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Transport properties of anisotropic superconductors: Influence of arbitrary electron-impurity phase shifts

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We investigate electron-impurity scattering in a number of unconventional superconducting states with a low impurity concentration, making no assumption about the phase shift in the normal state. The scattering amplitude in the superconducting states is found not to exhibit particle-hole symmetry, and not to be invariant under time reversal. As a consequence, we find that there can be large thermoelectric effects, and that some tensor components of transport coefficients that vanish in the normal state can be finite in the superconducting state. Detailed calculations of the quasiparticle relaxation time, thermoelectric coefficients, viscosities and thermal conductivity are performed for the axial and polar p-wave states and an axial d-wave state.

I. INTRODUCTION

Transport properties in heavy-fermion superconductors have attracted much attention since measurements of them can possibly shed light on the nature of the superconducting state and scattering mechanisms. At low temperatures, the transport coefficients of these compounds do not drop exponentially as functions of temperature as predicted by the BCS theory of superconductivity, but rather are more like power laws.[1-8] In UPt_3, the normal-state transport properties are those expected for electron quasiparticles scattering from impurities, while in other heavy-electron compounds it appears that both electron-impurity and electron-electron scattering are important. In this paper we shall confine our attention to the case of electron-impurity scattering alone.

Coffey, Rice, and Ueda,[9] and Pethick and Pines[10] showed that if electron-impurity scattering is treated in the Born approximation, the quasiparticle relaxation time at low temperatures in superconducting states with nodes of the gap on the Fermi surface varies as an inverse power of the temperature T. This behavior of the relaxation time was shown to give rise to transport coefficients that are in qualitative disagreement with experiment.[4-7] Pethick and Pines then went on to show that the temperature dependence of the transport coefficients could be understood qualitatively if the electron-impurity scattering in the normal state were close to resonant, corresponding to a phase shift $\delta_N \simeq \pi/2$, and therefore could not be treated in the Born approximation. Subsequently a number of authors[11-15] have performed calculations of transport properties of anisotropic superconductors assuming the electron-impurity scattering to be close to resonant. The calculations of Ref. 15 are based on the quasiparticle Boltzmann equation, while the others use the Green's-function formalism and allow for the effects of pair breaking. Comparison of the two sets of calculations shows that the effects of pair breaking are expected to be minor for experimentally realistic values of the quasiparticle mean free path, except at the very lowest temperatures.

In this paper we extend the calculations of Ref. 15 to phase shifts other than small ones and $\pi/2$. The motivation for this is twofold. First, Ott *et al.*[16] measured the specific heat of UBe_{13} for temperatures between 65 and 180 mK. Following the work of Hirschfeld *et al.*[11] on the effects of pair breaking on the specific heat, they could fit their experimental data using a phase shift $\delta_N = 0.9\pi/2$, hence giving experimental evidence for the importance of phase shifts different from $\pi/2$. Second, qualitatively new phenomena arise[8] when the phase shift is neither small nor $\pi/2$. Monien *et al.*[17] pointed out in connection with calculations of the ultrasonic attenuation that the quasiparticle-like and quasihole-like excitations of the same energy have different relaxation times. In Refs. 18 and 19 it was shown that the particle-hole asymmetry could lead to a thermoelectric coefficient in anisotropic superconductors orders of magnitude larger than in ordinary BCS superconductors. In addition it was demonstrated that angular asymmetries of the scattering cross section can occur if the gap has a nontrivial phase variation over the Fermi surface, and that, as a consequence, some components of the transport coefficient tensor can be finite in the superconducting state even though they vanish in the normal state. A brief report of some of our results has been given previously.[20] We begin by calculating, in Sec. II, the relaxation time and exhibit explicitly its behavior for quasiparticle-like and quasihole-like excitations. In Sec. III we calculate the thermal conductivity and the thermoelectric coefficient, and in Sec. IV we calculate the viscosity, which is related to the ultrasonic attenuation coefficient in the hydrodynamic regime. In Sec. V we compare our results with the available experimental data.

II. RELAXATION TIME

In this section we calculate the relaxation time for quasiparticle-like and quasihole-like excitations due to elastic scattering by nonmagnetic impurities, allowing the normal-state scattering phase shift δ_N to be arbitrary. In equilibrium, the quasiparticle energy spectrum is given by

$$E_{\mathbf{p}\sigma}=(\xi_{\mathbf{p}}^2+\hat{\Delta}_{\mathbf{p}}\hat{\Delta}_{\mathbf{p}}^{\dagger})^{1/2}\ , \tag{1}$$

where $\xi_{\mathbf{p}}=v_F(p-p_F)$ is the energy of a quasiparticle in the normal state relative to its value at the Fermi momentum p_F (if terms of higher order in $p-p_F$ are neglected), and $\hat{\Delta}_{\mathbf{p}}$ is the gap matrix, which is a 2×2 matrix in spin space. v_F is the Fermi velocity, which we shall assume to be isotropic. In this paper we shall restrict ourselves to superconducting states for which $\hat{\Delta}_{\mathbf{p}}$ is unitary, and therefore the energy spectrum $E_{\mathbf{p}\sigma}$ is independent of the spin. At impurity concentrations n_i so low that broadening of quasiparticle states is negligible, the relaxation rate for a quasiparticle of momentum $\mathbf{p}$ and spin σ is given by

$$\frac{1}{\tau_{\mathbf{p}\sigma}}=\frac{2\pi}{\hbar}n_i\sum_{\mathbf{p}',\sigma'}|t^s_{\mathbf{p}'\sigma',\mathbf{p}\sigma}|^2\delta(E_{\mathbf{p}\sigma}-E_{\mathbf{p}'\sigma'})\ . \tag{2}$$

Here $t^s_{\mathbf{p}'\sigma',\mathbf{p}\sigma}$ is the amplitude for scattering by a single impurity of a quasiparticle from a state with momentum $\mathbf{p}$ and spin σ to a state with momentum $\mathbf{p}'$ and spin σ', which was found in Ref. 15 to be

$$\begin{aligned}t^s_{\mathbf{p}'\mathbf{p}}=&u^{\dagger}_{\mathbf{p}'}(t_{11})_{\mathbf{p}'\mathbf{p}}u_{\mathbf{p}}+u^{\dagger}_{\mathbf{p}'}(t_{12})_{\mathbf{p}'\mathbf{p}}v_{\mathbf{p}}\\&+v^{\dagger}_{\mathbf{p}'}(t_{21})_{\mathbf{p}'\mathbf{p}}u_{\mathbf{p}}+v^{\dagger}_{\mathbf{p}'}(t_{22})_{\mathbf{p}'\mathbf{p}}v_{\mathbf{p}}\ ,\end{aligned} \tag{3}$$

where the spin indices have been suppressed for brevity. The quantities t_{ij} $(i,j=1,2)$ are the components of the T matrix in particle-hole space, and $(u_{\mathbf{p}},v^{\dagger}_{-\mathbf{p}})$, and $(u^{\dagger}_{\mathbf{p}},-v_{-\mathbf{p}})$ are the eigenvectors of the single-particle propagator.[21]

We consider the three superconducting states studied in Ref. 15, namely the polar and axial p-wave states with triplet pairing, and the axial d-wave singlet pairing state which is consistent with hexagonal and cubic symmetries. In the case of triplet states the gap matrix is given by[22]

$$\hat{\Delta}_{\mathbf{p}}=i\sigma_2\boldsymbol{\sigma}\cdot\boldsymbol{\Delta}(\hat{\mathbf{p}})\ , \tag{4}$$

where, for p-wave states

$$\boldsymbol{\Delta}(\hat{\mathbf{p}})=\Delta(\hat{\mathbf{p}})\mathbf{d}\ , \tag{5}$$

with $\Delta(\hat{\mathbf{p}})$ given for the polar state by

$$\Delta(\hat{\mathbf{p}})=\Delta(T)\cos\theta\ , \tag{6}$$

and for the axial state by

$$\Delta(\hat{\mathbf{p}})=\Delta(T)(\mathbf{i}+i\mathbf{j})\cdot\hat{\mathbf{p}}=\Delta(T)e^{i\phi}\sin\theta\ , \tag{7}$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors in the 1, 2, and 3 directions in momentum space, and $\mathbf{d}$ is a fixed unit vector in spin space satisfying the unitary condition $\mathbf{d}\times\mathbf{d}^*=0$. In the case of the d-wave state we have[23]

$$\hat{\Delta}_{\mathbf{p}}=i\sigma_2\Delta(\hat{\mathbf{p}})\ , \tag{8}$$

where

$$\Delta(\hat{\mathbf{p}})=2\Delta(T)e^{i\phi}\sin\theta\cos\theta\ . \tag{9}$$

In Eqs. (6), (7), and (9), $\Delta(T)$ is the maximum value of the energy gap on the Fermi surface, and θ and ϕ give the directions of $\mathbf{p}$ in polar coordinates.

As explained in detail in Refs. 10 and 15, the T matrix for scattering of a quasiparticle against a single impurity for the superconducting states considered here is given by

$$T=k_N\tau_3[1+i\pi N(0)k_Ng(E)\tau_3]^{-1}\ , \tag{10}$$

where τ_i $(i=1,2,3)$ are Pauli matrices in particle-hole space, $N(0)=m^*p_F/(2\pi^2\hbar^3)$ is the density of quasiparticle states at the Fermi surface for a single spin, and $k_N=-\tan\delta_N/\pi N(0)$. In these calculations we assume that the scattering in the normal state occurs only in the s-wave channel. The function $g(E)$ is

$$g(E)=\frac{i}{\pi}\int\frac{d\Omega_{\hat{\mathbf{p}}}}{4\pi}\int_{-E_0}^{E_0}d\xi_{\mathbf{p}}\frac{E}{E^2-E_{\mathbf{p}}^2}\ , \tag{11}$$

where E_0 is an energy cutoff which satisfies the condition $\Delta\ll E_0\ll E_F$, where E_F is the Fermi energy.

The components t_{ij} are then given by

$$t_{12}=t_{21}=0\ , \tag{12}$$

$$t_{11}=-\frac{1}{\pi N(0)}\frac{\sin\delta_N}{\cos\delta_N-ig(E)\sin\delta_N}\ , \tag{13}$$

and

$$t_{22}=\frac{1}{\pi N(0)}\frac{\sin\delta_N}{\cos\delta_N+ig(E)\sin\delta_N}\ . \tag{14}$$

In this paper we are not interested in spin-dependent properties, and therefore for calculating transport properties it is enough to know the squared scattering amplitude summed over final spin states and averaged over initial spin states. This may be obtained from Eq. (3) and is given by

$$\begin{aligned}\overline{|t^s_{\mathbf{p}'\mathbf{p}}|^2}=&\tfrac{1}{2}\,\mathrm{tr}[|t_{11}|^2u^{\dagger}_{\mathbf{p}'}u_{\mathbf{p}'}u^{\dagger}_{\mathbf{p}}u_{\mathbf{p}}+|t_{22}|^2v^{\dagger}_{\mathbf{p}'}v_{\mathbf{p}'}v^{\dagger}_{\mathbf{p}}v_{\mathbf{p}}\\&+2\,\mathrm{Re}(t_{11}t^*_{22}v_{\mathbf{p}'}u^{\dagger}_{\mathbf{p}'}u_{\mathbf{p}}v^{\dagger}_{\mathbf{p}})]\ ,\end{aligned} \tag{15}$$

where tr refers to the trace in spin space and Re(f) stands for real part of f. Using the usual definitions of $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ and introducing the quantities

$$a=\frac{|t_{11}|^2+|t_{22}|^2}{2|t_N|^2}\ , \tag{16}$$

$$b=\frac{|t_{11}|^2-|t_{22}|^2}{2|t_N|^2}\ , \tag{17}$$

and

$$c=\frac{t_{11}t^*_{22}}{2|t_N|^2}\ , \tag{18}$$

where

$$|t_N|=\frac{\sin\delta_N}{\pi N(0)}\ , \tag{19}$$

the squared scattering amplitude can be written as

$$\overline{|t^s_{\mathbf{p}'\mathbf{p}}|^2}=\left[a\left[1+\frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]+b\left[\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}+\frac{\xi_{\mathbf{p}'}}{E_{\mathbf{p}'}}\right]+\mathrm{Re}\left[c\,\mathrm{tr}\frac{\hat{\Delta}_{\mathbf{p}}\hat{\Delta}^{\dagger}_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]\right]\frac{|t_N|^2}{2}\ . \tag{20}$$

When expressed in terms of the phase shift and the function g, the quantities a, b, and c take the forms

$$a=\frac{\cos^2\delta_N+|g(x)|^2\sin^2\delta_N}{|\cos^2\delta_N+g^2(x)\sin^2\delta_N|^2}, \tag{21}$$

$$b=-2\frac{\mathrm{Im}[g(x)]\cos\delta_N\sin\delta_N}{|\cos^2\delta_N+g^2(x)\sin^2\delta_N|^2}, \tag{22}$$

and

$$c=-\tfrac{1}{2}\{\cos^2\delta_N-|g(x)|^2\sin^2\delta_N -2i\,\mathrm{Re}[g(x)]\sin\delta_N\cos\delta_N\}^{-1}, \tag{23}$$

where $x=E/\Delta$ and $\mathrm{Im}(f)$ stands for the imaginary part of f.

The squared scattering amplitude, Eq. (20), is then given for triplet states by

$$\overline{|t^s_{\mathrm{p'p}}|^2}=\left[a\left[1+\frac{\xi_{\mathrm{p}}\xi_{\mathrm{p'}}}{E_{\mathrm{p}}E_{\mathrm{p'}}}\right]+b\left[\frac{\xi_{\mathrm{p}}}{E_{\mathrm{p}}}+\frac{\xi_{\mathrm{p'}}}{E_{\mathrm{p'}}}\right] +2\,\mathrm{Re}\left[c\frac{\mathbf{\Delta}_{\mathrm{p}}\cdot\mathbf{\Delta}^*_{\mathrm{p'}}}{E_{\mathrm{p}}E_{\mathrm{p'}}}\right]\right]\frac{|t_N|^2}{2}, \tag{24}$$

and for singlet states by

$$\overline{|t^s_{\mathrm{p'p}}|^2}=\left[a\left[1+\frac{\xi_{\mathrm{p}}\xi_{\mathrm{p'}}}{E_{\mathrm{p}}E_{\mathrm{p'}}}\right]+b\left[\frac{\xi_{\mathrm{p}}}{E_{\mathrm{p}}}+\frac{\xi_{\mathrm{p'}}}{E_{\mathrm{p'}}}\right] +2\,\mathrm{Re}\left[c\frac{\Delta_{\mathrm{p}}\Delta^*_{\mathrm{p'}}}{E_{\mathrm{p}}E_{\mathrm{p'}}}\right]\right]\frac{|t_N|^2}{2}. \tag{25}$$

The scattering amplitudes (24) and (25) do not have a definite symmetry under the operation $\xi_{\mathrm{p}}\to-\xi_{\mathrm{p}}$, and consequently particle-hole symmetry is violated, and the scattering amplitudes for quasiparticle-like excitations ($p>p_F$) and quasihole-like excitations ($p<p_F$) of the same energy are different. The second terms on the right-hand sides of Eqs. (24) and (25), which are responsible for the asymmetry about the Fermi surface, vanish for phase shifts $\delta_N\ll\pi/2$, in which case $t_{11}=-t_{22}$, and for $\delta_N=\pi/2$, since then $t_{11}=t_{22}$: in both cases $b=0$. The physical reason for the asymmetry is that the amplitude $t_{11}(E)$ for a positive energy is the amplitude for scattering a normal-state quasiparticle of energy E, while $t_{22}(E)$ is the amplitude for scattering a quasiparticle of energy $-E$, that is a quasihole. These amplitudes are generally different because the basic interaction between a quasiparticle and an impurity is the opposite of that for a quasihole and an impurity. Only to lowest order in $\tan\delta_N$ and for resonant scattering ($|\delta_N|\to\pi/2$) are $|t_{11}(E)|$ and $|t_{22}(E)|$ equal. For other values of the phase shift a virtual bound state tends to form in the quasiparticle channel for positive energy if $\tan\delta_N>0$ and in the quasihole channel for negative energy if $\tan\delta_N<0$.

From Eq. (11), the expression for the function $g(E/\Delta)$ is given for the axial p-wave state by

$$g(x)=\begin{cases}\dfrac{x}{2}\ln\left[\dfrac{1+x}{1-x}\right]-i\dfrac{\pi}{2}x, & \text{for } |x|<1,\\[2ex] \dfrac{x}{2}\ln\left[\dfrac{x+1}{x-1}\right], & \text{for } |x|>1,\end{cases} \tag{26}$$

for the polar state by

$$g(x)=\begin{cases}\dfrac{\pi}{2}|x|-ix\ln\left[\dfrac{1+(1-x^2)^{1/2}}{x}\right], & \text{for } |x|<1,\\[2ex] x\arcsin\left[\dfrac{1}{x}\right], & \text{for } |x|>1,\end{cases} \tag{27}$$

and for the d-wave state by

$$g(x)=\begin{cases}\dfrac{|x|}{2}\left[\displaystyle\int_0^{\mu_1}+\int_{\mu_2}^{1}\right]\dfrac{d\mu}{(\mu^4-\mu^2+x^2/4)^{1/2}},\\ -i\dfrac{x}{2}\displaystyle\int_{\mu_1}^{\mu_2}\dfrac{d\mu}{(\mu^2-\mu^4-x^2/4)^{1/2}}, & \text{for } |x|<1,\\ \dfrac{|x|}{2}\displaystyle\int_0^1\dfrac{d\mu}{(\mu^4-\mu^2+x^2/4)^{1/2}}, & \text{for } |x|>1.\end{cases} \tag{28}$$

In Eq. (28)

$$\mu_1=\frac{1}{\sqrt{2}}[1-(1-x^2)^{1/2}]^{1/2}, \tag{29}$$

and

$$\mu_2=\frac{1}{\sqrt{2}}[1+(1-x^2)^{1/2}]^{1/2}. \tag{30}$$

For $E>\Delta$, we have for all superconducting states, $\mathrm{Im}[g(E/\Delta)]=0$, and consequently $|t_{11}|=|t_{22}|$ and $b=0$. Therefore, the asymmetry in the scattering amplitudes exists only for intermediate states for which $E<\Delta$, and vanishes for states with $E>\Delta$. The asymmetries vanish also for $E\to0$. Thus, for anisotropic superconductors having nodes in the energy gap on the Fermi surface, the transport properties at $T\sim T_c/2$, which are dominated by quasiparticles having energies comparable to the maximum of the energy gap, should be strongly affected by this asymmetry. Also we expect physical processes in which asymmetry about the Fermi surface plays an essential role, such as the thermoelectric effect, to be greatly enhanced. The study of such an effect is deferred to Sec. III.

