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Resonant-tunneling effects in a parabolic quantum well obtained by application of crossed magnetic and electric fields in a semiconductor quantum barrier

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By considering the conduction-band discontinuity and effective-mass characteristics of GaAs-$Al_xGa_{1-x}As$ and $In_{0.52}Ga_{0.48}As$-$In_{0.52}(Ga_{1-x}Al_x)_{0.48}As$ heterostructures, we show theoretically that it is possible to obtain a parabolic quantum well by applying crossed magnetic and electric fields in a single quantum barrier. Resonant-tunneling effects are then obtained through harmonic-oscillator energy levels in the well. We also discuss the resonant-tunneling conditions and the possible values for K_y transversal momentum of the incident electron.

The high quality of growth for different semiconductor materials has given possibilities for studies and discoveries of new devices and applications. In this manner the transport and optical properties have been verified and interesting and important effects have been reported. At the same time theoretical works have also been developed that sometimes open new fields for experimental investigations.

Following the pioneering work of Tsu and Esaki[1] we will show the possibility of obtaining a parabolic quantum well and consequently resonant-tunneling effects into the quasibound harmonic-oscillator states in the well. We will base our calculations on the GaAs-$Al_xGa_{1-x}As$-GaAs and quanternary $In_{0.52}Ga_{0.48}As$-$In_{0.52}(Ga_{1-x}$-$Al_x)_{0.48}As$-$In_{0.52}Ga_{0.48}As$ semiconductor barriers.

A generic parabolic potential can be obtained by the conjugated action of crossed magnetic (B) and electric (F) fields. By considering the magnetic field parallel to the heterolayers (Z direction), it is possible to obtain a parabolic potential profile of the barrier which acts as a quantum well for given values of B and F where the minimum position energy is lower than the Fermi energy E_F in the emitter. Thus, the harmonic-oscillator states with lower energies will give the conditions for a resonant tunneling in the system.

We will consider the energy origin at the top of the barrier. X is the growth direction of the sample. In the emitter and collector regions (1 and 3 in Fig. 1) the potentials are considered constants and equal to $-U_0$, where U_0 is the barrier height. If these regions are sufficiently doped we can neglect the field effects as also considered by Eaves and *et al.*[2] By the choice of the potential vector $A=(0,Bx,0)$ we will have the Hamiltonian in the barrier region given by

$$H=\frac{1}{2m^*}[\mathcal{P}_x^2+\mathcal{P}_z^2+(\mathcal{P}_y-eBx)^2]-eFx\,, \qquad (1)$$

where $\mathcal{P}$ is the momentum operator. The derived Schrödinger equation in the effective-mass approximation is given by

$$-(\hbar^2/2m^*)\frac{d^2\Psi}{dx^2}+[(\hbar^2/2m^*)K_z^2+(m^*\Omega^2/2)\times(x-x_0)^2-eFx]\Psi=E\Psi\,, \qquad (2)$$

where $x_0=\hbar K_y/eB$, $\Omega=eB/m^*$, and m^* is the electron effective mass in the barrier.

We can take $E=E_x+(\hbar^2/2m_0)(K_y^2+K_z^2)$ as the total energy of the electron in the emitter according to the Davies[3] statements for the three-dimensional case. The quantity m_0 is the electron effective mass in the emitter.

Thus, Eq. (2) takes the form

$$-(\hbar^2/2m^*)\frac{d^2\Psi}{dx^2}+V(x,K_y,K_z)\Psi=E_x\Psi\,, \qquad (3)$$

where

$$V(x,K_y,K_z)=V_{\min}+(m^*\Omega^2/2)(x-X_0)^2\,. \qquad (4)$$

The potential given by Eq. (4) could be understood as an effective potential in the barrier where $X_0=x_0+eF/m^*\Omega^2$ and $V_{\min}$ is the minimum energy of the parabolic quantum well, given by

$$V_{\min}=(\hbar^2K_z^2/2m_0)(m_0/m^*-1)-\hbar^2K_y^2/2m_0-eFx_0-e^2F^2/2m^*\Omega^2\,. \qquad (5)$$

By substituting Eq. (4) in Eq. (3) and performing the transformation $\eta=x-X_0$, we have

$$-(\hbar^2/2m^*)\frac{d^2\psi}{d\eta^2}+(m^*\Omega^2/2)\eta^2\psi=(E_x-V_{\min})\psi\,, \qquad (6)$$

