1)^{n+j}J_{k-n}(4/\tilde{z})J_{k-j}(4/\tilde{z}) ,$$
$$\int d\phi\, d\phi_1\, \psi_k(\phi)\psi_j(\phi)\psi_n(\phi_1)\psi_f(\phi_1)\cos(\phi-\phi_1)=(-1)^{j+f}J_{k-j}(4/\tilde{z})J_{n-f}(4/\tilde{z}) . \tag{17}$$

Now in order to obtain the quantity $\langle\dot{\phi}\rangle$ we shall use the quantum-mechanical formula for the velocity

$$\left\langle\frac{\partial\phi}{\partial t}\right\rangle=\frac{\partial}{\partial t}\int d\phi\,\hat{\rho}(t,\phi,\tilde{\phi})\phi . \tag{18}$$

Using the expression for the transition matrix element

$$\int d\phi\,\psi_n(\phi)\phi\psi_k(\phi)=\pi n\delta_{k,n}=\frac{\pi}{\tilde{z}}\delta_{k,n\pm 1} , \tag{19}$$

we get from Refs. 15, 18, and 19,

$$\left\langle\frac{\partial\phi}{\partial t}\right\rangle=-\left[\frac{\pi}{R_se^2}\right]\pi\sum_{k=-\infty}^{\infty}k\mathcal{K}(\varepsilon_k)J_k^2(4/\tilde{z})-\frac{\pi}{\tilde{z}}\frac{\partial}{\partial t}\sum_{k=-\infty}^{\infty}\left[\rho_{k+1}^k\exp\left[\frac{i\pi I}{e}t\right]+\rho_k^{k+1}\exp\left[\frac{-i\pi I}{e}t\right]\right] . \tag{20}$$

The first sum in (20) can be easily found as

$$\sum_{k=-\infty}^{\infty} k\mathcal{K}(\varepsilon_k)J_k^2(4/\tilde{z})=\frac{8I}{e\tilde{z}^2} . \tag{21}$$

The last two terms in the bracket depend on the initial conditions and decay in time exponentially. From Eq. (16) we get

$$\frac{\partial}{\partial t}\left[\sum_k \rho_{k\pm1}^k\right]=-\gamma\left[\sum_k \rho_{k\pm1}^k\right] , \tag{22}$$

where the decay rate γ is given by

$$\gamma=2\left[\frac{\pi}{R_s e^2}\right]\sum_{k=-\infty}^{\infty}\mathcal{K}(\varepsilon_k)J_k^2(4/\tilde{z}) . \tag{23}$$

At a current I equal to zero there exists a band of width δ. The value of δ is connected with the action S under the barrier

$$\delta=\frac{4\Omega}{2\pi^{1/2}}\exp(-S-\tfrac{1}{2}) ,$$
$$S=\frac{8E_J}{\Omega}-\frac{1}{2}\left[1+\ln\left[\frac{64E_J}{\Omega}\right]\right] . \tag{24}$$

In the region of large currents $I\gg\delta e$, it is necessary to take into account the dependence of the action S on the current. This leads to appearance of additional factor in expression (20) for $\langle\dot{\phi}\rangle$

$$\left\langle\frac{\partial\phi}{\partial t}\right\rangle=\left[\frac{\pi}{R_s e^2}\right]\frac{e\delta^2}{2\pi I}\cosh^2\left[\left[\frac{\pi I}{2e\Omega}\ln\left[\frac{64E_J}{\Omega}\right]\right]\right] . \tag{25}$$

In formula (25) we omitted the oscillating term, because its value depends on the initial condition and decays exponentially in time. Expression (25) for $\langle\dot{\phi}\rangle$ has a minimum at current value $I_{\min}$ given by

$$\frac{\pi I_{\min}}{2e\Omega}=\frac{x_0}{\ln(64E_J/\Omega)} ,$$
$$2x_0=\coth x_0,\quad x_0=0.7717 . \tag{26}$$