Another interesting feature of the scattering amplitudes (24) and (25) is the structure of the terms $\mathrm{Re}[c\mathbf{\Delta}_{\mathrm{p}}\cdot\mathbf{\Delta}^*_{\mathrm{p'}}/(E_{\mathrm{p}}E_{\mathrm{p'}})]$ and $\mathrm{Re}[c\Delta_{\mathrm{p}}\Delta^*_{\mathrm{p'}}/(E_{\mathrm{p}}E_{\mathrm{p'}})]$. Introducing $c=c_1+ic_2$, the last two terms can be written, respectively, as

$$\mathrm{Re}\left[c\frac{\mathbf{\Delta}_{\mathrm{p}}\cdot\mathbf{\Delta}^*_{\mathrm{p'}}}{E_{\mathrm{p}}E_{\mathrm{p'}}}\right]=c_1\,\mathrm{Re}\left[\frac{\mathbf{\Delta}_{\mathrm{p}}\cdot\mathbf{\Delta}^*_{\mathrm{p'}}}{E_{\mathrm{p}}E_{\mathrm{p'}}}\right]-c_2\,\mathrm{Im}\left[\frac{\mathbf{\Delta}_{\mathrm{p}}\cdot\mathbf{\Delta}^*_{\mathrm{p'}}}{E_{\mathrm{p}}E_{\mathrm{p'}}}\right], \tag{31}$$

and

$$\mathrm{Re}\left[c\frac{\Delta_{\mathbf{p}}\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]=c_1\,\mathrm{Re}\left[\frac{\Delta_{\mathbf{p}}\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]-c_2\mathrm{Im}\left[\frac{\Delta_{\mathbf{p}}\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]. \tag{32}$$

The second terms on the right-hand sides of Eqs. (31) and (32) exist only for phase shifts δ_N different from zero and $\pi/2$, since for $\delta_N \ll \pi/2$ and for $\delta_N=\pi/2$ the quantity c, Eq. (23), is real and $c_2=0$. They also exist only for superconducting states with an energy gap having a phase variation on the Fermi surface. Therefore, this term is nonzero for the axial p-wave and d-wave states, while it is zero for the polar state. For both axial p-wave and d-wave states, this term is proportional to $\sin(\phi-\phi')$, and consequently (31) and (32) do not have a definite parity under the operation $\phi-\phi'\to-(\phi-\phi')$, and the scattering amplitudes to two final quasiparticle states with momenta symmetric with respect to the plane formed by the incoming quasiparticle momentum and the energy-gap symmetry axis will be different. This absence of reflection symmetry will lead to new thermal-conductivity, thermoelectric, and viscosity coefficients. A study of such effects will be presented in Secs. III and IV.

In terms of $\overline{|t^s_{\mathbf{p}'\mathbf{p}}|^2}$, the relaxation rate is given by

$$\frac{1}{\tau_{\mathbf{p}}}=\frac{2\pi}{\hbar}n_i\sum_{\mathbf{p}'}\overline{|t^2_{\mathbf{p}'\mathbf{p}}|^2}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\ . \tag{33}$$

Using Eq. (24) or (25), we obtain the following expression for the relaxation rate for all superconducting states:

$$\frac{1}{\tau_{\mathbf{p}}}=\frac{2\pi}{\hbar}n_i|t_N|^2\left[a+b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]N_s(E_{\mathbf{p}})\ , \tag{34}$$

where $N_s(E)=\frac{1}{2}\sum_{\mathbf{p}}\delta(E-E_{\mathbf{p}})$ is the density of single-excitation states in the superconductor on a particular branch (particle-like or hole-like) and of a particular spin. In obtaining this equation we used the fact that on carrying out the sum, the contribution from the term $\xi_{\mathbf{p}}\xi_{\mathbf{p}'}/(E_{\mathbf{p}}E_{\mathbf{p}'})$ appearing in the scattering amplitudes (24) and (25) vanishes for all superconducting states, while the term coming from $\mathrm{Re}[c\boldsymbol{\Delta}_{\mathbf{p}}\cdot\boldsymbol{\Delta}^*_{\mathbf{p}'}/(E_{\mathbf{p}'}E_{\mathbf{p}})]$ in (24) vanishes because of the odd parity of $\boldsymbol{\Delta}_{\mathbf{p}}$, and the one coming from $\mathrm{Re}[c\Delta_{\mathbf{p}}\Delta^*_{\mathbf{p}'}/(E_{\mathbf{p}'}E_{\mathbf{p}})]$ in (25) vanishes when carrying out the integration over ϕ, since $\Delta_{\mathbf{p}}(\phi+\pi)=-\Delta_{\mathbf{p}}(\phi)$. Defining the average $\langle\cdots\rangle$ by

$$\langle\cdots\rangle=\frac{1}{2N(0)}\sum_{\mathbf{p}'}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\cdots\ , \tag{35}$$

the relaxation rate is then given by

$$\frac{1}{\tau_{\mathbf{p}}}=\frac{1}{\tau_N}\left[a+b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\langle 1\rangle\ , \tag{36}$$

where τ_N is the relaxation time in the normal state, which is given by

$$\frac{1}{\tau_N}=\frac{2\pi}{\hbar}n_iN(0)|t_N|^2\ . \tag{37}$$

In Eq. (36), $\xi_{\mathbf{p}}$ is given by $+(E^2_{\mathbf{p}}-|\Delta_p|^2)^{1/2}$ for quasiparticle-like excitations and by $-(E^2_{\mathbf{p}}-|\Delta_p|^2)^{1/2}$ for quasihole-like excitations. Therefore, the relaxation rates for excitations with positive and negative $\xi_{\mathbf{p}}$ are different. Particle-hole symmetry, usually taken for granted, is violated. The asymmetric term $b\xi_{\mathbf{p}}/E_{\mathbf{p}}$ disappears for very small phase shifts and for a phase shift $\delta_N=\pi/2$, since then $b=0$. It vanishes also for states with energy greater than the maximum of the energy gap and plays an important role only for intermediate quasiparticle states with energies less than the maximum of the energy gap.

For the scattering amplitude to violate particle-hole symmetry, the superconductor must have both real and imaginary parts of the density of states nonzero at the same energy. The effect can therefore not occur in a BCS superconductor with an isotropic gap in the absence of depairing processes, since the real part of the density of states vanishes for energies less than Δ, while the imaginary part of the density of states vanishes for energies greater than Δ. Particle-hole asymmetry can, however, occur if the superconductor is anisotropic, or if pair-breaking mechanisms exist. Consequences of these effects for BCS superconductors will be considered elsewhere.

The relaxation time obtained from Eq. (36) is given by

$$\tau_{\mathbf{p}}=\tau_N\frac{D}{\langle 1\rangle}\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\ , \tag{38}$$

where the quantity D is defined by

$$D=\frac{1}{a^2-b^2\tilde{v}^2}\ , \tag{39}$$

and

$$\tilde{v}=\frac{(E^2_{\mathbf{p}}-|\Delta_{\mathbf{p}}|^2)^{1/2}}{E_{\mathbf{p}}}\ . \tag{40}$$

For energies greater than the maximum of the energy gap we have $b=0$ and D reduces to $D=1/a^2$. Consequently the relaxation time takes the form

$$\tau_{\mathbf{p}}=\tau_N\frac{1}{\langle 1\rangle}\frac{1}{a}\ , \tag{41}$$

where in this case a is given by

$$a=(\cos^2\delta_N+\langle 1\rangle^2\sin^2\delta_N)^{-1}\ , \tag{42}$$

so that the relaxation time, Eq. (41), becomes

$$\tau_{\mathbf{p}}=\tau_N\left[\frac{\cos^2\delta_N}{\langle 1\rangle}+\langle 1\rangle\sin^2\delta_N\right]. \tag{43}$$

For phase shifts $\delta_N\ll\pi/2$ or $\delta_N=\pi/2$, both expressions (38) and (43) reduce to the results found in Ref. 15. The average over the Fermi surface of the relaxation time given by Eq. (38) is

$$\tau^<_s=\tau_N\frac{1}{\langle 1\rangle}(a\langle D\tilde{v}\rangle+b\langle D\tilde{v}^2\rangle)\ , \tag{44}$$

for quasihole-like excitations, and

$$\tau^>_s=\tau_N\frac{1}{\langle 1\rangle}(a\langle D\tilde{v}\rangle-b\langle D\tilde{v}^2\rangle)\ , \tag{45}$$

for quasiparticle-like excitations. The angular integrals

$\langle 1 \rangle$, $\langle D\bar{v} \rangle$, and $\langle D\bar{v}^2 \rangle$ are listed in the Appendix for the three superconducting states considered.

In Fig. 1, we have plotted the relaxation times τ_s/τ_N as functions of E/Δ for phase shifts $x\pi/2$ where $x=0$, 0.2, 0.5, 0.8, and 1.0, for the axial and polar p-wave states, and for the d-wave state. Note that by $\delta_N=0$ we mean the limit $\delta_N \rightarrow 0$. For energies so small that $|g(E/\Delta)| \ll |\tan\delta_N|$, in the case of the axial p-wave state, we have for both quasiparticles and quasiholes for a phase shift different from $\pi/2$

$$\frac{\tau_s}{\tau_N} \sim \left[\frac{\Delta}{E}\right]^2 \frac{\cos^2\delta_N \cot^2\delta_N}{S(\delta_N)} \ln\left|\frac{1+S(\delta_N)}{1-S(\delta_N)}\right| ,$$

where

$$S(\delta_N)=\left[1+\frac{\cot^2\delta_N}{\pi^2}\right]^{1/2} .$$

In the case of the polar state, we have

$$\frac{\tau_s}{\tau_N} \sim \cos^2\delta_N |\cot\delta_N| \left[\frac{\Delta}{E}\right] \frac{1}{|\ln(\Delta/E)|} ,$$

and for the d-wave state we have

$$\frac{\tau_s}{\tau_N} \sim \cos^2\delta_N |\cot\delta_N| \left[\frac{\Delta}{E}\right] \frac{1}{|\ln(2\Delta/E)|} .$$

For energies close to the energy-gap maximum we have in the case of the axial p-wave state for both quasiparticles and quasiholes

$$\frac{\tau_s}{\tau_N} \sim \ln\left[\frac{\Delta}{|E-\Delta|}\right] \sin^2\delta_N .$$

For the polar state we have

$$\frac{\tau_s}{\tau_N} = \frac{2}{\pi}\left[\cos^2\delta_N + \frac{\pi^2}{4}\sin^2\delta_N\right] ,$$

and for the d-wave state we have

$$\frac{\tau_s}{\tau_N} \sim \ln\left[\frac{\Delta}{|E-\Delta|}\right] \sin^2\delta_N .$$

A general feature of the relaxation times is that for $0<\delta_N<\pi/2$ and $E<\Delta$ the relaxation time for quasiholes is always greater than the one for quasiparticles. Results for negative phase shifts are obtained by interchanging quasiparticles and quasiholes in the results for positive phase shifts.

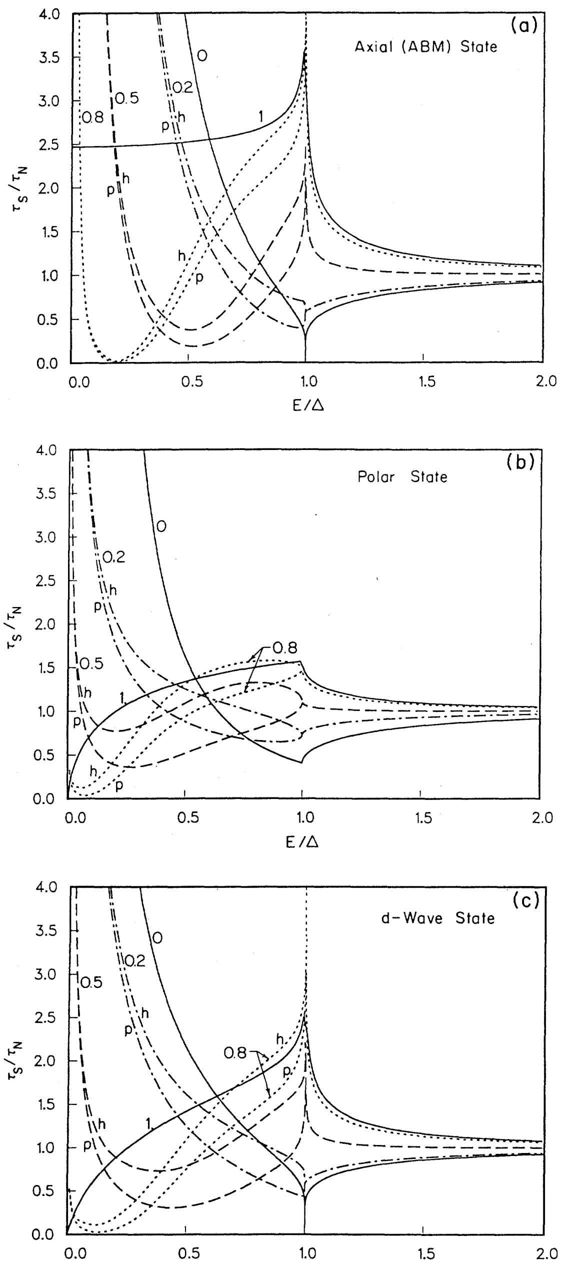

FIG. 1. Plots of the relaxation times for quasiparticle-like and quasihole-like excitations for phase shifts $\delta_N = x\pi/2$, where $x=0$, 0.2, 0.5, 0.8, and 1.0: (a) in the axial p-wave state, (b) in the polar p-wave state, and (c) in the d-wave state.

III. THERMAL CONDUCTION AND THERMOELECTRICITY

A. Thermal conduction

In this section we calculate the thermal conductivity for anisotropic superconductors using the Boltzmann equation approach in the hydrodynamic limit as developed in Refs. 24 and 15. As pointed out there, such an approach is valid provided frequencies are small compared with the gap frequency, length scales are large compared with the temperature-dependent coherence length, and the width, due to pair breaking processes, of the quasiparticle states is small compared with both the quasiparticle energies of interest and $\Delta(T)$.

We start from the linearized Boltzmann equation

$$-E_{\mathbf{p}}\mathbf{v}_{\mathbf{p}} \cdot \frac{\nabla T}{T}\left[\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}}\right] = \left[\frac{\partial n_{\mathbf{p}}}{\partial t}\right]_{\text{coll}} , \qquad (46)$$

where $\mathbf{v}_{\mathbf{p}}$ is the quasiparticle velocity given by

$$\mathbf{v}_{\mathbf{p}}=\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}v_F\hat{\mathbf{p}} ,$$

v_F is the Fermi velocity, and $n_{\mathbf{p}}^0$ is the equilibrium quasiparticle distribution function evaluated with the local equilibrium values of the energy, chemical potential, and temperature. The collision integral [the right-hand side of Eq. (46)] is given by

$$\left[\frac{\partial n_{\mathbf{p}}}{\partial t}\right]_{\text{coll}}=-\frac{2\pi}{\hbar}n_i\sum_{\mathbf{p}'}\overline{|t^s_{\mathbf{p}'\mathbf{p}}|^2}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})(n_{\mathbf{p}}-n_{\mathbf{p}'}) . \tag{47}$$

Here $E_{\mathbf{p}}$ is the quasiparticle energy including possible nonequilibrium contributions to it. In this calculation we are neglecting the counterflow contribution to the thermal conductivity, since the ratio of the conductivity due to counterflow to that due to normal fluid flow[25] is of the order of $\Delta T/E_F^2$. We linearize the quasiparticle distribution function $n_{\mathbf{p}}$ about the local equilibrium distribution function by writing

$$n_{\mathbf{p}}=n_{\mathbf{p}}^0+\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}}\Phi_{\mathbf{p}} , \tag{48}$$

where $\Phi_{\mathbf{p}}$ reflects the deviation from the local equilibrium Fermi-Dirac distribution function, $n_{\mathbf{p}}^0=n^0(E_{\mathbf{p}})$. The collision integral then takes the form

$$\left[\frac{\partial n_{\mathbf{p}}}{\partial t}\right]_{\text{coll}}=-\frac{2\pi}{\hbar}n_i\sum_{\mathbf{p}'}\overline{|t^s_{\mathbf{p}'\mathbf{p}}|^2}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\times\left[\frac{\partial n_{\mathbf{p}'}^0}{\partial E_{\mathbf{p}'}}\right](\Phi_{\mathbf{p}}-\Phi_{\mathbf{p}'}) . \tag{49}$$

Inserting this expression for the collision operator into the Boltzmann equation (46) leads to the equation

$$-E_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\cdot\frac{\nabla T}{T}=-\frac{1}{\tau_{\mathbf{p}}}\Phi_{\mathbf{p}}+\frac{2\pi}{\hbar}n_i\sum_{\mathbf{p}'}\overline{|t^s_{\mathbf{p}'\mathbf{p}}|^2}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\Phi_{\mathbf{p}'} . \tag{50}$$

Since we have linearized the transport equation, we may replace the quasiparticle energies in Eq. (50) by their global equilibrium values (1). Using the expressions given by Eqs. (24) and (25) for the scattering amplitude $\overline{|t^s_{\mathbf{p}'\mathbf{p}}|^2}$, and Eq. (36) for the relaxation time, we obtain for the p-wave states the equation

$$\langle 1\rangle\left[a+b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\Phi_{\mathbf{p}}-\frac{2\pi}{\hbar}n_i\frac{|t_N|^2}{2}\tau_N\sum_{\mathbf{p}'}\left[a\left[1+\frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]+b\left[\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}+\frac{\xi_{\mathbf{p}'}}{E_{\mathbf{p}'}}\right]+\text{Re}\left[2c\frac{\mathbf{\Delta}_{\mathbf{p}}\cdot\mathbf{\Delta}^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]\right]\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\Phi_{\mathbf{p}'}=\tau_N E_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\cdot\frac{\nabla T}{T} , \tag{51}$$

and for the d-wave state

$$\langle 1\rangle\left[a+b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\Phi_{\mathbf{p}}-\frac{2\pi}{\hbar}n_i\frac{|t_N|^2}{2}\tau_N\sum_{\mathbf{p}'}\left[a\left[1+\frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]+b\left[\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}+\frac{\xi_{\mathbf{p}'}}{E_{\mathbf{p}'}}\right]+\text{Re}\left[2c\frac{\Delta_{\mathbf{p}}\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]\right]\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\Phi_{\mathbf{p}'}=\tau_N E_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\cdot\frac{\nabla T}{T} . \tag{52}$$