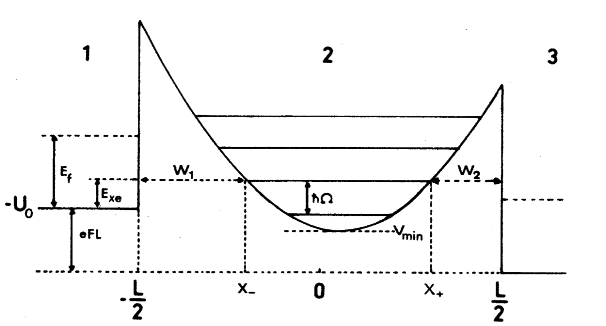

FIG. 1. Resonant-tunneling scheme in a parabolic quantum-well potential. X_- and X_+ are the classical turning points, W_1 and W_2 are the effective barrier widths, and L is the total width of the barrier.

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that is the Schrödinger equation for a one-dimensional harmonic oscillator. The solutions for the quantum energy levels are well known and we obtain

$$E_x = V_{\min} + (n + \tfrac{1}{2})\hbar\Omega \,. \tag{7}$$

An analytical solution for the wave function in Eq. (6) is given by a linear combination of the parabolic-cylinder functions.[4] The differential equation for these functions is given by $\Phi''(\xi) - (\xi^2/4 + a)\Phi(\xi) = 0$. We easily recover this equation considering $\xi = (x - X_0)/\alpha$ with $\alpha = (\hbar/2m^*\Omega)^{1/2}$ and $a = (V_{\min} - E_x)/\hbar\Omega$ in Eq. (6).

In our model the most interesting properties of the system are in the barrier region. By considering only this region we can define what are the necessary conditions to obtain a parabolic quantum well by application of crossed magnetic and electric fields. The effective potential barrier $V(x,K_y,K_z)$ and the minimum position X_0 of the parabolic quantum well depend explicitly on the K_y value, electric field F, and the magnetic field B.

By assuming the existence of a parabolic quantum well in the barrier region, for a determined set of B, F, and K_y value, then Eq. (7), which is an energy solution of the effective-mass Schrödinger equation, gives us the permissible energy states in the well. The effective barriers W_1 and W_2 (see Fig. 1) are large enough. The overlap between the wave functions inside and outside the barrier is small. Thus, the quasibound permissible states are well calculated by Eq. (7); in particular, the states with indices of $n \le 1$ (the cases where W_1 and W_2 are bigger), which are the principal contributors to the resonant tunneling process because of their low energies. Therefore, our problem is to find a permissible and reasonable range of K_y by utilizing the parameters F and B within the experimental range to obtain a resonant-tunneling effect in this system. Thus, we can state the following:

(i) X_0, the minimum potential position will have to be between the interfaces of the barrier. The potential value at this point will have to be deep enough to accommodate at least one permissible state with an energy smaller than the Fermi energy E_F.

(ii) The resonant condition will have to be satisfied; therefore, the energy $-U_0 + E_{xe}$ of the incident electron will have to be the same as the quasibound harmonic oscillator level in the parabolic quantum well given by Eq. (7).

(iii) The magnitude of K_y in the barrier region can only take the values that are in the allowed transversal momentum range in the emitter, i.e., $|K_y| \le [(m_0/\hbar^2)(E_F - E_{xe})]^{1/2}$. (For simplicity we have considered $K_z = K_y$).

To consider the third condition and a constant potential in a sufficiently doped emitter (conforming to Ref. 2) is nothing more than an approximation of the problem. The effect of the magnetic field will be much more reduced in the emitter than in the barrier region. An electron in a Landau orbit in the emitter will change its transversal momentum and consequently its centered X_0^* position constantly by scattering process. So, we could consider the existence of a K_y range obeying the third condition, corresponding to several incident transversal momentum of the emitter. In addition, the work done by Brey, Platero, and Tejedor[5] where the authors utilize a transfer Hamiltonian approach, demonstrates very clearly the conservation of transversal momentum in the transition process between right- and left-hand sides of the barrier in a high transverse applied magnetic field. This fact reinforces our approximations.

If one considers our three conditions valid, a good approximate method can be developed, considering the effective barrier $V(x,K_y,K_z)$ as a sequential step function,[6] which offers possibilities to obtain a numerical expression for the transmission coefficient in this problem. A comparison with the analytic result of Eq. (7) can be done.