The upper limit of validity for the current of expression (25) for $\langle\dot{\phi}\rangle$ is $e\Omega$,

$$I<e\Omega . \tag{27}$$

The lower boundary can be found from the simple condition that the distance between levels must be larger than damping γ, that is

$$\frac{I}{e}>\gamma . \tag{28}$$

For temperature T equal to zero and $I\ll\delta e$ we find from (23)

$$\gamma=\left[\frac{\pi}{R_s e^2}\right]\frac{4\delta}{\pi^2} , \tag{29}$$

and hence

$$\frac{I}{e\delta}>\left[\frac{\pi}{R_s e^2}\right]\frac{2}{\pi^2} . \tag{30}$$

III. DENSITY MATRIX IN BAND REPRESENTATION

For small values of current $I<\delta e$ it is convenient to go to the density-matrix representation in the band. In the case of narrow band that we considered here, the wave functions and spectrum are

$$E(k)=\frac{\Omega}{2}+\varepsilon(k) ,$$
$$\varepsilon(k)=-\frac{\delta}{2}\cos(\pi k) , \tag{31}$$

where k is a wave number

$$-1\le k\le 1 . \tag{32}$$

The wave functions $\psi_k(\phi)$ are normalized to $2\pi\delta(k-k')$. Moreover to find the equation for the density matrix it is necessary to calculate three transition matrix elements

$$\langle k_1|\phi|k\rangle=-i2\pi\frac{\partial}{\partial k}\delta(k-k_1) ,$$
$$\int d\phi\, d\phi_1\, \psi_{\tilde{k}}(\phi_1)\psi_{k_2}^*(\phi_1)\psi_{k_2}(\phi)\psi_k^*(\phi)\cos(\phi-\phi_1)$$
$$=(2\pi)^2\delta(k-\tilde{k})\delta(k_2-k\pm1) , \tag{33}$$
$$\int d\phi\, d\phi_1\, \psi_{\tilde{k}}(\phi)\psi_k^*(\phi)\psi_{\tilde{k}_1}^*(\phi_1)\psi_{k_1}(\phi_1)$$
$$=(2\pi)^2\delta(\tilde{k}-k\pm1)\delta(k_1-\tilde{k}_1\pm1) .$$

The density matrix $\hat{\rho}(t,\phi,\tilde{\phi})$ shall take the form

$$\hat{\rho}(t,\phi,\tilde{\phi})=\int_{-1}^{1}\frac{dk\, dk_1}{2\pi}\rho_{k_1}^k(t)\psi_k(\phi)\psi_{k_1}^*(\tilde{\phi}) . \tag{34}$$

Retaining only the first terms in the expansion of the exponent in formulas (1) on I and η and taking into account relation (33) we obtain the equation for the density matrix $\rho_{k_1}^k(t)$

$$\frac{\partial}{\partial t}\rho_{k_1}^k(t)=\left[\frac{\pi}{R_s e^2}\right]\left[i\rho_{k_1}^k\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\mathcal{K}(\varepsilon)\left[\frac{1}{2\varepsilon(k_1)-\varepsilon-i\nu}-\frac{1}{2\varepsilon(k)-\varepsilon+i\nu}\right]\right.$$
$$\left.+\rho_{k_1-\mathrm{sgn}k_1}^{k-\mathrm{sgn}k}\mathcal{K}[-2\varepsilon(k)]\frac{\exp[2i(t_f-t_i)][\varepsilon(k)-\varepsilon(k_1)]-1}{2i(t_f-t_i)}\right]$$
$$-\frac{I}{e}\left[\frac{\partial}{\partial k}\rho_{k_1}^k+\frac{\partial}{\partial k_1}\rho_{k_1}^k\right]+\frac{iI}{2e}(t_f+t_i)\rho_{k_1}^k\left[\frac{\partial\varepsilon(k)}{\partial k}-\frac{\partial\varepsilon(k_1)}{\partial k_1}\right] . \tag{35}$$

The last term in formula (35) for nondiagonal elements of density matrix obviously depends on (t_f+t_i). It means that nondiagonal elements essentially depend on the initial conditions and their dependence on time can be found only from Eq. (1). However, it is possible to search for the solution of Eq. (1) in the form,

$$\rho^k_{k_1(t)}=n(k,t)\left[\frac{2\pi}{L}\delta(k-k')\right] .$$

For the diagonal elements of the density matrix such difficulty is absent and from (35) it follows

$$\frac{\partial}{\partial t}n(k)=-\frac{I}{e}\frac{\partial n(k)}{\partial k}+\left[\frac{\pi}{R_s e^2}\right]\{-n(k)\mathcal{H}[2\varepsilon(k)+n(k-\mathrm{sgn}k)]\times\mathcal{H}[-2\varepsilon(k)]\} . \tag{36}$$

The boundary condition for the Eq. (36) is

$$n(1)=n(-1) . \tag{37}$$

From Eq. (36) it follows the important property

$$\left[\frac{\partial}{\partial t}+\frac{I}{e}\frac{\partial}{\partial k}\right][n(k)+n(k-\mathrm{sgn}k)]=0 . \tag{38}$$