The driving term in the two previous equations has odd parity, and since the collision integral preserves parity, $\Phi_{\mathbf{p}}$ will have odd parity. Consequently Eqs. (51) and (52) reduce, respectively, to

$$\langle 1\rangle\left[a+b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\Phi_{\mathbf{p}}-\frac{2\pi}{\hbar}n_i\frac{|t_N|^2}{2}\tau_N\sum_{\mathbf{p}'}2\,\text{Re}\left[c\frac{\mathbf{\Delta}_{\mathbf{p}}\cdot\mathbf{\Delta}^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\Phi_{\mathbf{p}'}=\tau_N E_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\cdot\frac{\nabla T}{T} , \tag{53}$$

and

$$\langle 1\rangle\left[a+b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\Phi_{\mathbf{p}}=\tau_N E_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\cdot\frac{\nabla T}{T} , \tag{54}$$

where we have used the fact that the d-wave state has even parity. One notes that the second term on the left-hand side of Eq. (53), which corresponds to the vertex corrections of the microscopic calculations, does not vanish here for the p-wave states as opposed to the cases of phase shifts $\delta_N \ll \pi/2$ and $\delta_N=\pi/2$ treated in Ref. 15, for which it does vanish. This is due to the fact that, because of the particle-hole asymmetry for general phase shifts, $\Phi_{\mathbf{p}}$ is not an odd function of $\xi_{\mathbf{p}}$. For the d-wave state the vertex corrections vanish for any value of the phase shift. The solution of Eq. (54) for the d-wave state is then given simply by

$$\Phi_{\mathbf{p}}=\tau_N E_{\mathbf{p}}D\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\mathbf{v}_{\mathbf{p}}\cdot\frac{\nabla T}{T} . \tag{55}$$

Since the collision integral conserves energy we may add to this solution of the transport equation an arbitrary function of the energy. Such a function will not affect the heat current, so we shall not mention it further. Using the definition (35) of the average, we can rewrite Eq. (53) as

$$\langle 1\rangle\Phi_{\mathbf{p}}-2D\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\left\langle \mathrm{Re}\left[c\frac{\Delta_{\mathbf{p}}\cdot\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]\Phi_{\mathbf{p}'}\right\rangle' =\tau_N E_{\mathbf{p}} D\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\mathbf{v}_{\mathbf{p}}\cdot\frac{\nabla T}{T} . \quad (56)$$

where the prime on $\langle\ \rangle'$ indicates that the sum is over $\mathbf{p}'$. For the polar state, this equation reduces to

$$\langle 1\rangle\Phi_{\mathbf{p}}-2Dc_1\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\frac{\Delta(\hat{\mathbf{p}})}{E_{\mathbf{p}}}\left\langle\left[\frac{\Delta(\hat{\mathbf{p}}')}{E_{\mathbf{p}'}}\right]\Phi_{\mathbf{p}'}\right\rangle' =\tau_N E_{\mathbf{p}} D\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\mathbf{v}_{\mathbf{p}}\cdot\frac{\nabla T}{T} . \quad (57)$$

Multiplying this equation by $\Delta(\hat{\mathbf{p}})/E_{\mathbf{p}}$, taking the average, using the relation

$$\left\langle D\frac{|\Delta(\hat{\mathbf{p}})|^2}{E^2_{\mathbf{p}}}\right\rangle=\langle D\rangle-\langle D\bar{v}^2\rangle , \quad (58)$$

and solving the resulting equation for $\langle\Phi\Delta(\hat{\mathbf{p}})/E_{\mathbf{p}}\rangle$ gives

$$\left\langle\Phi\frac{\Delta(\hat{\mathbf{p}})}{E_{\mathbf{p}}}\right\rangle =-\sum_i\tau_N v_F E_{\mathbf{p}} b\frac{\left\langle D\bar{v}^2\mu_i\frac{\Delta(\hat{p})}{E_{\mathbf{p}}}\right\rangle}{\langle 1\rangle-2ac_1(\langle D\rangle-\langle D\bar{v}^2\rangle)}\frac{\nabla_i T}{T} , \quad (59)$$

so that the function $\Phi_{\mathbf{p}}$ for the polar state is given by

$$\Phi_{\mathbf{p}}=\sum_i\tau_N v_F\frac{E_{\mathbf{p}}}{\langle 1\rangle}D\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\left[\mu_i\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}-2bc_1\frac{\left\langle D\bar{v}^2\mu_i\frac{\Delta(\hat{\mathbf{p}}')}{E_{\mathbf{p}'}}\right\rangle'\frac{\Delta(\hat{\mathbf{p}})}{E_{\mathbf{p}}}}{\langle 1\rangle-2ac_1(\langle D\rangle-\langle D\bar{v}^2\rangle)}\right]\frac{\nabla_i T}{T} . \quad (60)$$

The quantities μ_i, $i=1$, 2, or 3, are the direction cosines of $\mathbf{p}$. For the axial p-wave state we have

$$\Delta(\hat{\mathbf{p}})=\Delta(T)(\mu_x+i\mu_y) , \quad (61)$$

and Eq. (56) takes the form

$$\langle 1\rangle\Phi_{\mathbf{p}}-2D\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\frac{\Delta^2}{E^2_{\mathbf{p}}}[(c_1\mu_x-c_2\mu_y)\langle\Phi\mu_x\rangle+(c_2\mu_x+c_1\mu_y)\langle\Phi\mu_y\rangle]=\sum_i\tau_N E_{\mathbf{p}} v_F D\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\mu_i\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\frac{\nabla_i T}{T} . \quad (62)$$

Multiplying this equation by μ_x, or by μ_y, and taking the average, we obtain the following pair of equations for the two unknowns, $\langle\Phi\mu_x\rangle$ and $\langle\Phi\mu_y\rangle$:

$$[\langle 1\rangle-ac_1(\langle D\rangle-\langle D\bar{v}^2\rangle)]\langle\Phi\mu_x\rangle-ac_2(\langle D\rangle-\langle D\bar{v}^2\rangle)\langle\Phi\mu_y\rangle=-\sum_i\tau_N E_{\mathbf{p}} v_F b\langle D\bar{v}^2\mu_i\mu_x\rangle\frac{\nabla_i T}{T} , \quad (63)$$

and

$$ac_2(\langle D\rangle-\langle D\bar{v}^2\rangle)\langle\Phi\mu_x\rangle+[\langle 1\rangle-ac_1(\langle D\rangle-\langle D\bar{v}^2\rangle)]\langle\Phi\mu_y\rangle=-\sum_i\tau_N E_{\mathbf{p}} v_F b\langle D\bar{v}^2\mu_i\mu_y\rangle\frac{\nabla_i T}{T} . \quad (64)$$

In obtaining Eqs. (63) and (64) we used the relations

$$\langle D\mu_x\mu_y\rangle=0 , \quad (65)$$

and

$$\langle D\mu_x^2\rangle=\langle D\mu_y^2\rangle=\tfrac{1}{2}\langle D(\mu_x^2+\mu_y^2)\rangle . \quad (66)$$

The solution of Eqs. (63) and (64) are then given simply by

$$\langle\Phi\mu_x\rangle=-\sum_i\tau_N E_{\mathbf{p}} v_F bL[\langle 1\rangle\langle D\bar{v}^2\mu_i\mu_x\rangle-a(\langle D\rangle-\langle D\bar{v}^2\rangle)\langle D\bar{v}^2\mu_i(c_1\mu_x-c_2\mu_y)\rangle]\frac{\nabla_i T}{T} , \quad (67)$$

and

$$\langle \Phi\mu_y \rangle = -\sum_i \tau_N E_{\mathbf{p}} v_F b L [\langle 1 \rangle \langle D\tilde{v}^2 \mu_i \mu_y \rangle - a(\langle D \rangle - \langle D\tilde{v}^2 \rangle)\langle D\tilde{v}^2 \mu_i (c_1\mu_y + c_2\mu_x)\rangle] \frac{\nabla_i T}{T} , \tag{68}$$

where the quantity L is given by

$$L^{-1} = [\langle 1 \rangle - ac_1(\langle D \rangle - \langle D\tilde{v}^2 \rangle)]^2 + a^2 c_2^2 (\langle D \rangle - \langle D\tilde{v}^2 \rangle)^2 . \tag{69}$$

Therefore the solution of Eq. (62) takes the form

$$\Phi_{\mathbf{p}} = \sum_i \tau_N v_F E_{\mathbf{p}} \frac{D}{\langle 1 \rangle} \left[a - b \frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} \right] \left[\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} \mu_i - 2bL \frac{\Delta^2}{E_{\mathbf{p}}^2} (c_2 \beta_i^a + \gamma_i^s) \right] \frac{\nabla_i T}{T} , \tag{70}$$

where

$$\beta_i^a = \langle 1 \rangle [\langle D\tilde{v}^2 \mu_i \mu_y \rangle \mu_x - \langle D\tilde{v}^2 \mu_i \mu_x \rangle \mu_y] \tag{71}$$

and

$$\gamma_i^s = [c_1 \langle 1 \rangle - a|c|^2 (\langle D \rangle - \langle D\tilde{v}^2 \rangle)][\langle D\tilde{v}^2 \mu_i \mu_x \rangle \mu_x + \langle D\tilde{v}^2 \mu_i \mu_y \rangle \mu_y] . \tag{72}$$

Here one notes that the quantities β_i^a and γ_i^s are, respectively, antisymmetric and symmetric with respect to the exchange of the indices x and y. As will be shown below, this affects the symmetry properties of the thermal conductivity and thermoelectric coefficient tensors. The deviation $\delta n_{\mathbf{p}}$ from local equilibrium of the quasiparticle distribution function is then given for the axial p-wave state by

$$\delta n_{\mathbf{p}} = \sum_i \tau_N v_F E_{\mathbf{p}} \frac{D}{\langle 1 \rangle} \left[\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] \left[a - b \frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} \right] \left[\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} \mu_i - 2bL \frac{\Delta^2}{E_{\mathbf{p}}^2} (c_2 \beta_i^a + \gamma_i^s) \right] \frac{\nabla_i T}{T} , \tag{73}$$

for the polar state by

$$\delta n_{\mathbf{p}} = \sum_i \tau_N v_F E_{\mathbf{p}} \frac{1}{\langle 1 \rangle} \left[\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] D \left[a - b \frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} \right] \left[\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} \mu_i - 2bc_1 \frac{\Delta^2}{E_{\mathbf{p}}^2} \mu_z \frac{\langle D\tilde{v}^2 \mu_i \mu_z \rangle}{\langle 1 \rangle - 2ac_1(\langle D \rangle - \langle D\tilde{v}^2 \rangle)} \right] \frac{\nabla_i T}{T} , \tag{74}$$

and for the d-wave state by

$$\delta n_{\mathbf{p}} = \sum_i \tau_N v_F E_{\mathbf{p}} \frac{1}{\langle 1 \rangle} \left[\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] D \left[a - b \frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} \right] \frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}} \mu_i \frac{\nabla_i T}{T} . \tag{75}$$

A common feature of all these results is that $\delta n_{\mathbf{p}}$ has odd parity but does not have a definite symmetry under the operation $\xi_{\mathbf{p}} \to -\xi_{\mathbf{p}}$ for all superconducting states considered. The heat current is given by

$$J_E^i = \sum_{\mathbf{p},\sigma} E_{\mathbf{p}} (\mathbf{v}_{\mathbf{p}})_i \delta n_{\mathbf{p}} = -K_{ij} \nabla_j T , \tag{76}$$

where K_{ij} is the thermal-conductivity tensor. All superconducting states considered have uniaxial symmetry, so we calculate the thermal conductivity for heat conduction along the axis of symmetry and perpendicular to it. For very small phase shifts $\delta_N \ll \pi/2$ and for $\delta_N = \pi/2$, K_{ij} was found to be diagonal in Ref. 15 for this particular choice of geometry. Because of the reflection asymmetry previously mentioned in Sec. II, we expect some of the off-diagonal elements of K_{ij} in the plane perpendicular to the symmetry axis of the energy gap to be nonvanishing for either the axial p-wave state or the d-wave state. From Eq. (76) we see that only the odd part of $\delta n_{\mathbf{p}}$ under the operation $\xi_{\mathbf{p}} \to -\xi_{\mathbf{p}}$ contributes to the heat current. Using Eqs. (73), (74), and (75) the thermal-conductivity tensor is found to be

$$K_{ij} = \tau_N \frac{v_F^2}{T} \sum_{\mathbf{p},\sigma} E_{\mathbf{p}}^2 \left[-\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] \frac{D\tilde{v}^2}{\langle 1 \rangle} \left[a\mu_i \mu_j + 2b^2 L \frac{\Delta^2}{E_{\mathbf{p}}^2} \mu_i (c_2 \beta_j^a + \gamma_j^s) \right] , \tag{77}$$

for the axial p-wave state, and

$$K_{ij} = \tau_N \frac{v_F^2}{T} \sum_{\mathbf{p},\sigma} E_{\mathbf{p}}^2 \left[-\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] \frac{D\tilde{v}^2}{\langle 1 \rangle} \left[a\mu_i \mu_j + 2b^2 c_1 \frac{\Delta^2}{E_{\mathbf{p}}^2} \frac{\mu_z \mu_j \langle D\tilde{v}^2 \mu_i \mu_z \rangle}{\langle 1 \rangle - 2ac_1(\langle D \rangle - \langle D\tilde{v}^2 \rangle)} \right] . \tag{78}$$

for the polar p-wave state. For the d-wave state we obtain

$$K_{ij} = \tau_N \frac{v_F^2}{T} \sum_{\mathbf{p},\sigma} E_{\mathbf{p}}^2 \left[-\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] \frac{D\tilde{v}^2}{\langle 1 \rangle} a\mu_i \mu_j . \tag{79}$$

On carrying out the angular integrations, we see that all off-diagonal components of the thermal-conductivity tensor K_{ij}

vanish for the polar and d-wave states, but for the axial p-wave state the component K_{xy} and K_{yx} $(=-K_{xy})$ do not vanish. This means that a temperature gradient in the plane perpendicular to the symmetry axis of the energy gap in the superconducting state will generate a heat current which has a component in the plane but perpendicular to the temperature gradient. This is a new effect which can exist only for a superconducting state with an odd-parity energy gap having a structure similar to that of the axial p-wave state considered here, that is an energy gap with a phase varying on the Fermi surface, and when the normal state scattering phase shift is neither equal to $\pi/2$ nor very small.

The thermal-conductivity tensor can be written for the polar state as

$$K_{ij}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_i^2\rangle}{\langle 1\rangle}+2\delta_{iz}\int_0^\Delta dE\frac{\Delta^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b^2c_1}{\langle 1\rangle}\frac{\langle D\bar{v}^2\mu_z^2\rangle^2}{\langle 1\rangle-2ac_1(\langle D\rangle-\langle D\bar{v}^2\rangle)} \tag{80}$$

for the axial state as

$$\begin{aligned}K_{ij}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\Bigg[&\delta_{ij}\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_i^2\rangle}{\langle 1\rangle}\\&-2\int_0^\Delta dE\frac{\Delta^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]b^2Lc_2(\langle D\bar{v}^2\mu_i\mu_y\rangle\langle D\bar{v}^2\mu_j\mu_x\rangle-\langle D\bar{v}^2\mu_i\mu_x\rangle\langle D\bar{v}^2\mu_j\mu_y\rangle)\\&+2\int_0^\Delta dE\frac{\Delta^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b^2}{\langle 1\rangle}L[c_1\langle 1\rangle-a|c|^2(\langle D\rangle-\langle D\bar{v}^2\rangle)]\\&\times[\langle D\bar{v}^2\mu_i\mu_x\rangle\langle D\bar{v}^2\mu_j\mu_x\rangle+\langle D\bar{v}^2\mu_i\mu_y\rangle\langle D\bar{v}^2\mu_j\mu_y\rangle]\Bigg],\end{aligned} \tag{81}$$

and for the d-wave state as

$$K_{ij}=\delta_{ij}K_{ii}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\delta_{ij}\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_i^2\rangle}{\langle 1\rangle}, \tag{82}$$

where $K_N(T_c)=2\pi^2N(0)\tau_N v_F^2T_c/9$ is the normal state thermal conductivity evaluated at the transition temperature T_c. The nonvanishing components of the thermal-conductivity tensor are given for the polar state by

$$K_{zz}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\left[\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_z^2\rangle}{\langle 1\rangle}+2\int_0^\Delta dE\frac{\Delta^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b^2c_1}{\langle 1\rangle}\frac{\langle D\bar{v}^2\mu_z^2\rangle^2}{\langle 1\rangle-2ac_1(\langle D\rangle-\langle D\bar{v}^2\rangle)}\right], \tag{83}$$

$$K_{xx}=K_{yy}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_x^2\rangle}{\langle 1\rangle}, \tag{84}$$

for the axial state by

$$K_{zz}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_z^2\rangle}{\langle 1\rangle}, \tag{85}$$

$$\begin{aligned}K_{xx}=K_{yy}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\Bigg[&\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_x^2\rangle}{\langle 1\rangle}\\&+2\int_0^\Delta dE\frac{\Delta^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b^2L}{\langle 1\rangle}[c_1\langle 1\rangle-a|c|^2(\langle D\rangle-\langle D\bar{v}^2\rangle)]\langle D\bar{v}^2\mu_x^2\rangle^2\Bigg],\end{aligned} \tag{86}$$

$$K_{xy}=-K_{yx}=\frac{36}{\pi^2}K_N(T_c)\frac{T}{T_c}\int_0^\Delta dE\frac{\Delta^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]b^2Lc_2\langle D\bar{v}^2\mu_x^2\rangle^2, \tag{87}$$

and for the d-wave state by

$$K_{zz}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_z^2\rangle}{\langle 1\rangle}, \tag{88}$$

$$K_{xx}=K_{yy}=\frac{18}{\pi^2}K_N(T_c)\frac{T}{T_c}\int_0^\infty dE\frac{E^2}{T^2}\left[-\frac{\partial n^0}{\partial E}\right]a\frac{\langle D\bar{v}^2\mu_x^2\rangle}{\langle 1\rangle}. \tag{89}$$