By considering the sequential step approach, the wave functions at each step potential region can be written

$$\phi_j(x) = A_j e^{\sigma_j x} + B_j e^{-\sigma_j x} \,, \tag{8}$$

where $j = 0, 1, 2, \ldots, (N+1)$ is the index of each potential step and N is the total number of steps in the barrier region, and

$$\sigma_j = \begin{cases} i[(2m_j^*/h^2)(E_{xe} - S_j)]^{1/2}, & E_{xe} > S_j \,, \\ [(2m_j^*/h^2)(S_j - E_{xe})]^{1/2}, & E_{xe} < S_j \,. \end{cases} \tag{9}$$

In the emitter and collector region ($j = 0$ and $j = N+1$) the effective mass must be changed for m_0.

$$S_j = \begin{cases} -U_0, \quad x < -L/2, \; j = 0 \,, \\ [V(x_{j-1},K_y,K_z) + V(x_j,K_y,K_z)]/2, \quad x_0 \le x_j \le x_N, \; j = 1, \ldots, N \,, \\ -U_0 - eFL, \quad x > L/2, \; j = N+1 \,, \end{cases} \tag{10}$$

where L is the barrier width, $x_0 = -L/2$, and $x_N = L/2$.

By verifying the boundary conditions of the wave functions $\phi_j(x)$ and their derivates $(1/m_j^*)\phi_j'$ at each j interface we can find a matrix relation connecting the emitter and collector wave-function coefficients,

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \left[\prod_{j=0}^{N} \mathbf{M}_j \right] \begin{pmatrix} A_{N+1} \\ B_{N+1} \end{pmatrix} , \tag{11}$$

where $\mathbf{M}_j$ is the iteration matrix given by

$$\mathbf{M}_j = \frac{1}{2} \begin{pmatrix} (1+\theta_j) e^{(\sigma_{j+1} - \sigma_j)x_j} & (1-\theta_j) e^{-(\sigma_{j+1} + \sigma_j)x_j} \\ (1-\theta_j) e^{(\sigma_{j+1} + \sigma_j)x_j} & (1+\theta_j) e^{(\sigma_j - \sigma_{j+1})x_j} \end{pmatrix} , \tag{12}$$

and $\theta_j = (m_j^* \sigma_{j+1})/(m_{j+1}^* \sigma_j)$.

By associating $A_0 = 1$, $B_0 = R$, $A_{N+1} = T$, and $B_{N+1} = 0$, where R and T are respectively the reflection and transmission amplitudes, we obtain

$$\begin{pmatrix} 1 \\ R \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} T \\ 0 \end{pmatrix}, \qquad (13)$$

with $\mathbf{L} = \prod_{j=0}^{N} \mathbf{M}_j$. The transmission coefficient is then given by

$$T^* T(E_{xe}, K_y, F, B) = (\sigma_{N+1}/\sigma_0) \, | 1/L_{11} |^2 . \qquad (14)$$

The viability of our numerical method is then verified when we recover the energy values of the quasibound states given by Eq. (7). The energy values are now calculated by the peak positions in the transmission coefficients which are shown in Figs. 2(b) and 3(b), respectively, for $Al_xGa_{1-x}As$-GaAs and $In_{0.52}(Ga_{1-x}Al_x)_{0.48}As$-$In_{0.52}Ga_{0.48}As$ single quantum barriers. The parameters utilized by us[7,8] and results are specified in the captions.

These two semiconductor systems, by their characteristics of conduction-band discontinuities and effective masses, enable us to obtain resonant tunneling conditions. If we consider a reasonable Fermi energy in the emitter, equal to 40 meV, there will be a level from which we will get a transmitted resonant current.

The minimum position of the parabolic well potential is given by $X_0 = \hbar K_y/eB + Fm^*/eB^2$. We can see that it depends fundamentally on F, B, and K_y. A good relationship between F and B fields guarantees that X_0 is between the barrier interfaces. This condition is obtained for a limited range of F (or voltage) when we consider B fixed. Thus, for F values in this limited range, resonant tunneling occurs with an increase of the tunneling current. The results shown in Figs. 2(a) and 3(a) are for just one of these cases where we have a parabolic well in the barrier region. Outside of this range, by increasing F, the minimum position X_0 can move to the extremity of the barrier and consequent quench of the parabolic well in this region. The effective barrier will be very large (~500 Å) and practically impenetrable. There will be a decrease in the tunneling current characterizing a differential negative resistance region. Note that the three basic conditions depend on K_y and they will have to be satisfied simultaneously. Thus, by fixing the parameters B, F, L, U_0, and E_F, the variation of the possible K_y values turns out to be limited. The relationship between F and B turns out to be the principal factor which controls the position X_0. This justifies our above statements for the formation of a differential negative resistance.