The average velocity $v=\langle\dot{\phi}\rangle$ can be found from the quantum-mechanical formula

$$\left\langle\frac{\partial\phi}{\partial t}\right\rangle=\left[-\frac{i}{2m}\right]\int_{-\infty}^{\infty}d\phi\left[\left[\frac{\partial}{\partial\phi}-\frac{\partial}{\partial\phi_1}\right]\hat{\rho}(t,\phi,\phi_1)\right]_{\phi=\phi_1}=\int_{-1}^{1}dk\, n(k)\frac{\partial\varepsilon}{\partial k} . \tag{39}$$

Now we shall solve Eq. (36) for two limiting cases: low- $(T\ll\delta)$ and high- $(T\gg\delta)$ temperature. At stationary flow $\partial_n(k)/\partial t=0$ and for $T\ll\delta$ from relations (36) and (38) we get

$$n(k)=\begin{cases} c\exp[x\sin(\pi k)], & |k|>\frac{1}{2} \\ 2c\cosh(x)-c\exp[-x\sin(\pi k)], & |k|<\frac{1}{2} ,\end{cases} \tag{40}$$

where

$$x=\frac{2e\delta}{\pi^2 I}\left[\frac{\pi}{R_s e^2}\right] . \tag{41}$$

The constant c can be found from the normalization condition

$$\int_{-1}^{1}dk\, n(k)=1$$

that gives

$$c=\frac{1}{2\cosh x} .$$

Inserting expression (40) for $n(k)$ into (39) we get

$$\left\langle\frac{\partial\phi}{\partial t}\right\rangle=\frac{\pi\delta}{2}\frac{I_1(x)}{\cosh(x)} . \tag{42}$$

Here $I_1(x)$ is the Bessel function.

The function $I_1(x)/\cosh(x)$ has a smooth maximum at

$$x_0=1.83\,476$$

that is equal to

$$I_1(x_0)/\cosh(x_0)=0.42\,393 . \tag{43}$$

For large current values $(I>\eta\delta e)$ the parameter x is small and relation (42) gives the same answer for $\langle\dot{\phi}\rangle$ as formula (25). For small value of the current $(I\ll\eta\delta e)$ the parameter x is large and for $\langle\dot{\phi}\rangle$ we get

$$\langle\dot{\phi}\rangle=\left[\frac{\pi}{R_s e^2}\right]^{-1/2}\left[\frac{\pi\delta}{2}\right]\left[\frac{\pi^2 I}{2e}\right]^{1/2} ,\quad T\ll\delta \tag{44}$$

that is for small value of current $\langle\dot{\phi}\rangle\sim I^{1/2}$.

In the region of high temperature $(T\gg\delta)$ the function $n(k)$ satisfies the equation

$$n(k)=\frac{c}{2}+F(k),\quad x_1=\frac{2eT}{\pi I}\left[\frac{\pi}{R_s e^2}\right] ,\qquad \frac{\partial F(k)}{\partial k}=-2x_1F(k)-\frac{cx_1}{T}\varepsilon(k) . \tag{45}$$

The solution of this equation is

$$F(k)=\frac{c\delta x_1}{2\pi T}\left[\sin(\pi k)+\frac{2x_1}{\pi}\cos(\pi k)\right]\Big/\left[1-\left[\frac{2x_1}{\pi}\right]^2\right] . \tag{46}$$

Finally, from formulas (39) and (46) we find

$$\left\langle\frac{\partial\phi}{\partial t}\right\rangle=\frac{x_1\delta^2}{4T}\frac{1}{1+\left[\frac{2x_1}{\pi}\right]^2},\quad T\gg\delta . \tag{47}$$

From formula (47) it follows that temperature smears the maximum in the dependence of $\langle\dot{\phi}\rangle$ versus I. At $T\gg\delta$

$$\langle\dot{\phi}\rangle_{\max}=\pi\delta^2/16T .$$

The value of the current in this point is

$$I=\frac{4eT}{\pi^2}\left[\frac{\pi}{R_s e^2}\right] .$$

The condition (38) enables to solve Eq. (36) in the full temperature region. Omitting the intermediate formulas for the average velocity value we get