In the case of the p-wave states it is the "in scattering" terms in the Boltzmann equation, corresponding to vertex corrections in the Green's-function calculations, which give rise to the second and third terms on the right-hand side of Eq. (81), and the second term on the right-hand side of Eq. (80). At low temperatures they give substantial contributions to the diagonal elements of the thermal-conductivity tensor, but near the transition temperature T_c they become very small. The off-diagonal elements $K_{xy}=-K_{yx}$ in the case of the axial p-wave state, Eq. (87), are completely due to the vertex corrections at any temperature. The angular averages are given by

$$\langle D\tilde{v}^2\mu_i^2\rangle = \begin{cases} \dfrac{1}{b^2}\langle G\tilde{v}^2\mu_i^2\rangle, & \text{for } E<\Delta , \\ \dfrac{1}{a^2}\langle \tilde{v}^2\mu_i^2\rangle, & \text{for } E>\Delta , \end{cases} \tag{90}$$

where

$$G=\frac{1}{r^2-\tilde{v}^2} , \tag{91}$$

and $r=a/b$. The averages $\langle G\tilde{v}^2\mu_i^2\rangle$ and $\langle \tilde{v}^2\mu_i^2\rangle$ are given in the Appendix as functions of the variable E/Δ. In order to calculate the temperature dependence of the transport coefficients we need an expression for the magnitude of the gap as a function of temperature. We employ the form used in Ref. 15, which was based on an expression used by Wölfle and Koch:[26]

$$\Delta(T)=\Delta(0)\tanh\left\{\frac{\pi T_c}{\Delta(0)}\left[\frac{2}{3f}\left[\frac{\Delta C}{C}\right]\frac{T_c-T}{T}\right]^{1/2}\right\} . \tag{92}$$

This tends to $\Delta(0)$ for $T\rightarrow 0$, and its behavior close to T_c is such that the specific-heat jump at T_c calculated from it is equal to $\Delta C/C$. Here

$$f=\frac{1}{\Delta^2}\int\frac{d\Omega_{\hat{p}}}{4\pi}|\Delta_{\hat{p}}|^2 , \tag{93}$$

is the mean-square gap compared with its maximum value, $\Delta C/C$ is the specific-heat jump at T_c, and $\Delta(0)$ is the zero-temperature gap. f is equal to $\frac{2}{3}$ for the axial state, $\frac{1}{3}$ for the polar state, and $\frac{8}{15}$ for the d-wave state. For $\Delta(0)$ we adopted the weak-coupling values $\Delta(0)=2.02T_c$ (axial), $\Delta(0)=2.45T_c$ (polar), and $\Delta(0)=2.10T_c$ (d-wave). We adopted the value $\Delta C/C=0.86$, the "idealized" value extracted by Sulpice *et al.*[4] from their measurements for UPt_3. The thermal-conductivity tensor is diagonal in the cases of the polar and d-wave states, but has off-diagonal elements K_{xy} and K_{yx} in the case of the axial p-wave state. For these off-diagonal elements of the thermal-conductivity tensor to exist, three conditions must be fulfilled: (i) The normal-state scattering phase shift has to be different from $\pi/2$ and not very small; (ii) the energy gap must have a phase variation on the Fermi surface; and (iii) the superconducting state must have odd parity. The order of magnitude of the ratio of the off-diagonal elements to the diagonal ones is a few percent. In Fig. 2, we show $K_{zz}T_c/K_NT$, $K_{xx}T_c/K_NT$, and K_{xy}/K_{xx}, for the axial state for phase shifts $\delta_N=x\pi/2$ for $x=0.1$, 0.3, 0.5, 0.7, 0.8, 0.9, and 1.0, where $\hat{z}$ is the direction of the symmetry axis of the energy gap. In Fig. 3, we show $K_{zz}T_c/K_NT$ and $K_{xx}T_c/K_NT$, for the polar state, for the same phase shifts as above, while in Fig. 4 we show those correspond-

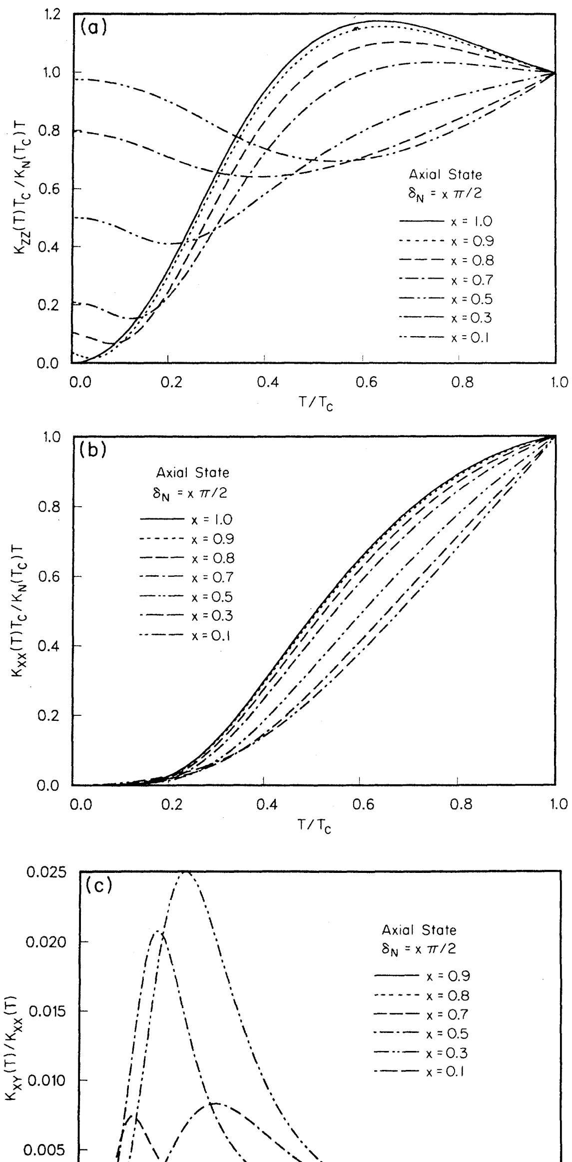

FIG. 2. Thermal conductivity divided by the temperature as a function of temperature for the axial p-wave state, and for $\delta_N=x\pi/2$, where $x=0.1$, 0.3, 0.5, 0.7, 0.8, 0.9, and 1.0: (a) the component along the symmetry axis and (b) the diagonal component in the plane perpendicular to the symmetry axis. (c) Off-diagonal component of the thermal-conductivity tensor divided by the diagonal component of the thermal conductivity in the plane perpendicular to the symmetry axis.

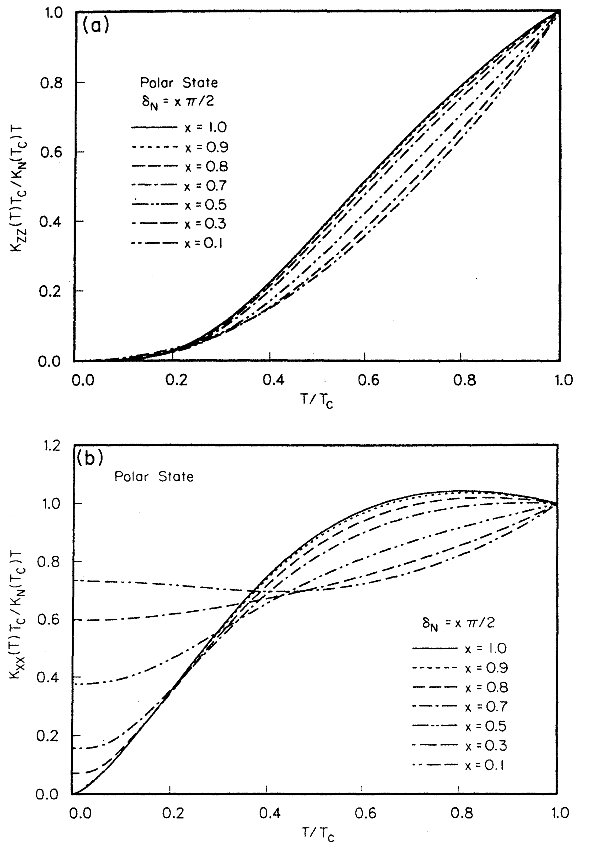

FIG. 3. Thermal conductivity divided by the temperature as a function of temperature for the polar p-wave state, and for $\delta_N = x\pi/2$, where $x = 0.1$, 0.3, 0.5, 0.7, 0.8, 0.9, and 1.0: (a) the component along the symmetry axis and (b) the component in the plane perpendicular to the symmetry axis.

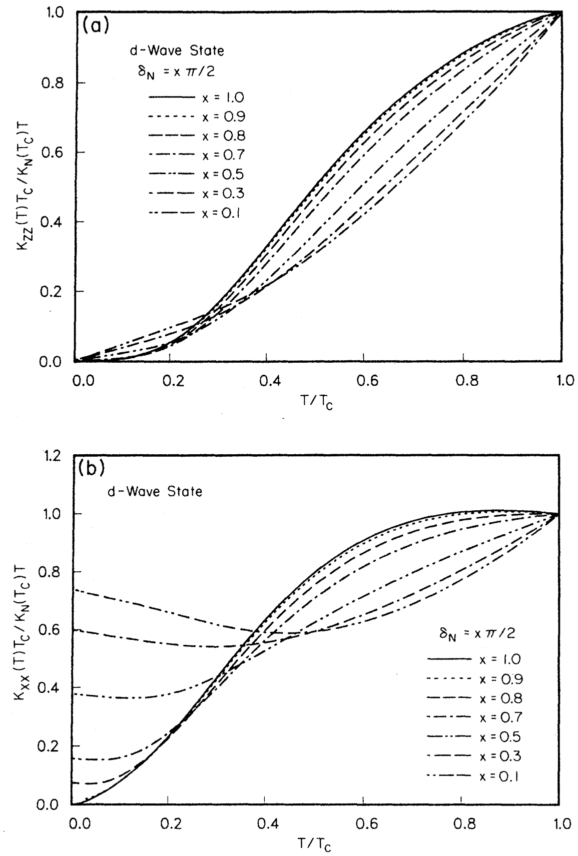

FIG. 4. Thermal conductivity divided by the temperature as a function of temperature for the d-wave state, and for $\delta_N = x\pi/2$, where $x = 0.1$, 0.3, 0.5, 0.7, 0.8, 0.9, and 1.0: (a) the component along the symmetry axis and (b) the component in the plane perpendicular to the symmetry axis.

ing to the d-wave state.

For all superconducting states, and for all phase shifts the thermal-conductivity tensor shows a strong anisotropy as a function of temperature, since its components corresponding to a heat flow along the direction of nodes (axial state) or in the plane of nodes (polar and d-wave states) is enhanced compared to the ones corresponding to the orthogonal direction. A common feature of the diagonal components of the thermal-conductivity tensor for all superconducting states is that the results for a phase shift $\delta_N = 0.9\pi/2$ or greater are almost indistinguishable from those obtained in the unitarity limit except at low temperatures. At low temperatures, for all superconducting states, the diagonal components of the thermal-conductivity tensor (relative to the normal-state conductivity at T_c) increase as the phase shift decreases, reaching eventually the Born-approximation result for very small phase shifts, whereas at intermediate temperatures, they decrease with decreasing phase shift. In the case of the axial state, the off-diagonal components K_{xy} and K_{yx} go to zero when $T \rightarrow 0$ or $T \rightarrow T_c$, reaching a maximum in between. This maximum increases up to a certain value with decreasing phase shift, then decreases with phase shift, in accordance with the fact that the thermal-conductivity tensor is diagonal in the unitarity limit or for very small phase shifts (see Ref. 15). For $T \rightarrow 0$ the results for all states and all components tend to $\cos^2\delta_N$ times the corresponding result for small phase shifts.

B. Thermoelectric effect

The existence of thermoelectric effects in superconductors was first predicted by Ginzburg as long ago as in 1944,[27] but it is only within the past 15 years or so that techniques have been developed that have made it possible to measure the thermoelectric coefficient experimentally. The basic effect is a flow of normal current $\mathbf{J}^n$ in response to an applied temperature gradient, which is expressed phenomenologically by the equation

$$J_i^n = -L_{ij} \nabla_j T \ , \tag{94}$$

where L_{ij} is the thermoelectric coefficient tensor. The thermoelectric coefficient in a superconductor cannot be measured in the same way as in normal metals because any thermoelectric potential differences developed in the superconductor are shorted out by motion of the superfluid component. Nevertheless, the effect is predicted to give rise to changes in the magnetic flux in a ring made up of two superconductors when a temperature difference is applied between the two junctions,[28–31] and to charge imbalance voltages in the vicinity of the boundary of a superconductor when a temperature gradient exists perpendicular to the surface of the superconductor.[32] Other possible manifestations of the thermoelectric coefficient have been proposed,[32] but experiments have been performed mainly using these two methods. It is difficult to interpret results of the first class of experiment because the measured flux changes have neither the temperature dependence nor the magnitude predicted theoretically, while the second class of experiment gives results for Al which are in accord with theoretical expectations.

Microscopically the normal current is given by

$$\mathbf{J}_n = e \sum_{\mathbf{p},\sigma} \frac{\mathbf{p}}{m^*} \delta n_{\mathbf{p}} , \tag{95}$$

where e is the electronic charge and m^* is the quasiparticle effective mass. The thermoelectric current is a consequence of asymmetries about the Fermi surface. Usually in both normal and superconducting metals these occur on energy scales comparable to the Fermi energy, and the thermoelectric coefficient is of order $(\sigma_N/e)(T/T_F)$ where $\sigma_N = 2e^2 N(0) v_F^2 \tau_N/3$ is the electrical conductivity in the normal state and T_F the Fermi temperature. We have seen in Sec. II that in anisotropic superconductors the asymmetries of the relaxation time occur on energy scales of order the gap energy rather than the Fermi energy, and therefore in calculating the leading terms in T_c/T_F one may neglect the asymmetries of p and m^* about the Fermi surface. The response of the quasiparticle distribution to a temperature gradient is given by Eqs. (73)–(75), and therefore from Eqs. (94) and (95) we find for the axial p-wave state

$$L_{ij} = \frac{3\sigma_N}{eN(0)} \sum_{\mathbf{p},\sigma} \frac{E_{\mathbf{p}}}{T} \left[-\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] \frac{D}{\langle 1 \rangle} b \left[-\bar{v}^2 \mu_i \mu_j - 2aL \frac{\Delta^2}{E_{\mathbf{p}}^2} \mu_i (c_2 \beta_j^a + \gamma_j^s) \right] , \tag{96}$$

for the polar p-wave state

$$L_{ij} = \frac{3\sigma_N}{eN(0)} \sum_{\mathbf{p},\sigma} \frac{E_{\mathbf{p}}}{T} \left[-\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] \frac{D}{\langle 1 \rangle} b \left[-\bar{v}^2 \mu_i \mu_j - 2ac_1 \frac{\Delta^2}{E_{\mathbf{p}}^2} \mu_i \mu_z \frac{\langle D\bar{v}^2 \mu_z \mu_j \rangle}{\langle 1 \rangle - 2ac_1(\langle D \rangle - \langle D\bar{v}^2 \rangle)} \right] , \tag{97}$$

and for the d-wave state

$$L_{ij} = -\frac{3\sigma_N}{eN(0)} \sum_{\mathbf{p},\sigma} \frac{E_{\mathbf{p}}}{T} \left[-\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}} \right] \frac{D}{\langle 1 \rangle} b \bar{v}^2 \mu_i \mu_j . \tag{98}$$

Throughout this paper we use units in which Boltzmann's constant is unity. Since the quantity b, which is a function of energy only, vanishes for energies greater than the maximum of the energy gap, only intermediate states with energies less than the maximum of the energy gap contribute to the coefficients L_{ij}. Consequently for all superconducting states considered we have $L_{ij}(T_c)=0$. This is not in contradiction with the well-known fact that thermoelectric effects exist in the normal state, but instead shows that to the zeroth order in T/T_F, to which we are working, the value of the normal state thermoelectric coefficient at $T=T_c$, is negligible. In usual superconductors, the thermoelectric coefficient is equal to the normal state one at the transition temperature, and is of the order of T/T_F. Therefore, there is an enhancement of the thermoelectric coefficient as calculated here compared to the one for conventional BCS superconductors by a factor $\sim T_F/T_c$.

The expressions for the thermoelectric coefficient tensor can be rewritten as

$$\begin{aligned} L_{ij} = \frac{6\sigma_N}{e} \int_0^\Delta dE \frac{E}{T} \left[-\frac{\partial n^0}{\partial E} \right] \frac{b}{\langle 1 \rangle} \Bigg[& -\langle D\bar{v}^2 \mu_i \mu_j \rangle + 2aL \frac{\Delta^2}{E^2} c_2 \langle 1 \rangle (\langle D\bar{v}^2 \mu_i \mu_y \rangle \langle D\mu_j \mu_x \rangle - \langle D\bar{v}^2 \mu_i \mu_x \rangle \langle D\mu_j \mu_y \rangle) \\ & -2aL \frac{\Delta^2}{E^2} [c_1 \langle 1 \rangle - a|c|^2 (\langle D \rangle - \langle D\bar{v}^2 \rangle)] \\ & \times (\langle D\bar{v}^2 \mu_i \mu_x \rangle \langle D\mu_j \mu_x \rangle + \langle D\bar{v}^2 \mu_i \mu_y \rangle \langle D\mu_j \mu_y \rangle) \Bigg] \quad \text{(axial state)} , \end{aligned} \tag{99}$$

$$L_{ij} = \frac{6\sigma_N}{e} \int_0^\Delta dE \frac{E}{T} \left[-\frac{\partial n^0}{\partial E} \right] \frac{b}{\langle 1 \rangle} \left[-\langle D\bar{v}^2 \mu_i \mu_j \rangle - 2ac_1 \frac{\Delta^2}{E^2} \frac{\langle D\mu_i \mu_z \rangle \langle D\bar{v}^2 \mu_j \mu_z \rangle}{\langle 1 \rangle - 2ac_1(\langle D \rangle - \langle D\bar{v}^2 \rangle)} \right] \quad \text{(polar state)} , \tag{100}$$

$$L_{ij} = -\frac{6\sigma_N}{e} \int_0^\Delta dE \frac{E}{T} \left[-\frac{\partial n^0}{\partial E} \right] \frac{b}{\langle 1 \rangle} \langle D\bar{v}^2 \mu_i \mu_j \rangle \quad (d\text{-wave state}) . \tag{101}$$

The nonvanishing components are explicitly given for the axial state by

$$L_{zz}=-\frac{6\sigma_N}{e}\int_0^\Delta dE\frac{E}{T}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b}{\langle 1\rangle}\langle D\bar{v}^2\mu_z^2\rangle , \tag{102}$$

$$L_{xx}=L_{yy}=-\frac{6\sigma_N}{e}\int_0^\Delta dE\frac{E}{T}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b}{\langle 1\rangle}\langle D\bar{v}^2\mu_x^2\rangle\{1+aL[c_1\langle 1\rangle-a|c|^2(\langle D\rangle-\langle D\bar{v}^2\rangle)](\langle D\rangle-\langle D\bar{v}^2\rangle)\} , \tag{103}$$

$$L_{xy}=-L_{yx}=-\frac{6\sigma_N}{e}\int_0^\Delta dE\frac{E}{T}\left[-\frac{\partial n^0}{\partial E}\right]abLc_2\langle D\bar{v}^2\mu_x^2\rangle(\langle D\rangle-\langle D\bar{v}^2\rangle) , \tag{104}$$

for the polar state by

$$L_{zz}=-\frac{6\sigma_N}{e}\int_0^\Delta dE\frac{E}{T}\left[-\frac{\partial n^0}{\partial E}\right]b\frac{\langle D\bar{v}^2\mu_z^2\rangle}{\langle 1\rangle-2ac_1(\langle D\rangle-\langle D\bar{v}^2\rangle)} , \tag{105}$$

$$L_{xx}=L_{yy}=-\frac{6\sigma_N}{e}\int_0^\Delta dE\frac{E}{T}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b}{\langle 1\rangle}\langle D\bar{v}^2\mu_x^2\rangle , \tag{106}$$

and for the d-wave state by

$$L_{zz}=-\frac{6\sigma_N}{e}\int_0^\Delta dE\frac{E}{T}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b}{\langle 1\rangle}\langle D\bar{v}^2\mu_z^2\rangle , \tag{107}$$

$$L_{xx}=L_{yy}=-\frac{6\sigma_N}{e}\int_0^\Delta dE\frac{E}{T}\left[-\frac{\partial n^0}{\partial E}\right]\frac{b}{\langle 1\rangle}\langle D\bar{v}^2\mu_x^2\rangle . \tag{108}$$

One remarks that L_{ij} is diagonal in the cases of polar and d-wave states, but has off-diagonal elements L_{xy} and L_{yx} in the case of the axial p-wave state. The off-diagonal elements of L_{ij}, Eq. (104), are completely due to the vertex correction contributions. As in the case of the thermal-conductivity tensor K_{ij}, the tensor L_{ij} has off-diagonal elements only when the phase shift is neither very small nor equal to $\pi/2$, the energy gap has a phase variation on the Fermi surface, and the superconducting state has an odd parity.