We could also notice that a more accurate solution of this problem in the case of the $In_{0.52}(Ga_{1-x}Al_x)_{0.48}As$-$In_{0.52}Ga_{0.48}As$ system would involve the effect of effective-mass nonparabolicity.

In conclusion we have shown the possibility of obtaining a parabolic quantum well by applying a high magnetic field perpendicular to the electric field on GaAs-$Al_xGa_{1-x}As$-GaAs and $In_{0.52}Ga_{0.48}As$-$In_{0.52}(Ga_{1-x}Al_x)_{0.48}As$-$In_{0.52}Ga_{0.48}As$ semiconductor single barriers.

The analytical energy levels in the parabolic quantum well, given by Eq. (7), are in a very good agreement with the results obtained by our transmission-coefficient approach.

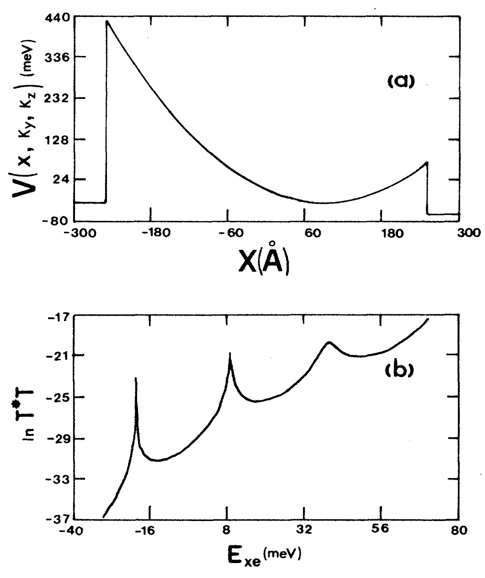

FIG. 2. (a) Effective potential $V(x,K_y,K_z)$ for a GaAs-$Al_xGa_{1-x}As$-GaAs semiconductor quantum barrier; (b) transmission coefficient giving the resonant harmonic-oscillator energy levels at $E_0 = -20.2$ meV, $E_1 = 9.2$ meV, $E_2 = 39.9$ meV. The following parameters are used: aluminum concentration $x = 0.05$, $U_0 = 35.0$ meV, $m_0 = 0.067 m_e$, $m^* = 0.071 m_e$ (m_e is the free-electron mass), $B = 18$ T, $F = 2.2 \times 10^6$ V/m, $L = 500$ Å, $V_{min} = -34.8$ meV, and $K_y = 1.726 \times 10^8$ m^{-1}.

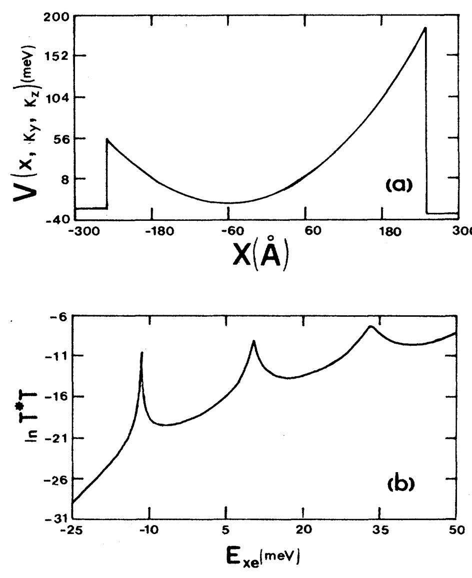

FIG. 3. (a) Effective potential $V(x,K_y,K_z)$ for a $In_{0.52}Ga_{0.48}As$-$In_{0.52}(Ga_{1-x}Al_x)_{0.48}As$-$In_{0.52}Ga_{0.48}As$ quaternary semiconductor quantum barrier; (b) transmission coefficient giving the resonant harmonic-oscillator energy levels at $E_0 = -11.5$ meV, $E_1 = 10.4$ meV, $E_2 = 32.3$ meV. The following parameters are used: $x = 0.05$, $U_0 = 26.5$ meV, $m_0 = 0.042 m_e$, $m^* = 0.069 m_e$, $B = 13$ T, $F = 0.5 \times 10^6$ V/m, $L = 500$ Å, $V_{min} = -22.4$ meV, and $K_y = -1.428 \times 10^8$ m^{-1}.

Though we do not consider the magnetic field in the emitter region and the exact way of counting the allowed K_y in a transmission process, our treatment leads us to think that this phenomenon can occur and gives us a good idea of the physical parameters to be used to observe this resonant-tunneling effect experimentally.

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