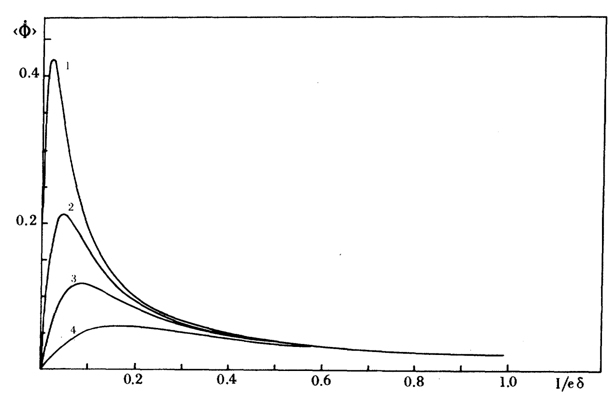

FIG. 1. Dependence of $\langle\psi\rangle$ on the current I for four temperature values: curve (1) $T=0$; curve (2) $T=\delta/2$; curve (3) $T=\delta$; curve (4) $T=2\delta$. $\alpha=0.1$.

$$\left\langle \frac{\partial \phi}{\partial t} \right\rangle = \left[\frac{\pi \delta}{2} \right] \pi x \left[\frac{1}{2} \int_0^1 dk F_0(k) \int_0^k dk_1 F_0(-k_1) \sin[\pi(k-k_1)] + \frac{1}{1+F_0(1)} \left[\int_0^1 dk \sin(\pi k) F_0(k) \right] \left[\int_0^1 dk_1 F_0(k_1) \cos(\pi k_1) \right] \right] \tag{48}$$

where

$$F_0(k) = \exp \left[-\pi x \int_0^k dk_1 \cos(\pi k_1) \coth \left[\frac{\delta \cos(\pi k_1)}{2T} \right] \right] \tag{49}$$

and the quantity x is defined by formulas (41). The dependence of $\langle \dot{\phi} \rangle$ on the current I for four temperature values is given in Fig. 1. In the dynamics of the two state system the validity of the first order expansion on the viscosity parameter η at high temperatures ($T \gg \delta$) was restricted by the condition $\eta T \ll \delta$. Apparently the same restriction exists in our case.

IV. THE INFLUENCE OF A HIGH-FREQUENCY FIELD ON THE DYNAMICS OF QUANTUM PARTICLE

We shall study the influence of a high-frequency field on the motion of a quantum particle in the region where the condition (28) is fulfilled. In this region the distances between levels is larger than their width. In the opposite limiting case strong resonant phenomena are impossible. The fulfillment of the condition (28) enables us to use the representation of density matrix in quasilocalized states. The action of the high-frequency field on the quantum particle can be described with the help of an additional term in the potential $V(\phi)$ given by

$$-\frac{I_1}{e} \phi \cos(\omega t) , \tag{50}$$

where I_1 is the amplitude of induced current. As before, we retain only the first term in the expansion of the exponent in formula (1) in I_1. As a result in the representation of quasilocalized states, we get the equation for the density matrix

$$\frac{\partial \rho_f^j}{\partial t} = \left[\frac{\pi}{R_s e^2} \right] \left[\sum_{k=-\infty}^{\infty} \left[-\mathcal{K}(\varepsilon_k) J^2(4/\tilde{z}) \rho_f^j + (-1)^{j-f} \mathcal{K}(\varepsilon(k)) J_k^2(4/\tilde{z}) \rho_{k+f}^{k+j} \right] \right] - \frac{i \pi I_1}{2e\tilde{z}} \left\{ (\rho_f^{j+1} - \rho_{f-1}^j) \exp \left[i \left[\omega - \frac{\pi I}{e} \right] t \right] + (\rho_f^{j-1} - \rho_{f+1}^j) \exp \left[-i \left[\omega - \frac{\pi I}{e} \right] t \right] \right\} . \tag{51}$$

We suppose, that the frequency ω is near the resonant value. Since only the transition matrix elements for neighboring levels are nonzero [formula (19)] the resonance can take place only for

$$\omega = \pi I / e .$$

The velocity $\langle \dot{\phi} \rangle$ is defined as before by the expression (18) and in a weak high-frequency field is equal to

$$\left\langle \frac{\partial \phi}{\partial t} \right\rangle = - \left[\frac{\pi}{R_s e^2} \right] \pi \sum_{k=-\infty}^{\infty} k \mathcal{K}(\varepsilon(k)) J_k^2(4/\tilde{z}) - \frac{\pi}{\tilde{z}} \frac{\partial}{\partial t} \sum_{k=-\infty}^{\infty} \left[\rho_{k+1}^k \exp \left[i \frac{\pi I}{e} t \right] + \rho_k^{k+1} \exp \left[-i \frac{\pi I}{e} t \right] \right] + i\pi \left[\frac{\pi I_1}{2e\tilde{z}} \right] \sum_{k=-\infty}^{\infty} \left\{ \rho_k^{k+1} \exp \left[i \left[\omega - \frac{\pi I}{e} \right] t \right] - \rho_{k+1}^k \exp \left[i \left[\omega - \frac{\pi I}{e} \right] t \right] \right\} . \tag{52}$$

From the Eq. (51) it follows that for the sum of near diagonal elements we have as before Eq. (22). It means that a high-frequency field does not lead to the resonant change of velocity $\langle \dot{\phi} \rangle$. According to Ref. 12 it must appear at ω close to $\pi I / e$ a step in the dependence of $\langle \dot{\phi} \rangle$ on I.