It is quite remarkable that the off-diagonal term L_{xy} for the axial p-wave state is of the same order of magnitude as L_{xx}. This is a consequence of the fact that all contributions to the components of the thermoelectric coefficient tensor come from quasiparticles with energy less than Δ, while in the case of thermal conductivity, the off-diagonal components again come from quasiparticles with energy less than Δ, while the diagonal components have contributions from quasiparticles of all energies.

In Fig. 5, we show the coefficients $L_{zz}/(\sigma_N/e)$, $L_{xx}/(\sigma_N/e)$, and $L_{xy}/(\sigma_N/e)$ as functions of T/T_c for phase shifts $\delta_N=x\pi/2$, where $x=0.1$, 0.3, 0.5, 0.7, 0.8, and 0.9, for the axial p-wave state. In Fig. 6, we show $L_{zz}/(\sigma_N/e)$, and $L_{xx}/(\sigma_N/e)$ for the polar state for the same phase shifts as above, and in Fig. 7, we show $L_{zz}/(\sigma_N/e)$, and $L_{xx}/(\sigma_N/e)$ for the d-wave state. A common feature of all results is that L_{ij} goes to zero when $T\to 0$, and $T\to T_c$, reaching a maximum in between. This maximum increases up to a certain value as the phase shift decreases from $\pi/2$, and then decreases with further decrease in the phase shift, for all components and all superconducting states. This is in accordance with the fact that the thermoelectric coefficients vanish to zeroth order in T/T_F for phase shifts $\delta_N\ll\pi/2$ and $\delta_N=\pi/2$.

The striking feature of the predictions of thermoelectric coefficients are that they are typically T_F/T_c times larger than usual estimates. This result is a consequence of the quasiparticle relaxation time being energy-dependent on an energy scale of order Δ. We remark in passing that Kon[33] has earlier predicted an enhancement of the thermoelectric coefficient in an isotropic BCS superconductor with resonant scatterers. However, in the case he considered the energy scale was set by the resonant scattering in the *normal* state, whereas in the problem we consider the energy scale is set by the energy scale for the virtual bound state in the anisotropic superconductor.

IV. ULTRASONIC ATTENUATION

In this section we calculate the attenuation of ultrasound in anisotropic superconductors. All experiments on ultrasonic attenuation in heavy fermion superconductors that have been performed to date appear to have been done in the hydrodynamic regime, since the ultrasonic attenuation shows an ω^2 dependence,[5–7] where ω is the sound frequency. Following Ref. 15, we study the response of the metal to a homogeneous strain using the Boltzmann equation approach, which is valid under the conditions stated in Sec. III of this paper. The Boltzmann equation for this situation is

$$\frac{\partial n_{\mathbf{p}}^{\mathrm{le}}}{\partial t}=\left[\frac{\partial n_{\mathbf{p}}}{\partial t}\right]_{\mathrm{coll}} , \tag{109}$$

where $n_{\mathbf{p}}^{\mathrm{le}}$ is the local equilibrium distribution function of the quasiparticles. Equation (109) can be rewritten as

$$\left[\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}}\right]\frac{\partial E_{\mathbf{p}}}{\partial u_{ij}}\dot{u}_{ij}=\left[\frac{\partial n_{\mathbf{p}}}{\partial t}\right]_{\mathrm{coll}} , \tag{110}$$

where $u_{ij}=\partial u_i/\partial x_j$ is the strain tensor, $\mathbf{u}$ being the displacement vector and $\dot{u}$ its time derivative. Neglecting the modulation of the gap by the strain,[15] Eq. (110) takes the form

$$\frac{\partial n_{\mathbf{p}}^0}{\partial E_{\mathbf{p}}}\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\mathcal{D}_{ij}\dot{u}_{ij} = \left[\frac{\partial n_{\mathbf{p}}}{\partial t}\right]_{\text{coll}} , \tag{111}$$

where $\mathcal{D}_{ij}$, given by

$$\frac{\partial E_{\mathbf{p}}}{\partial u_{ij}} = \frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\mathcal{D}_{ij} \tag{112}$$

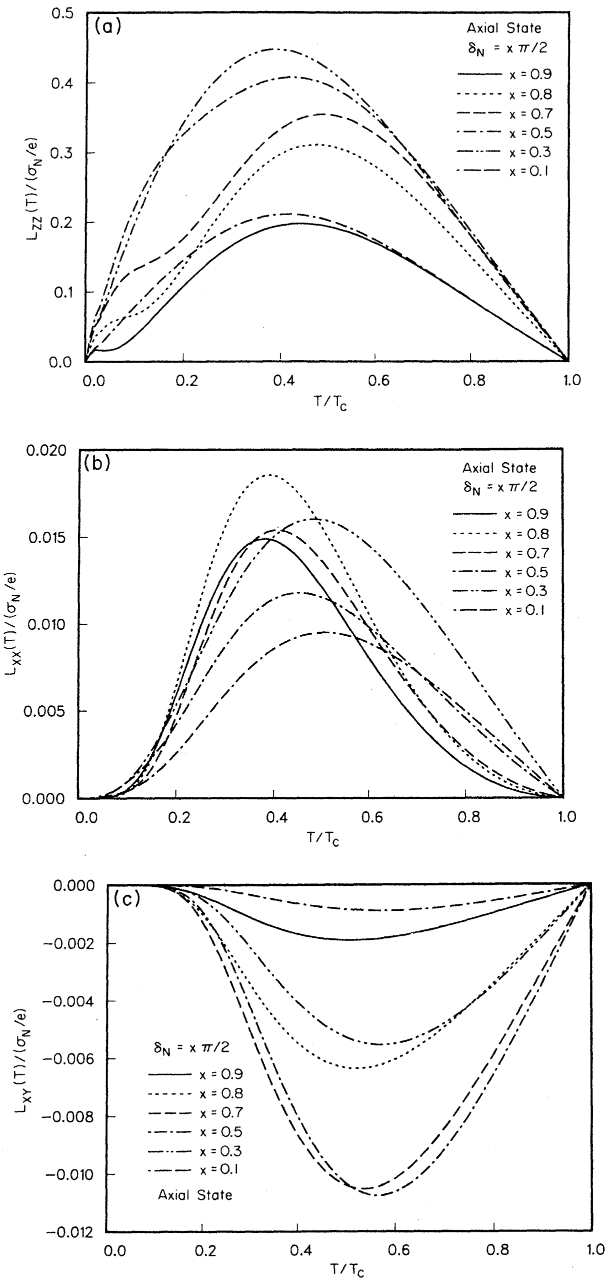

FIG. 5. Thermoelectric coefficient as a function of temperature for the axial p-wave state, and for $\delta_N = x\pi/2$, where $x = 0.1$, 0.3, 0.5, 0.7, 0.8, and 0.9: (a) the coefficient for a temperature gradient along the symmetry axis, (b) the diagonal component for a temperature gradient perpendicular to the symmetry axis, and (c) the off-diagonal component for a temperature gradient along the x axis.

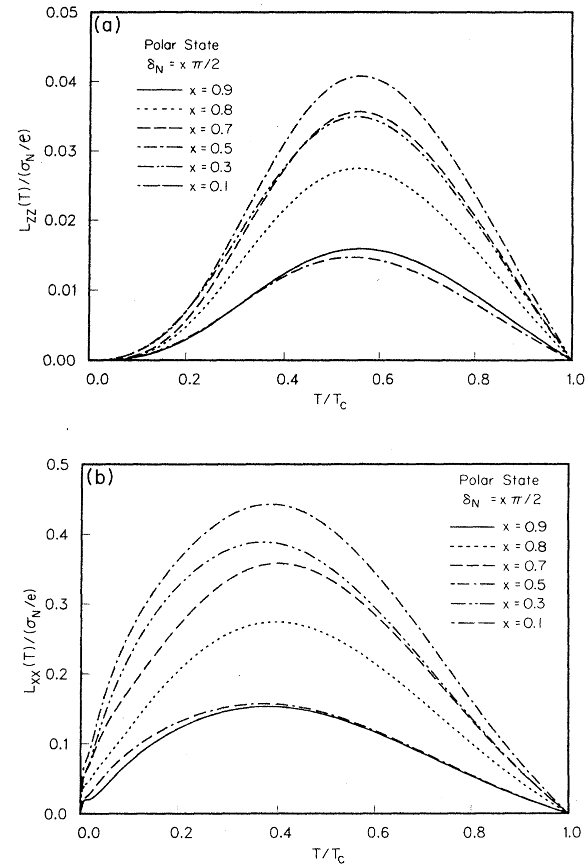

FIG. 6. Thermoelectric coefficient as a function of temperature for the polar p-wave state, and for $\delta_N = x\pi/2$, where $x = 0.1$, 0.3, 0.5, 0.7, 0.8, and 0.9: (a) the coefficient for a temperature gradient along the symmetry axis and (b) the diagonal component for a temperature gradient perpendicular to the symmetry axis.

is the usual deformation potential which gives the strain dependence of the normal-state quasiparticle energy.[34] The linearized transport equation is thus

$$\frac{1}{\tau_{\mathbf{p}}}\Phi_{\mathbf{p}} - \frac{2\pi}{\hbar} n_i \sum_{\mathbf{p}'} \overline{|t^s_{\mathbf{p}'\mathbf{p}}|^2}\,\delta(E_{\mathbf{p}} - E_{\mathbf{p}'})\Phi_{\mathbf{p}'} = -\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\mathcal{D}_{ij}\dot{u}_{ij} . \tag{113}$$

In our calculation we have taken the deformation potential $\mathcal{D}_{ij}$ to have the same angular dependence as for a normal Fermi liquid and write $\mathcal{D}_{ij} = d_{ij}\lambda_{ij}$, where d_{ij} is a quantity that is independent of direction on the Fermi surface, and

$$\lambda_{ij} = \mu_i\mu_j - \tfrac{1}{3}\delta_{ij} . \tag{114}$$

Using Eqs. (24), (25), and (36), we can rewrite Eq. (113) for the axial and polar p-wave states as

$$\langle 1\rangle\left[a+b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\Phi_{\mathbf{p}}-\frac{1}{2N(0)}\sum_{\mathbf{p}'}\left[a\left[1+\frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]+b\left[\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}+\frac{\xi_{\mathbf{p}'}}{E_{\mathbf{p}'}}\right]+2\,\mathrm{Re}\left[c\frac{\mathbf{\Delta}_{\mathbf{p}}\cdot\mathbf{\Delta}^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]\right]\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\Phi_{\mathbf{p}'}=-\tau_N\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}d_{ij}\lambda_{ij}\dot{u}_{ij}\ , \qquad (115)$$

and for the d-wave state as

$$\langle 1\rangle\left[a+b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\Phi_{\mathbf{p}}-\frac{1}{2N(0)}\sum_{\mathbf{p}'}\left[a\left[1+\frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]+b\left[\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}+\frac{\xi_{\mathbf{p}'}}{E_{\mathbf{p}'}}\right]+2\,\mathrm{Re}\left[c\frac{\Delta_{\mathbf{p}}\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right]\right]\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})\Phi_{\mathbf{p}'}=-\tau_N\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}d_{ij}\lambda_{ij}\dot{u}_{ij}\ . \qquad (116)$$

The driving term in the two previous equations has even parity, and since the collision operator preserves parity, $\Phi_{\mathbf{p}}$ must have even parity. Consequently the term containing $\mathrm{Re}(c\mathbf{\Delta}_{\mathbf{p}}\cdot\mathbf{\Delta}^*_{\mathbf{p}'})/(E_{\mathbf{p}}E_{\mathbf{p}'})$ in Eq. (115) vanishes, since $\mathbf{\Delta}_{\mathbf{p}}$ has odd parity. Equation (115) can then be rearranged in the following form:

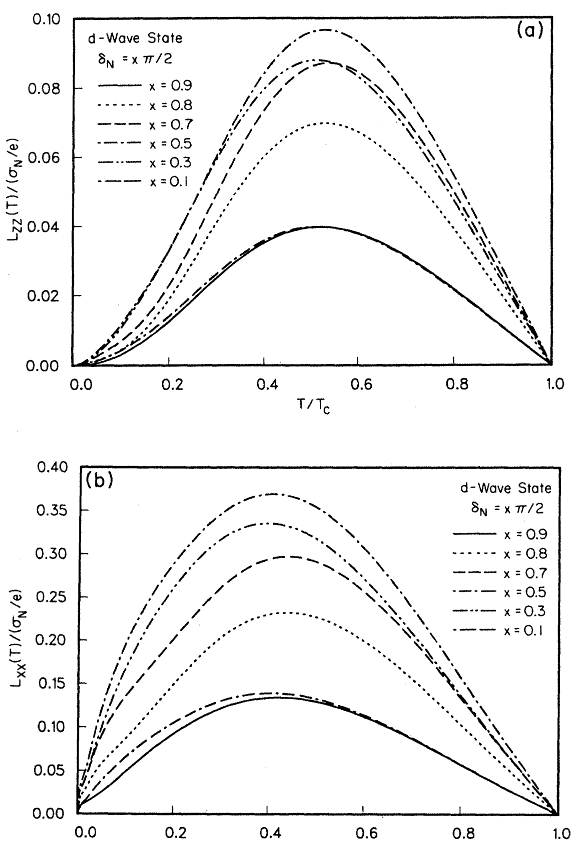

FIG. 7. Thermoelectric coefficient as a function of temperature for the d-wave state, and for $\delta_N = x\pi/2$, where $x = 0.1$, 0.3, 0.5, 0.7, 0.8, and 0.9: (a) the coefficient for a temperature gradient along the symmetry axis and (b) the diagonal component for a temperature gradient perpendicular to the symmetry axis.

$$\Phi_{\mathbf{p}}=Q(E)-\frac{\dfrac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}}{a+b\dfrac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}}\left[\frac{(b^2-a^2)}{a}\frac{\left\langle\Phi\dfrac{\xi}{E}\right\rangle}{\langle 1\rangle}+\frac{\tau_N}{\langle 1\rangle}d_{ij}\dot{u}_{ij}\lambda_{ij}\right], \qquad (117)$$

where the quantity $Q(E)$, which depends only on energy, is defined by

$$Q(E)=\frac{\langle\Phi\rangle}{\langle 1\rangle}+\left[\frac{b}{a}\right]\frac{\left\langle\Phi\dfrac{\xi}{E}\right\rangle}{\langle 1\rangle}\ . \qquad (118)$$

Since we are considering elastic scattering only, the energy of a quasiparticle is conserved in a collision. This is reflected by the presence of the Dirac δ function in the collision operator. Consequently $Q(E)$ is an eigenfunction of the collision operator with a zero eigenvalue. Since it is independent of the sign of $\xi_{\mathbf{p}}$ it will not contribute to the stress tensor and we may neglect it. The relevant contribution to $\Phi_{\mathbf{p}}$ thus takes the form

$$\Phi_{\mathbf{p}}=-\frac{\dfrac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}}{a+b\dfrac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}}\left[\frac{(b^2-a^2)}{a}\frac{\left\langle\Phi\dfrac{\xi}{E}\right\rangle}{\langle 1\rangle}+\frac{\tau_N}{\langle 1\rangle}d_{ij}\dot{u}_{ij}\lambda_{ij}\right]. \qquad (119)$$

Multiplying this equation by $\xi_{\mathbf{p}}/E_{\mathbf{p}}$, taking the average, and solving the resulting equation for $\langle\Phi\xi/E\rangle$ gives

$$\left\langle\Phi\frac{\xi}{E}\right\rangle=-\tau_N d_{ij}\dot{u}_{ij}a\frac{\langle D\overline{v}^{\,2}\lambda_{ij}\rangle}{\langle 1\rangle-(a^2-b^2)\langle D\overline{v}^{\,2}\rangle}\ . \qquad (120)$$

Inserting this expression for $\langle\Phi\xi/E\rangle$ into Eq. (119) gives the deviation function for the polar and axial p-wave states:

$$\Phi_{\mathbf{p}}=-\tau_N d_{ij}\dot{u}_{ij}\frac{D}{\langle 1\rangle}\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\times\left[\lambda_{ij}+\frac{(a^2-b^2)\langle D\overline{v}^{\,2}\lambda_{ij}\rangle}{\langle 1\rangle-(a^2-b^2)\langle D\overline{v}^{\,2}\rangle}\right]. \qquad (121)$$

Equation (116) for the d-wave state can be rewritten as

$$\Phi_{\mathbf{p}}=Q(E)-\frac{\dfrac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}}{a+b\dfrac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}}\left[\frac{(b^2-a^2)}{a}\frac{\left\langle\Phi\dfrac{\xi}{E}\right\rangle}{\langle 1\rangle}+\frac{\tau_N}{\langle 1\rangle}d_{ij}\dot{u}_{ij}\lambda_{ij}\right]$$

$$+\frac{c}{\langle 1\rangle}\frac{\dfrac{\Delta_{\mathbf{p}}}{E_{\mathbf{p}}}}{\left[a+b\dfrac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]}\left\langle\Phi_{\mathbf{p}'}\frac{\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}'}}\right\rangle'+\frac{c^*}{\langle 1\rangle}\frac{\dfrac{\Delta^*_{\mathbf{p}}}{E_{\mathbf{p}}}}{\left[a+b\dfrac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]}\left\langle\Phi_{\mathbf{p}'}\frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}}\right\rangle' . \quad (122)$$

The quantity $Q(E)$ can again be dropped from this equation for the same reason as in the p-wave state case. Multiplying Eq. (122) by $\Delta^*_{\mathbf{p}}/E_{\mathbf{p}}$, taking the average, and solving the resulting equation for $\langle\Phi\Delta^*_{\mathbf{p}}/E_{\mathbf{p}}\rangle$, we obtain

$$\left\langle\Phi\frac{\Delta^*_{\mathbf{p}}}{E_{\mathbf{p}}}\right\rangle=-\tau_N d_{ij}\dot{u}_{ij}b\frac{\left\langle D\tilde{v}^2\dfrac{\Delta^*_{\mathbf{p}}}{E_{\mathbf{p}}}\lambda_{ij}\right\rangle}{\langle 1\rangle-ac(\langle D\rangle-\langle D\tilde{v}^2\rangle)} . \quad (123)$$

Since Φ is real it follows that

$$\left\langle\Phi\frac{\Delta_{\mathbf{p}}}{E_{\mathbf{p}}}\right\rangle=\left[\left\langle\Phi\frac{\Delta^*_{\mathbf{p}}}{E_{\mathbf{p}}}\right\rangle\right]^* . \quad (124)$$

The quantity $\langle\Phi\xi/E\rangle$ is again given by Eq. (120). Using these results with Eq. (122), the deviation function for the d-wave state takes the form

$$\Phi_{\mathbf{p}}=-\tau_N d_{ij}\dot{u}_{ij}\frac{D}{\langle 1\rangle}\left[a-b\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\left\{\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\left[\lambda_{ij}+\frac{(a^2-b^2)\langle D\tilde{v}^2\lambda_{ij}\rangle}{\langle 1\rangle-(a^2-b^2)\langle D\tilde{v}^2\rangle}\right]+b\,\mathrm{Re}\left[c\left[\frac{\Delta_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\frac{\left\langle D\tilde{v}^2\dfrac{\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}'}}\lambda_{ij}\right\rangle'}{\langle 1\rangle-ac(\langle D\rangle-\langle D\tilde{v}^2\rangle)}\right]\right\} . \quad (125)$$

The stress produced by a superconducting quasiparticle is minus the derivative of the quasiparticle energy with respect to the corresponding strain, and it is therefore equal to the negative of the deformation potential in the superconductor, Eq. (112). The nonequilibrium contribution to the stress is then given by

$$\delta\pi_{ij}=-\sum_{\mathbf{p},\sigma}\frac{\xi_{\mathbf{p}}}{E_{\mathbf{p}}}\mathcal{D}_{ij}\left[\frac{\partial n^0_{\mathbf{p}}}{\partial E_{\mathbf{p}}}\right]\Phi_{\mathbf{p}} . \quad (126)$$

The viscosity tensor $\eta_{ij,kl}$ for an anisotropic superconductor is defined by

$$\delta\pi_{ij}=-\eta_{ij,kl}\dot{u}_{kl} . \quad (127)$$

Using Eqs. (121), (125), (126), and (127), and noting that only the part of $\Phi_{\mathbf{p}}$ odd in $\xi_{\mathbf{p}}$ contributes to the sum in Eq. (126), we obtain the viscosity tensor

$$\eta_{ij,kl}=2\tau_N d_{ij}d_{kl}\sum_{\mathbf{p}}\left[-\frac{\partial n^0_{\mathbf{p}}}{\partial E_{\mathbf{p}}}\right]\frac{D}{\langle 1\rangle}a\tilde{v}^2\left[\lambda_{ij}\lambda_{kl}+\lambda_{ij}\frac{(a^2-b^2)\langle D\tilde{v}^2\lambda_{kl}\rangle}{\langle 1\rangle-(a^2-b^2)\langle D\tilde{v}^2\rangle}\right] , \quad (128)$$

for the p-wave states, and

$$\eta_{ij,kl}=2\tau_N d_{ij}d_{kl}\sum_{\mathbf{p}}\left[-\frac{\partial n^0_{\mathbf{p}}}{\partial E_{\mathbf{p}}}\right]\frac{1}{\langle 1\rangle}aD\tilde{v}^2\left\{\lambda_{ij}\lambda_{kl}+\lambda_{ij}\frac{(a^2-b^2)\langle D\tilde{v}^2\lambda_{kl}\rangle}{\langle 1\rangle-(a^2-b^2)\langle D\tilde{v}^2\rangle}+\lambda_{ij}b^2\,\mathrm{Re}\left[c\left[\frac{\Delta_{\mathbf{p}}}{E_{\mathbf{p}}}\right]\frac{\left\langle D\tilde{v}^2\dfrac{\Delta^*_{\mathbf{p}'}}{E_{\mathbf{p}'}}\lambda_{kl}\right\rangle'}{\langle 1\rangle-ac(\langle D\rangle-\langle D\tilde{v}^2\rangle)}\right]\right\} . \quad (129)$$

for the d-wave state.

The viscous contribution to the attenuation of a sound wave with wave vector $\mathbf{q}$ and polarization $\boldsymbol{\epsilon}$ is given by

$$\alpha(\mathbf{q},\epsilon)=\frac{q^2}{\rho c_s}\eta_{ij,kl}\hat{\epsilon}_i\hat{q}_j\hat{\epsilon}_k\hat{q}_l\ , \tag{130}$$

where c_s is the sound velocity of the mode and ρ is the mass density. In writing Eq. (130), we have neglected the effects of gap relaxation on the attenuation of longitudinal sound, as well as the thermal expansion contribution.[35] The former has been considered by a number of authors,[36–39] and the latter is usually small in superconductors.

As an application of the above results, we consider the heavy fermion compound UPt_3. We shall assume that the symmetry axis of the superconducting state is parallel to the hexagonal axis of the crystal, an assumption consistent with the measured anisotropies in the attenuation of transverse sound[7] and of H_{c_2}.[40] If we assume that the anisotropy axis is along the c axis of the crystal, we find five independent components of the viscosity tensor, which may be taken to be $\eta_{zz,zz}$, $\eta_{xx,xx}$, $\eta_{xz,xz}$, $\eta_{xy,xy}$, and $\eta_{xz,yz}$. All other components of the viscosity tensor can be determined from these by symmetry. These coefficients determine the attenuation of longitudinal and transverse sound waves propagating along the symmetry axis of the crystal or in the basal plane. The component $\eta_{xz,yz}$ is nonzero only if the phase shift is neither very small nor equal to $\pi/2$, the superconducting state has even parity, and the energy gap has a nontrivial phase variation on the Fermi surface.

Since the deformation potential coefficients d_{ij} are not known, we can only calculate the viscosity coefficients relative to normal-state values. These are

$$\frac{\eta_{ij,kl}}{\eta^N_{ij,kl}}=H_{ij,kl}\left[\frac{T}{T_c}\right] , \tag{131}$$

where

$$\eta^N_{ij,kl}=2\tau_N N(0)d_{ij}d_{kl}\overline{\lambda_{ij}\lambda_{kl}}\ ,$$

and the function H is given for the polar and axial states by

$$H_{ij,kl}\left[\frac{T}{T_c}\right]=\frac{2}{\overline{\lambda_{ij}\lambda_{kl}}}\int_0^\infty dE\left[-\frac{\partial n^0}{\partial E}\right]\frac{a}{\langle 1\rangle}\left[\langle D\bar{v}^2\lambda_{ij}\lambda_{kl}\rangle+(a^2-b^2)\frac{\langle D\bar{v}^2\lambda_{ij}\rangle\langle D\bar{v}^2\lambda_{kl}\rangle}{\langle 1\rangle-(a^2-b^2)\langle D\bar{v}^2\rangle}\right] , \tag{132}$$

and, for the d-wave state, by

$$\begin{aligned}H_{ij,kl}\left[\frac{T}{T_c}\right]=\frac{2}{\overline{\lambda_{ij}\lambda_{kl}}}\int_0^\infty dE\left[-\frac{\partial n^0}{\partial E}\right]\frac{a}{\langle 1\rangle}&\left[\langle D\bar{v}^2\lambda_{ij}\lambda_{kl}\rangle+(a^2-b^2)\frac{\langle D\bar{v}^2\lambda_{ij}\rangle\langle D\bar{v}^2\lambda_{kl}\rangle}{\langle 1\rangle-(a^2-b^2)\langle D\bar{v}^2\rangle}\right.\\&+4b^2L\left[\frac{\Delta}{E}\right]^2\mathrm{Re}\{c\langle D\bar{v}^2(\lambda_{xz}+i\lambda_{yz})\lambda_{kl}\rangle\langle D\bar{v}^2(\lambda_{xz}-i\lambda_{yz})\lambda_{ij}\rangle\\&\left.\times[\langle 1\rangle-ac^*(\langle D\rangle-\langle D\bar{v}^2\rangle)]\}\right] ,\end{aligned} \tag{133}$$

where L is given by Eq. (69) and

$$\overline{\lambda_{ij}\lambda_{kl}}=\int\frac{d\Omega}{4\pi}\lambda_{ij}\lambda_{kl}\ . \tag{134}$$

Since $\eta_{xz,yz}=0$ in the normal state—a result of the fact that $\overline{\lambda_{xz}\lambda_{yz}}=0$—the function $H_{xz,yz}$ is ill defined. Therefore we calculate the ratio $\eta_{xz,yz}/\eta_{xz,xz}$ using the fact that, because of the hexagonal symmetry, we have $d_{xz}=d_{yz}$. Hence, we can write

$$\frac{\eta_{xz,yz}}{\eta_{xz,xz}}=\frac{120}{H_{xz,xz}\left[\frac{T}{T_c}\right]}\int_0^\Delta dE\frac{\Delta^2}{E^2}\left[-\frac{\partial n^0}{\partial E}\right]\times ab^2Lc_2\langle D\bar{v}^2\lambda_{xz}^2\rangle^2\ . \tag{135}$$

The third term in the expression in curly brackets on the right-hand side of Eq. (133) is different from zero only for $H_{xz,xz}$ and $\eta_{xz,yz}$, and for all components related to these by symmetry. This term is finite only for energies smaller than the energy gap and for phase shifts neither very small nor equal to $\pi/2$. It comes exclusively from the in scattering term in the collision integral of the Boltzmann equation.

In the expressions (132), (133), and (135), for the function H and $\eta_{xz,yz}$, the integrals over the energy variable can only be done numerically. In the Appendix, we give the expressions for the various integrals needed in the numerical calculations. To predict the temperature dependence of the viscosity we use the same expression for the gap as a function of the temperature as in the case of the thermal conductivity.

In Figs. 8, 9, and 10, we show the four viscosity coefficients relative to their normal state values $H_{zz,zz}$, $H_{xx,xx}$, $H_{xz,xz}$, $H_{xy,xy}$, and the ratio $\eta_{xz,yz}/\eta_{xz,xz}$ as functions of T/T_c, for the same superconducting states and the same phase shifts we considered in the calculation of the thermal conductivity. The dependence of various viscosity coefficients $H_{ii,ii}$ ($i=x$, y, or z), and $H_{xy,xy}$ on the phase shift is very similar to the dependence of the di-

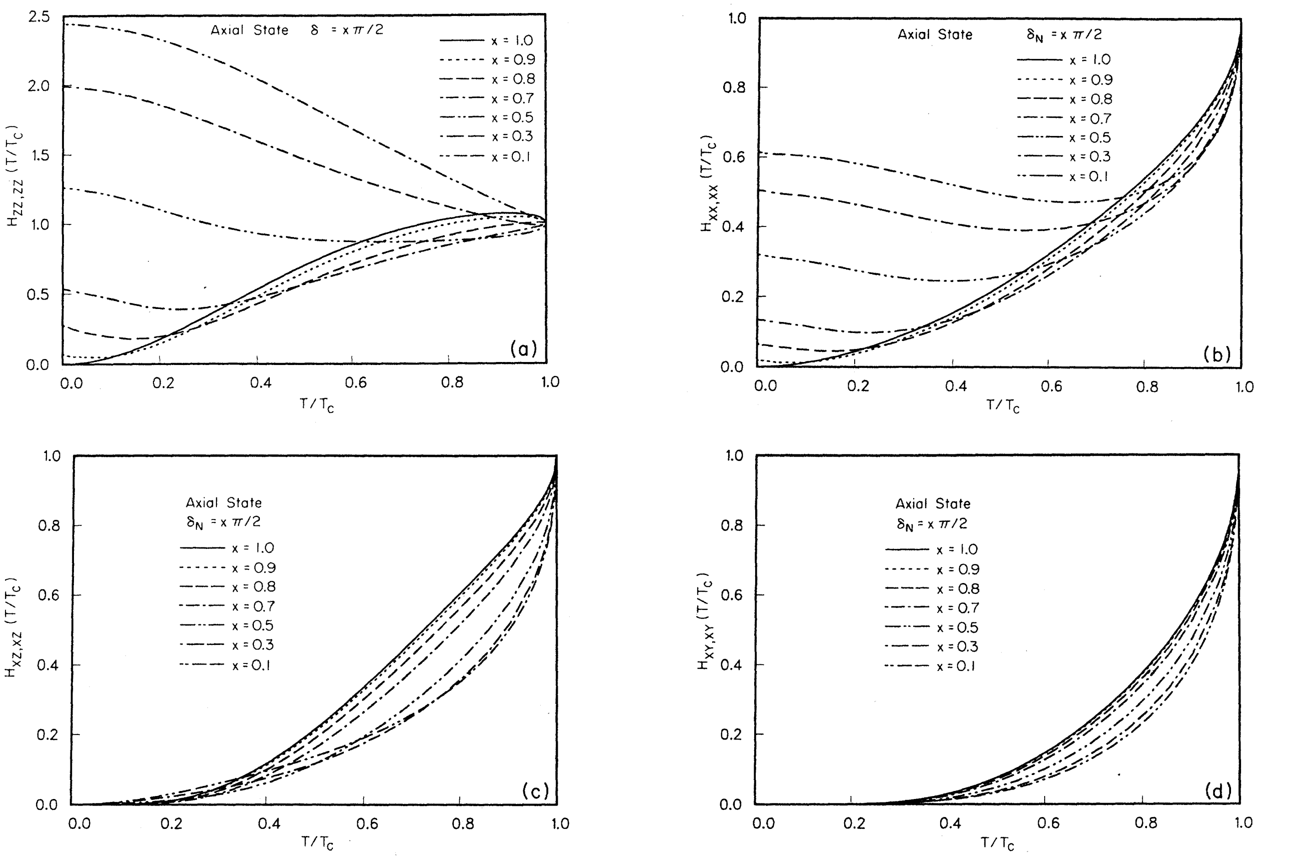

FIG. 8. Components of the ultrasonic attenuation relative to its normal state value at T_c for the axial p-wave state, and for $\delta_N = x\pi/2$, where $x = 0.1$, 0.3, 0.5, 0.7, 0.8, 0.9, and 1.0: (a) $H_{zz,zz}$, (b) $H_{xx,xx}$, (c) $H_{xz,xz}$, and (d) $H_{xy,xy}$.

agonal components of the thermal-conductivity tensor. For the component $H_{xz,xz}$ the situation is different, since it decreases slowly with decreasing phase shift.

One remarks here that the ratio $\eta_{xz,yz}/\eta_{xz,xz}$ is as large as 10% at low temperatures for phase shifts $\sim 0.8\pi/2$, but negligible for temperatures close to the superconducting transition temperature T_c. This is a consequence of the fact that $\eta_{xz,yz}$ is exclusively due to contributions from energies $E < \Delta$, while such energies dominate the contributions to $\eta_{xz,xz}$ only at low temperatures. At temperatures much less than Δ, $\eta_{xz,yz}/\eta_{xz,xz}$ is again small, because the particle-hole asymmetry is of order ξ/Δ.

Calculations of the attenuation of longitudinal ultrasound for the axial and polar p-wave states have also been carried out by Monien *et al.*,[17] who used field-theoretic methods and allowed for pair breaking. Our results agree well with theirs for the case of small pair breaking.

V. DISCUSSION

In this paper we have considered the effect of arbitrary normal-state scattering phase shifts on the transport properties of anisotropic superconductors with either odd or even parity and having nodes in the energy gap on the Fermi surface. We have explicitly calculated the relaxation time, the thermal conductivity, the thermoelectric coefficients, and the viscosity coefficients.

We have shown that particle-hole symmetry, which is frequently taken for granted, is violated on energy scales of order the gap energy, for phase shifts which are neither small nor equal to $\pi/2$, and we have shown explicitly that relaxation times for quasiparticles above and below the Fermi surface are different. This particle-hole asymmetry leads to an enhancement of the thermoelectric coefficient by a few orders of magnitude compared with that for isotropic superconductors. For ordinary superconductors T_c/T_F is of order 10^{-4}, and therefore one expects the thermoelectric coefficient to be of order $10^{-4}\sigma_N/e$, while our calculations show that for anisotropic superconductors it could be as large as $0.1\sigma_N/e$. We remark that even without the particle-hole asymmetry in the scattering process we have considered, one would expect thermoelectric effects in heavy fermion superconductors and high T_c materials to be larger in units of σ_N/e than in ordinary superconductors because T_c/T_F for these materials is higher, typically by a factor 10^2, than in ordinary superconductors. However, in these cases the thermoelectric coefficient in the superconducting state would be comparable to its normal-state value at T_c.