Note that if at $t=0$ the particle was localized in one well (for example at $\phi=0$), then its wave function is

$$\psi(t=0,\phi) = \left[\frac{m\Omega}{\pi} \right]^{1/4} \mathcal{D}_0(\phi(2m\Omega)^{1/2}) = \sum_{N=-\infty}^{\infty} (-1)^N J_N(2/\tilde{z}) \psi_N(\phi) \tag{53}$$

and for the sum of near-diagonal elements of density matrix (which are those of our interest) we get

$$\sum_n \rho^n_{n+1}(t=0) = -\sum_n J_n(2/\tilde{z})J_{n+1}(2/\tilde{z}) = 0 \ . \qquad (54)$$

The main reason why there exists a large difference in the dependence of $\langle \dot{\phi} \rangle$ on the current I in our paper and that of Ref. 12 is the following. At a small value of current transition with large change levels number N,

$$N \sim \frac{e\delta}{2\pi I} \gg 1 \qquad (55)$$

are essential. It means that essential coordinate difference is

$$\phi - \tilde{\phi} \sim (e\delta/I) \gg 2\pi \qquad (56)$$

and therefore the exact form of effective action is important.

V. CONCLUSION

We investigated the dependence of the velocity of the motion of a quantum particle as a function of the slope of the washboard potential and of temperature. To impose to the Josephson junction the restrictions on the temperature, capacitance, and normal state resistance is very hard. If we neglect the self-capacitance of the junction, and take into account only the effect of renormalization of capacitance, then the value of the band width δ is $\delta\Omega \exp[-6^{1/2}(4R_N e^2)]$. It means that the quantity $R_N e^2$ must be larger or of the order of 1. The actual parameter α of the expansion on viscosity is $\alpha = \pi/2R_s e^2$. As it is well known from the dynamics of the two-states system 11, the behavior of the system strongly changes at $\alpha = \frac{1}{2}$, which means that the parameter $R_s e^2$ must be larger than 1. As we have seen at $T > \delta$ the maximum in the dependence of $\langle \dot{\phi} \rangle$ versus current I smears and decreases.

There exist two different additional restrictions on the temperature. One is connected (as it was seen before) to the validity of the first-order perturbation theory on the viscosity $-\eta T < \delta$, the other is connected to the circumstance that we neglect interband transitions. This is possible only if $T < \Omega/\ln(64E_J/\Omega)$. In the opposite case the particle shall tunnel in the excited state. The relaxation processes in this case are stronger (on the parameter Ω/δ), and it is very unlikely to see some deviation from the quasiclassical picture in this temperature region.

*Permanent address: Institute for Theoretical Physics, Academy of Sciences of U.S.S.R., Kossigin St. 2, Moscow, U.S.S.R.

[1]A. O. Caldeira and A. I. Leggett, Phys. Rev. Lett. **46**, 211 (1981).
[2]A. Schmid, J. Low Temp. Phys. **49**, 609 (1982).
[3]V. A. Ambegaokar, V. Eckern, and G. Schon, Phys. Rev. Lett. **48**, 1745 (1982).
[4]A. I. Larkin and Yu. N. Ovchinnikov, Phys. Rev. B **28**, 6281 (1983).
[5]John H. Martinis, Michel H. Devoret, and John Clarke, Phys. Rev. Lett. **55**, 1543 (1985).
[6]Michel H. Devoret, John H. Martinis, and John Clarke, Phys. Rev. Lett. **55**, 1908 (1985).
[7]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **91**, 318 (1986).
[8]K. S. Chow, D. A. Browne, and V. A. Ambegaokar, Phys. Rev. B **37**, 1624 (1988); Peter Kopietz and Sudip Chakravarty, Harvard University report, 1988 (unpublished).
[9]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **86**, 719 (1984).
[10]M. Grabert and V. Weiss, Phys. Rev. Lett. **53**, 1787 (1984).
[11]A. I. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fischer, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
[12]K. K. Licharev and A. B. Zorin, J. Low Temp. Phys. **59**, 347 (1985).
[13]L. D. Landau and E. M. Lifshiz (MIR, Moscow, 1967).
[14]A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **63**, 315 (1986).