For superconducting states having an energy gap with

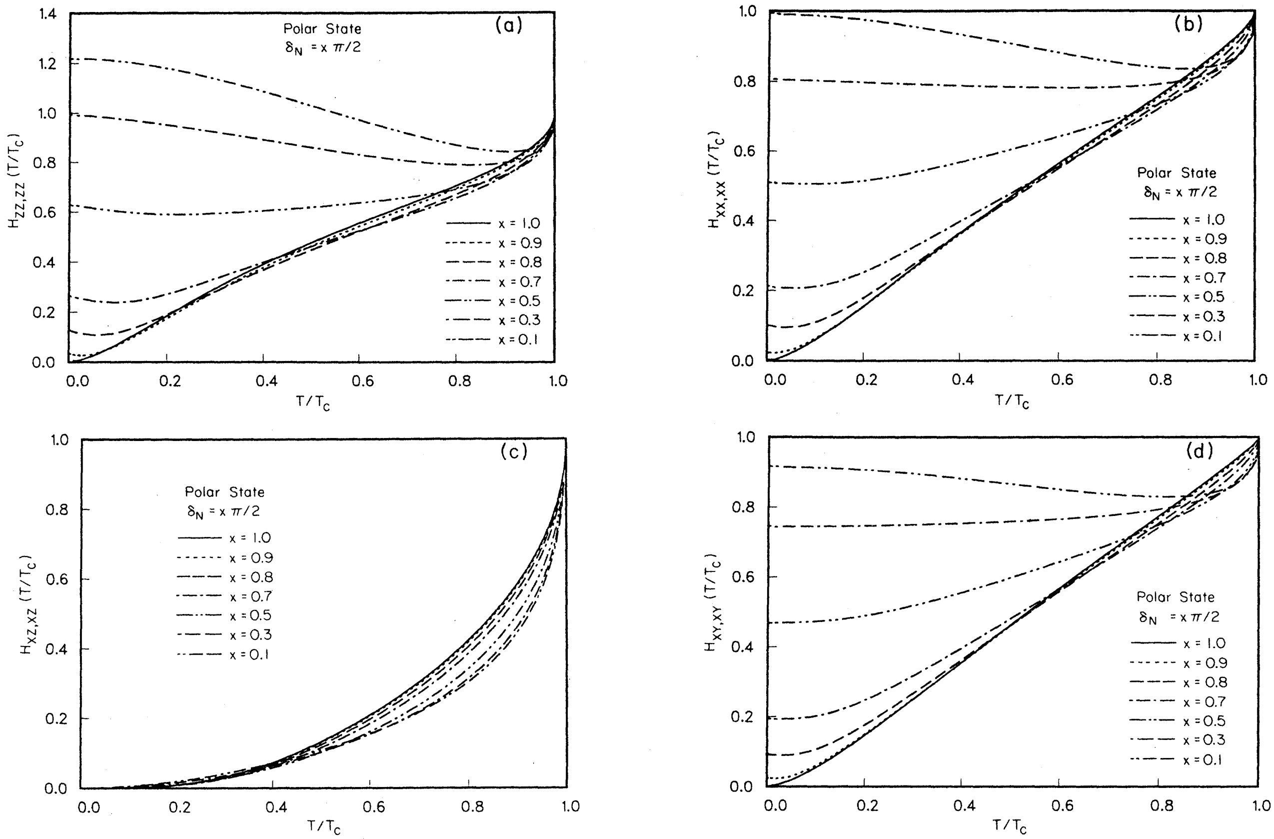

FIG. 9. Components of the ultrasonic attenuation relative to its normal state value at T_c for the polar p-wave state, and for $\delta_N = x\pi/2$, where $x = 0.1$, 0.3, 0.5, 0.7, 0.8, 0.9, and 1.0: (a) $H_{zz,zz}$, (b) $H_{xx,xx}$, (c) $H_{xz,xz}$, and (d) $H_{xy,xy}$.

a phase variation over the Fermi surface, such as the axial p-wave state and the d-wave state consistent with the hexagonal and cubic crystal symmetries, a reflection asymmetry in the quasiparticle-impurity scattering amplitude exists in the plane orthogonal to the energy-gap symmetry axis. In the case of the odd-parity axial state, this absence of reflection symmetry leads to new components of the thermal-conductivity and thermoelectric coefficient tensors which vanish in the normal state. In the case of the even-parity d-wave state, it leads to new components of the viscosity tensor, which are zero in the normal state.

The basic origin of this reflection asymmetry is the breaking of time-reversal invariance due to the existence of a condensate of pairs with finite angular momentum. Because the gap matrix is not invariant under time reversal, the Onsager relations must be written in a form which allows for this. If we neglect the effects of external magnetic fields and bulk rotation of the superconductor, the Onsager relations for a transport coefficient γ have the form

$$\gamma_{ij}(\Delta) = \gamma_{ji}(\mathcal{T}\Delta) ,$$

where $\mathcal{T}$ is the time-reversal operator, and Δ is the gap matrix. This is a natural generalization of the results given by Landau and Lifshitz.[41] We note that in the case of the viscosity, the indices i and j refer to *pairs* of Cartesian indices. If one uses for $\mathcal{T}\Delta$ the results given, for example, by Ueda and Rice,[42] one finds the results $K_{xy}(\Delta) = -K_{yx}(\Delta)$, $L_{xy}(\Delta) = -L_{yx}(\Delta)$, and $\eta_{xz,yz}(\Delta) = -\eta_{yz,xz}(\Delta)$, which are satisfied explicitly by our expressions.

New components of transport coefficients occur only if the parity of the gap is the same as that of the relevant current entering the transport coefficient. The heat current and the electrical current have odd parity, and therefore the thermal conductivity and thermoelectric coefficient can have additional components for certain p-wave states but not for d-wave ones. The conclusion for the viscosity is the opposite of this, since the momentum flux has even parity. Experimental observation of new components of transport coefficients would provide conclusive evidence for states with nontrivial phase variations over the Fermi surface, and would also determine the parity of the gap.

To facilitate comparison with experiment it is helpful to give the results of our calculations in the low-temperature limit. For the polar and d-wave states one finds for the thermal conductivity $K_{xx}/T \simeq \frac{3}{4}\cos^2\delta_N K_N(T_c)/T_c$, and for the axial state $K_{zz}/T \simeq \cos^2\delta_N K_N(T_c)/T_c$. The results for the viscosity in the polar and d-wave states are $\eta_{zz,zz} \simeq \frac{5}{4}\cos^2\delta_N \eta^N_{zz,zz}$, $\eta_{xx,xx} \simeq \cos^2\delta_N \eta^N_{xx,xx}$, and $\eta_{xy,xy} \simeq \frac{15}{16}\cos^2\delta_N \eta^N_{xy,xy}$, while for the axial state the results are $\eta_{zz,zz} \simeq \frac{5}{2}\cos^2\delta_N \eta^N_{zz,zz}$,

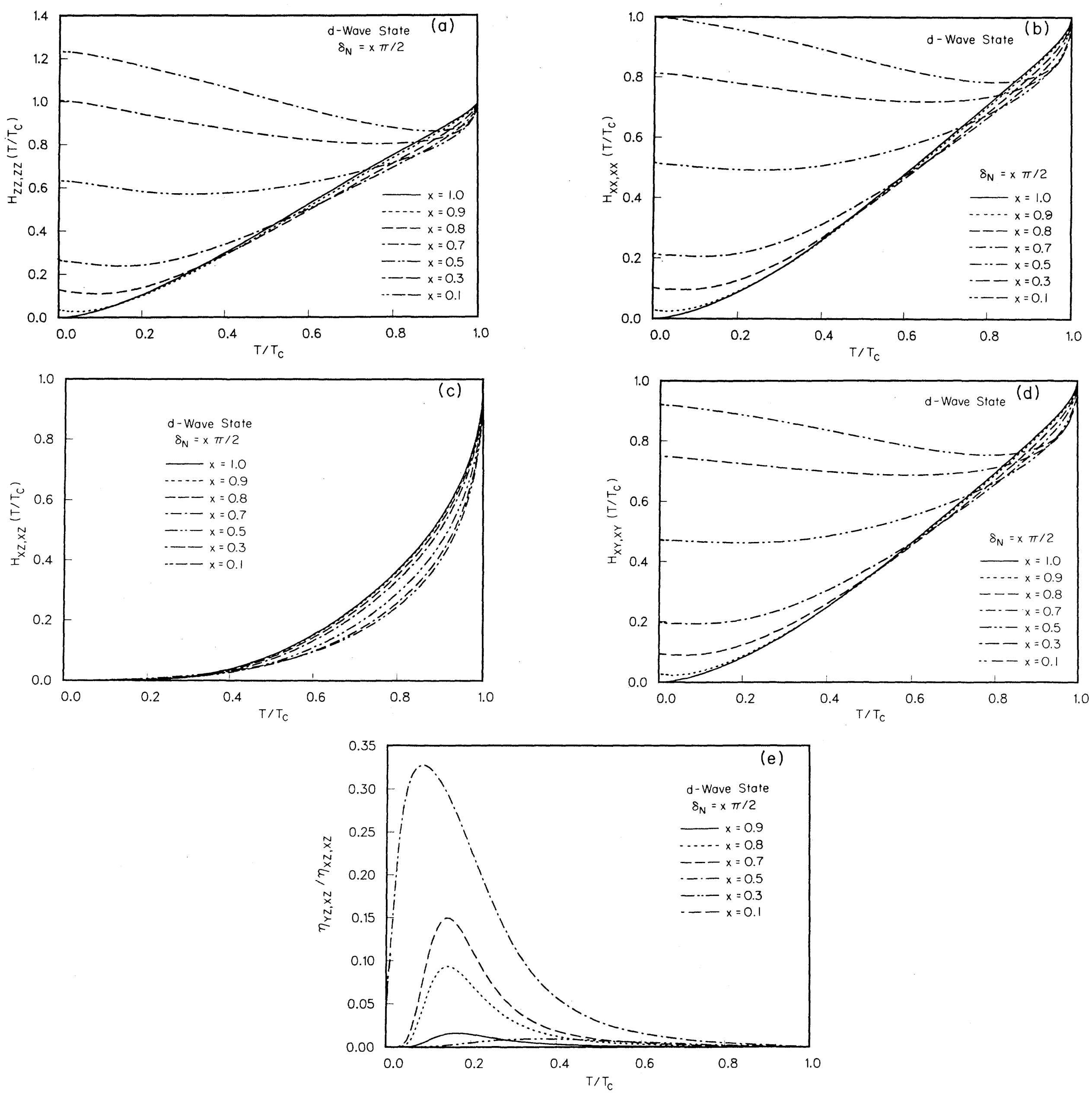

FIG. 10. Components of the ultrasonic attenuation relative to its normal-state value at T_c for the d-wave state, and for $\delta_N = x\pi/2$, where $x = 0.1, 0.3, 0.5, 0.7, 0.8, 0.9$, and 1.0: (a) $H_{zz,zz}$, (b) $H_{xx,xx}$, (c) $H_{xz,xz}$, (d) $H_{xy,xy}$, and (e) the ratio $\eta_{xz,yz}/\eta_{xz,xz}$.

$\eta_{xx,xx} \simeq \frac{5}{8}\cos^2\delta_N \eta^N_{xx,xx}$. All other components of the thermal-conductivity and viscosity tensors are smaller than these by a factor at least of order $(T/T_c)^2$. The fact that the results for the polar p-wave and d-wave states behave essentially identically in the low-temperature limit is a consequence of the fact that the properties of both states are dominated by the nodal line at the equator of the Fermi surface. In fact at all temperatures the thermal conductivity and viscosity of the two states are qualitatively similar, since they have common values in both the limits $T \to T_c$ and $T \to 0$.

Let us now turn to the experimental data. Sulpice *et al.*[4] have measured the thermal conductivity of a UPt_3 polycrystal at temperatures above about 30 mK (approximately $0.06T_c$), but unfortunately no data is available on single crystals as far as we are aware. If we assume that the quantity measured in the experiments is just an average thermal conductivity $K = \frac{1}{3}\sum_i K_{ii}$, the theoretical results for all superconducting states considered in this paper resemble the experimental ones for phase shifts greater than about $0.7\pi/2$. For smaller values of the phase shift, the theoretical values of the low-temperature thermal conductivity exceed the measured one. The lack of data on single crystals makes it difficult to discriminate among the various possible states.

The ultrasonic attenuation of UPt_3 single crystals has been measured by several groups.[5-7] Since the attenuation of longitudinal waves may have contributions from physical processes not considered here, such as gap relaxation[36,37] and order-parameter collective modes,[38,39] we will not attempt to make a careful comparison of our results with measurements of the attenuation of longitudinal waves, but instead will focus on the attenuation of transverse sound, which was measured by Shivaram *et al.*[7] for temperatures between 35 mK and T_c. For sound propagating in the basal plane along the b axis, they found the attenuation to be proportional to T for waves polarized along the a axis but to vary roughly as a power law of the temperature, with an exponent between 2 and 3, for waves polarized along the c axis. Qualitatively the measured attenuation resembles most closely the theoretical calculations for the polar and d-wave states, with the nodal line lying in the plane perpendicular to the c axis, and for phase shifts close to $\pi/2$. This fact was first pointed out by Schmitt-Rink *et al.*[13]

There are, however, a number of points that must be borne in mind in making the comparison with experiment. The first is that experimentally the attenuation is determined only up to a temperature-dependent additive constant, since the importance of end losses and similar effects is not known quantitatively. The plots of the experimental data given in Ref. 7 have been drawn so that the attenuation tends to zero at $T=0$, but the data do not rule out the possibility that the intrinsic attenuation is nonzero at $T=0$. A second experimental remark is that the width of the superconducting transition is ~ 50 mK or $T_c/10$, which makes difficult the deduction from experiment of what the initial drop in the attenuation just below T_c would be in a material with a sharp transition.

There are also uncertainties on the theoretical side. The first of these is that our basic model, with a spherical Fermi surface and s-wave scattering, is an oversimplification of the real band structure and scattering processes in UPt_3. However, allowing for band-structure effects and for higher partial waves, is not difficult, in principle, and can be done straightforwardly at the expense of more algebra and numerical work. A second simplification is that we have neglected the effects of pair breaking. These are important whenever characteristic excitation energies of importance become small compared with $\hbar/\tau$, where τ is the excitation lifetime. We may estimate τ at T_c by using the measurements of Shivaram *et al.*[7] to deduce a shear viscosity. If we assume that the deformation potential is comparable to the value for a Fermi liquid, of order $p_F v_F$, take the Fermi velocity to be $\sim 6\times 10^5$ cm/s, which is closer to the value found for all pieces of the Fermi surface in the de Haas–van Alphen experiments of Taillefer *et al.*,[43] and take for γ, the coefficient of the term in the specific heat linear in T, the value 420 mJ/K^2 mol,[4] we find for the sample of Shivaram *et al.* $\tau_N \simeq 3.3\times 10^{-11}$ s, which corresponds to a mean free path of approximately 2000 Å. This value is similar to the values deduced by the authors of Ref. 41 for their sample. From our estimate of the collision time above we find the characteristic scattering rate at T_c, $\Gamma(T_c)=\hbar/2\tau_N$ is 110 mK. This estimate of the scattering rate is uncertain because of our assumption that the deformation potential is equal to its value for a Fermi liquid, which may not be the case for heavy-fermion materials where band-structure effects can be important. We may obtain an independent estimate by noting that the transition temperature T_c of the sample used in the experiments of Ref. 7 is certainly less than 50 mK below the value found in the purest samples. According to the calculations of Walker[44] and of Hirschfeld, Wölfle, and Einzel[45] the depression of the transition temperature is given by

$$\frac{\delta T_c}{T_c(\Gamma=0)} \simeq -\frac{2\Gamma}{\pi T_c(\Gamma)} \quad \text{for} \quad \frac{\Gamma}{\pi T_c(\Gamma)} \leq 0.1 \ .$$

Thus a depression of T_c by 10% would correspond to $\Gamma \simeq \pi T_c(\Gamma)/20 \simeq 70$ mK, which is certainly an overestimate of Γ for the sample. Given the uncertainties in the two methods of determining Γ, this value is in good agreement with the one obtained from the ultrasonic attenuation.

Pair breaking is important in two temperature ranges—close to T_c, where the gap can be less than or of the order of the pair-breaking rate, and at low temperatures, where the energy of thermal excitations becomes comparable to Γ. A crude estimate for the polar state in the unitarity limit shows that pair-breaking effects play an important role only for temperatures such that $T/T_c \leq 1/10$ and $1-T/T_c \leq 1/100$. This estimate agrees very well with what is found in detailed calculations of Walker,[44] who studied the ultrasonic attenuation in p-wave superconducting states including explicitly the effects of pair breaking. We therefore conclude that the neglect of pair breaking is a very good first approximation for the conditions under which detailed experiments have been performed on UPt_3.

Another effect we have neglected is inelastic scattering processes. Their importance is clearly indicated by the temperature dependence of the attenuation in the normal state. Fits by Hess[46] to the experimental data of Shivaram *et al.*[7] on the attenuation of transverse sound in UPt_3 using a Fermi liquid picture in which both electron-electron and electron-impurity scattering processes contribute to the normal-state attenuation reveals that, for the sample used in Ref. 7, the contribution due to inelastic processes is 40% of the total attenuation at the transition temperature. Below the transition temperature the rate of inelastic processes will be reduced for two reasons: First, without pairing correlations, the inelastic scattering rate would be reduced as a consequence of the smaller number of thermal excitations to scatter from and the reduced phase space, and second, the superconducting correlations reduce even further the density of states. For a superconducting state with a nodal line, we expect the scattering rate to decrease roughly as $(T/T_c)^4$ compared to its value at T_c, making the dropoff of the attenuation below T_c less rapid than it would be in the absence of inelastic processes. Another assumption we have made is that all impurities have the same phase shift. With a distribution of phase shifts one would ex-

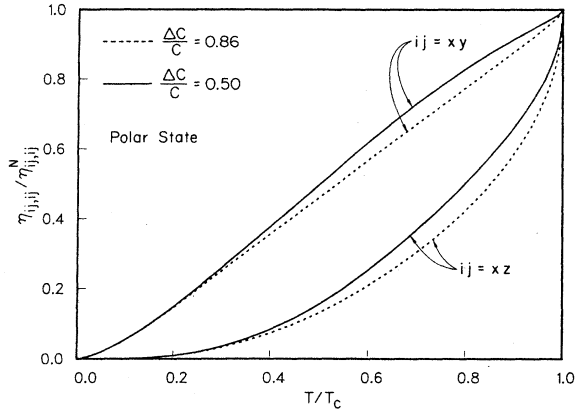

FIG. 11. Plots of $\eta_{xz,xz}$ and $\eta_{xy,xy}$ showing the effect of different values of $\Delta C/C$ on the viscosity coefficients for a normal-state phase shift of $\pi/2$.

pect features in the temperature dependence of the transport coefficients to be smoothed out.

In predicting the temperature dependence of the various transport coefficients the temperature dependence of the gap is an important ingredient. For this we used an expression introduced by Wölfle and Koch[26] in the context of superfluid ^{3}He. Some strong-coupling corrections to the energy gap were allowed for by using in this expression the experimental value of the specific-heat jump $\Delta C/C$ at the transition temperature. For the value of the energy gap at $T=0$, $\Delta(0)$, we adopted the weak-coupling value specific to each superconducting state. The value adopted for $\Delta C/C$ has most effect on the behavior of transport coefficients for T close to T_c, while the value of $\Delta(0)$ affects low-temperature properties. However, we note that any components of K/T and η that are finite at $T=0$ are unaffected by the value of $\Delta(0)$. For the specific heat jump we used the idealized value given by Sulpice *et al.*[4] $\Delta C/C \simeq 0.86$. It is difficult to determine $\Delta C/C$ experimentally, and the authors of Ref. 4 give 3 values, 0.5, 0.66, and 0.86. The uncertainty in the value of $\Delta C/C$ makes difficult the comparison of our results with experiment since the shapes of the curves of the transport coefficients as functions of temperature depend on the value of $\Delta C/C$. To illustrate this effect we show in Fig. 11 calculations of the coefficients $\eta_{xz,xz}$ and $\eta_{xy,xy}$ in the case of the polar state for the two extreme values of $\Delta C/C$.

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APPENDIX

Here we give results for the angular integrals required in the calculation of the transport coefficients. These are $\langle 1\rangle$, $g(E)$, $\langle D\rangle$, $\langle D\tilde{v}\rangle$, $\langle D\tilde{v}^2\rangle$, $\langle D\tilde{v}^2\mu_i^2\rangle$, $\langle D\tilde{v}^2\lambda_{ij}\rangle$, and $\langle D\tilde{v}^2\lambda_{ij}^2\rangle$. The integrals involving μ_i^2 and λ_{ij} may be expressed in terms of the basic integrals

$$I_{2n}=\langle D\tilde{v}^2\mu^{2n}\rangle \quad (n=0,\ 1,\ \text{or}\ 2)\ , \tag{A1}$$

where μ is a shorthand notation for μ_z.

The integrals needed in the thermal-conductivity and thermoelectric coefficients calculations are

$$\langle D\tilde{v}^2\mu_x^2\rangle=\langle D\tilde{v}^2\mu_y^2\rangle=\tfrac{1}{2}(I_0-I_2)\ , \tag{A2}$$

and

$$\langle D\tilde{v}^2\mu_z^2\rangle=I_2\ . \tag{A3}$$

For the components of the viscosity that we evaluate, we need the integrals $\langle D\tilde{v}^2\lambda_{ij}\rangle$ and $\langle D\tilde{v}^2\lambda_{ij}^2\rangle$, which may be expressed as

$$\langle D\tilde{v}^2\lambda_{ij}\rangle=b_{ij}^{(0)}I_0+b_{ij}^{(2)}I_2\ , \tag{A4}$$

and

$$\langle D\tilde{v}^2\lambda_{ij}^2\rangle=c_{ij}^{(0)}I_0+c_{ij}^{(2)}I_2+c_{ij}^{(4)}I_4\ , \tag{A5}$$

where the coefficients b_{ij} and c_{ij} are shown in Table I. The integrals involving the D function may be written as

$$\langle D\cdots\rangle=\begin{cases}\dfrac{1}{b^2}\langle G\cdots\rangle, & |x|<1\ , \quad \text{(A6)}\\[2ex] \dfrac{1}{a^2}\langle\cdots\rangle, & |x|>1\ , \quad \text{(A7)}\end{cases}$$

where

$$G=\frac{1}{r^2-\tilde{v}^2}\ , \tag{A8}$$

and $r=a/b$. The quantities needed have the following forms.

Axial state

For $|x|<1$

$$|g(x)|^2=\left[\frac{\pi}{2}x\right]^2+\left[\frac{x}{2}\ln\left|\frac{1+x}{1-x}\right|\right]^2\ , \tag{A9}$$

$$\langle 1\rangle=\frac{x}{2}\ln\left|\frac{1+x}{1-x}\right|\ , \tag{A10}$$

$$\langle G\rangle=\frac{x^2}{2(1-x^2)r^2}\frac{1}{Z}\ln\left|\frac{1+Z}{1-Z}\right|\ , \tag{A11}$$

$$\langle G\tilde{v}\rangle=\frac{x^2}{2rZ}\ln\left|\frac{rZ+1}{rZ-1}\right|\ , \tag{A12}$$

TABLE I. Coefficients b_{ij} and c_{ij}.

ij	$b^{(0)}$	$b^{(2)}$	$c^{(0)}$	$c^{(2)}$	$c^{(4)}$
zz	$-\frac{1}{3}$	1	$\frac{1}{9}$	$-\frac{2}{3}$	1
xx	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{11}{72}$	$-\frac{5}{12}$	$\frac{3}{8}$
xz	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
xy	0	0	$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$

$$\langle G\tilde{v}^2\rangle = -\frac{x}{2}\ln\left|\frac{1+x}{1-x}\right| + \frac{x^2}{2Z}\ln\left|\frac{Z+1}{Z-1}\right| , \quad \text{(A13)}$$

$$\langle G\tilde{v}^2\mu^2\rangle = -\frac{x^2}{2} - \frac{x}{2}\left[r^2x^2 - \frac{x^2}{2} + \frac{1}{2}\right]\ln\left|\frac{1+x}{1-x}\right| + \frac{1}{2}x^2r^2Z\ln\left|\frac{Z+1}{Z-1}\right| , \quad \text{(A14)}$$

$$\langle G\tilde{v}^2\mu^4\rangle = -\frac{x^2}{2}\left[r^2x^2 - \frac{3}{4}x^2 + \frac{5}{4}\right] + \frac{x}{2}\left[\frac{1}{8}(1-x^2)^2 - r^2Z^2\left[r^2x^2 - \frac{x^2}{2} + \frac{1}{2}\right]\right] \times\ln\left|\frac{1+x}{1-x}\right| + \frac{1}{2}x^2r^4Z^3\ln\left|\frac{Z+1}{Z-1}\right| , \quad \text{(A15)}$$

where

$$Z = \left[\frac{1}{r^2} + \left[1 - \frac{1}{r^2}\right]x^2\right]^{1/2} . \quad \text{(A16)}$$

For $|x| > 1$

$$|g(x)|^2 = \left[\frac{x}{2}\ln\left[\frac{x+1}{x-1}\right]\right]^2 , \quad \text{(A17)}$$

$$\langle 1\rangle = \frac{x}{2}\ln\left[\frac{x+1}{x-1}\right] , \quad \text{(A18)}$$

$$\langle\tilde{v}^2\rangle = \frac{1}{2} + \frac{1}{4}\frac{(x^2-1)}{x}\ln\left[\frac{x+1}{x-1}\right] , \quad \text{(A19)}$$

$$\langle\tilde{v}^2\mu^2\rangle = \frac{1}{4} + \frac{1}{8}(x^2-1) - \frac{1}{16}\frac{(x^2-1)^2}{x}\ln\left[\frac{x+1}{x-1}\right] , \quad \text{(A20)}$$

and

$$\langle\tilde{v}^2\mu^4\rangle = \frac{1}{6} + \frac{1}{16}(x^2-1)\left[\frac{5}{3} - x^2\right] + \frac{1}{32}\frac{(x^2-1)^3}{x}\ln\left[\frac{x+1}{x-1}\right] . \quad \text{(A21)}$$

Polar state

For $|x| < 1$

$$|g(x)|^2 = \left[\frac{\pi}{2}x\right]^2 + \left[x\ln\left[\frac{1+(1-x^2)^{1/2}}{x}\right]\right]^2 , \quad \text{(A22)}$$

$$\langle 1\rangle = \frac{\pi}{2}|x| , \quad \text{(A23)}$$

$$\langle G\rangle = \frac{\pi|x|}{2r}\frac{1}{(r^2-1)^{1/2}} , \quad \text{(A24)}$$

$$\langle G\tilde{v}\rangle = \frac{x}{(r^2-1)^{1/2}}\tan^{-1}\left[\frac{1}{x(r^2-1)^{1/2}}\right] , \quad \text{(A25)}$$

$$\langle G\tilde{v}^2\rangle = \frac{\pi}{2}|x|\left[\frac{r}{(r^2-1)^{1/2}} - 1\right] , \quad \text{(A26)}$$

$$\langle G\tilde{v}^2\mu^2\rangle = \frac{\pi}{2}|x|^3\left[r^2 - \frac{1}{2} - r(r^2-1)^{1/2}\right] , \quad \text{(A27)}$$

and

$$\langle G\tilde{v}^2\mu^4\rangle = \frac{\pi}{2}|x|^5\left[-\frac{3}{8} + \frac{3}{2}r^2 - r^4 + r(r^2-1)^{3/2}\right] . \quad \text{(A28)}$$

For $|x| > 1$

$$|g(x)|^2 = \left[x\sin^{-1}\frac{1}{x}\right]^2 , \quad \text{(A29)}$$

$$\langle 1\rangle = x\sin^{-1}\frac{1}{x} , \quad \text{(A30)}$$

$$\langle\tilde{v}^2\rangle = \frac{1}{2}\left[1 - \frac{1}{x^2}\right]^{1/2} + \frac{x}{2}\sin^{-1}\frac{1}{x} , \quad \text{(A31)}$$

$$\langle\tilde{v}^2\mu^2\rangle = \frac{1}{2}\left[-\frac{x^2}{4} + \frac{1}{2}\right]\left[1 - \frac{1}{x^2}\right]^{1/2} + \frac{x^3}{8}\sin^{-1}\frac{1}{x} , \quad \text{(A32)}$$

and

$$\langle\tilde{v}^2\mu^4\rangle = \frac{1}{2}\left[-\frac{x^4}{8} - \frac{x^2}{12} + \frac{1}{3}\right]\left[1 - \frac{1}{x^2}\right]^{1/2} + \frac{x^5}{16}\sin^{-1}\frac{1}{x} . \quad \text{(A33)}$$

d-wave state

For $|x| < 1$

$$|g(x)|^2 = \left[\frac{x}{2}\right]^2\left\{\left[\left[\int_0^{\mu_1} + \int_{\mu_2}^1\right]\frac{d\mu}{(\mu^4-\mu^2+x^2/4)^{1/2}}\right]^2 + \left[\int_{\mu_1}^{\mu_2}\frac{d\mu}{(\mu^2-\mu^4-x^2/4)^{1/2}}\right]^2\right\} , \quad \text{(A34)}$$

$$\langle 1\rangle = \frac{|x|}{2}\left[\int_0^{\mu_1} + \int_{\mu_2}^1\right]\frac{d\mu}{(\mu^4-\mu^2+x^2/4)^{1/2}} , \quad \text{(A35)}$$

$$\langle G\rangle=\frac{|x|^3}{8}\left[\int_0^{\mu_1}+\int_{\mu_2}^{1}\right]\frac{d\mu}{(\mu^4-\mu^2+x^2/4)^{1/2}}\left[\frac{1}{\mu^2-\mu^4+x^2(r^2-1)/4}\right], \tag{A36}$$

$$\langle G\tilde{v}\rangle=\frac{x^2}{4}\int_0^1\frac{d\mu}{\mu^2-\mu^4+x^2(r^2-1)/4}, \tag{A37}$$

and

$$\langle G\tilde{v}^2\mu^{2n}\rangle=\frac{|x|}{2}\left[\int_0^{\mu_1}+\int_{\mu_2}^{1}\right]\times d\mu\frac{(\mu^4-\mu^2+x^2/4)^{1/2}}{\mu^2-\mu^4+x^2(r^2-1)/4}\mu^{2n}, \tag{A38}$$

where

$$\mu_1=\frac{1}{\sqrt{2}}[1-(1-x^2)^{1/2}]^{1/2},$$

$$\mu_2=\frac{1}{\sqrt{2}}[1+(1-x^2)^{1/2}]^{1/2},$$

and $n=0,1,2$.

For $|x|>1$

$$|g(x)|^2=\left[\frac{x}{2}\right]^2\left[\int_0^1\frac{d\mu}{(\mu^4-\mu^2+x^2/4)^{1/2}}\right]^2, \tag{A39}$$

$$\langle 1\rangle=\frac{|x|}{2}\int_0^1\frac{d\mu}{(\mu^4-\mu^2+x^2/4)^{1/2}}, \tag{A40}$$

and

$$\langle\tilde{v}^2\mu^{2n}\rangle=\frac{2}{|x|}\int_0^1 d\mu\,\mu^{2n}(\mu^4-\mu^2+x^2/4)^{1/2}. \tag{A41}$$

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[1]G. R. Stewart, Rev. Mod. Phys. **56**, 755 (1984).
[2]Z. Fisk, H. R. Ott, T. M. Rice, and J. L. Smith, Nature (London) **320**, 124 (1986).
[3]H. R. Ott, in *Progress of Low Temperature Physics Vol. XI,* edited by D. F. Brewer (Elsevier, Amsterdam, 1987), p. 215.
[4]A. Sulpice, P. Gandit, J. Chaussy, J. Flouquet, D. Jaccard, P. Lejay, and J. L. Tholence, J. Low Temp. Phys. **62**, 39 (1986).
[5]D. J. Bishop, C. M. Varma, B. Batlogg, E. Bucher, Z. Fisk, and J. L. Smith, Phys. Rev. Lett. **53**, 1009 (1984).
[6]V. Müller, D. Maurer, E. W. Scheidt, Ch. Roth, K. Lüders, E. Bucher, and H. E. Bömmel, Solid State Commun. **57**, 319 (1986).
[7]B. S. Shivaram, Y. H. Jeong, T. F. Rosenbaum, and D. G. Hinks, Phys. Rev. Lett. **56**, 1078 (1986).
[8]C. J. Pethick and D. Pines, in Proceedings of the 4th International Conference on Recent Progress in Many Body Theories, San Francisco, 1985, edited by P. Siemens and R. A. Smith (unpublished).
[9]L. Coffey, T. M. Rice, and K. Ueda, J. Phys. C **18**, L813 (1985).
[10]C. J. Pethick and D. Pines, Phys. Rev. Lett. **57**, 118 (1986).
[11]P. Hirschfeld, D. Vollhardt, and P. Wölfle, Solid State Commun. **59**, 111 (1986).
[12]K. Scharnberg, D. Walker, H. Monien, L. Tewordt, and R. A. Klemm, Solid State Commun. **60**, 535 (1986).
[13]S. Schmitt-Rink, K. Miyake, and C. M. Varma, Phys. Rev. Lett. **57**, 2575 (1986).
[14]H. Monien, K. Scharnberg, L. Tewordt, and D. Walker, Solid State Commun. **61**, 581 (1987).
[15]B. Arfi and C. J. Pethick, Phys. Rev. B **38**, 2312 (1988).
[16]H. R. Ott, E. Felder, C. Bruder, and T. M. Rice, Europhys. Lett. **3**, 113 (1987).
[17]H. Monien, K. Scharnberg, and D. Walker, Solid State Commun. **63**, 263 (1987).
[18]C. J. Pethick and D. Pines (unpublished).
[19]C. J. Pethick and D. Pines, in *Recent Progress in Many-Body Theories,* edited by A. J. Kallio, E. Pajanne, and R. F. Bishop (Plenum, New York, 1988), Vol. I, p. 17.
[20]B. Arfi, H. Bahlouli, C. J. Pethick, and D. Pines, Phys. Rev. Lett. **60**, 2206 (1988).
[21]M. Salomaa, C. J. Pethick, and G. Baym, J. Low Temp. Phys. **40**, 297 (1980).
[22]A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).
[23]P. W. Anderson and P. Morel, Phys. Rev. **123**, 1911 (1961).
[24]H. Bahlouli and B. Arfi, J. Low Temp. Phys. **69**, 327 (1987).
[25]W. F. Vinen, in *Superconductivity,* edited by R. D. Parks (Marcel Dekker, New York, 1969), Vol. 2, p. 1167.
[26]P. Wölfle and V. E. Koch, J. Low Temp. Phys. **30**, 61 (1978).
[27]V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **14**, 177 (1944) [Sov. Phys.—JETP **8**, 148 (1944)].
[28]Y. M. Galperin, V. L. Gurevich, and V. I. Kozub, Zh. Eksp. Teor. Fiz. **66**, 1387 (1974) [Sov. Phys.—JETP **39**, 680 (1974)].
[29]D. J. Van Harlingen and J. C. Garland, Solid State Commun. **25**, 419 (1978).
[30]D. J. Van Harlingen, D. F. Heidel, and J. C. Garland, Phys. Rev. B **21**, 1842 (1980).
[31]D. J. Van Harlingen, Physica **109/110B&C**, 1710 (1982).
[32]H. J. Mamin, J. Clarke, and D. J. Van Harlingen, Phys. Rev. B **29**, 3881 (1984).
[33]L. Z. Kon, Zh. Eksp. Teor. Fiz. **70**, 286 (1976) [Sov. Phys.—JETP **43**, 149 (1976)].
[34]F. S. Khan and P. B. Allen, Phys. Rev. B **35**, 1002 (1987).
[35]L. D. Landau and E. M. Lifshitz, *Theory of Elasticity,* 3rd ed. (Pergamon, Oxford, 1986), p. 137.
[36]K. Miyake and C. M. Varma, Phys. Rev. Lett. **57**, 1627 (1986).
[37]P. Wölfle, Phys. Lett. A **119**, 40 (1986).
[38]H. Monien, L. Tewordt, and K. Scharnberg, Solid State Commun. **63**, 1027 (1987).
[39]St. Lenck, H. Monien, and L. Tewordt, J. Low Temp. Phys. **70**, 309 (1988).

[40]B. S. Shivaram, T. F. Rosenbaum, and D. G. Hinks, Phys. Rev. Lett. **57**, 1259 (1986).
[41]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, Oxford, 1980), Pt. I, p. 367.
[42]K. Ueda and T. M. Rice, Phys. Rev. B **31**, 7114 (1985).
[43]L. Taillefer, R. Newbury, G. G. Lonzarich, Z. Fisk, and J. L. Smith, J. Magn. Magn. Mater. **63-64**, 372 (1987).
[44]D. Walker, Ph.D. thesis, University of Hamburg, 1986.
[45]P. J. Hirschfeld, P. Wölfle, and D. Einzel, Phys. Rev. B **37**, 83 (1988).
[46]D. W. Hess, J. Low Temp. Phys. **68**, 311 (1987).