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Scattering of a surface-skimming bulk transverse wave by an elastic ridge

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We study theoretically the scattering of a surface-skimming bulk transverse wave that propagates along the planar surface of a semi-infinite substrate and impinges normally on a rectangular ridge in its path. The ridge is fabricated from the same material as the substrate, or from a different material. The differential scattering cross section, as a function of frequency, displays peaks at the real parts of the frequencies of the acoustic surface shape resonances supported by the ridge. This result suggests that acoustic surface shape resonances can be studied experimentally by the kind of scattering experiment analyzed here.

In a recent paper,[1] we presented a theory of acoustic surface shape resonances. These are vibrational excitations that are localized in the vicinity of an isolated protuberance on the otherwise planar, stress-free surface of a semi-infinite elastic medium. The protuberance may be fabricated from the same material as the substrate, or from a different material. In general, there is an infinite number of such resonances associated with a given protuberance. Their frequencies are discrete because of the loss of translational symmetry caused by the presence of the protuberance; they are complex because they overlap the range of frequencies allowed the vibrations of the substrate, into which they can decay; and they depend on the shape of the protuberance and on the relation of its material properties (mass density, elastic moduli) to those of the substrate.

In Ref. 1, the simplest structure supporting acoustic surface shape resonances was considered. This consists of a rectangular ridge of width $2l$ and height d, fabricated from a material of mass density $\rho^{(1)}$ and elastic moduli $\{c_{ij}^{(1)}\}$, bonded to a semi-infinite substrate consisting of a material whose mass density is $\rho^{(2)}$ and whose elastic moduli are $\{c_{ij}^{(2)}\}$. The ridge is oriented parallel to the x_2 axis (Fig. 1).

The elastic displacement field was assumed to have only a single, nonzero component, along the x_2 direction, and to be independent of the coordinate x_2 (shear horizontal polarization). The frequencies of the first few lowest-frequency acoustic surface shape resonances were calculated for modes of even and odd symmetry in x_1 in this structure, for several different sets of values of the material parameters describing it.

It was suggested in Ref. 1 that an experimental method for studying these acoustic surface shape resonances is provided by the scattering of a surface-skimming bulk transverse wave incident normally on the ridge depicted in Fig. 1. A surface-skimming bulk transverse wave is a wave of shear horizontal polarization that satisfies both the equation of motion of elasticity theory and a stress-free boundary condition at the planar surface of the medium supporting it. It is not, however, a surface acoustic wave and, indeed, is described by a displacement field that is independent of the coordinate normal to the surface.[2] The differential scattering cross section for the surface-skimming bulk transverse wave, examined as a function of its frequency at a fixed scattering angle, should possess structure at the (real part of the) frequencies of the acoustic surface shape resonances that would demonstrate their existence.

In this note we study theoretically the scattering of a surface-skimming bulk transverse wave incident normally on the ridge depicted in Fig. 1, and show that the suggestion advanced in Ref. 1 concerning the possibility of observing these resonances in this fashion is confirmed by the results.

The starting point for the scattering calculation is Eq. (I2.12), in which we replace the amplitude $A(k,\omega)$ by the sum $2\pi\delta(k-(\omega/c_2))+R(k,\omega)$, where c_2 is defined in Eq. (I2.14). The first term in this sum corresponds to the incident surface-skimming bulk transverse wave, while the second corresponds to the scattered wave. The use of this form for $A(k,\omega)$ in Eqs. (I2.16) and (I2.21), together with the recognition that the surface-skimming bulk transverse wave satisfies the stress-free boundary condition on the plane $x_3=0$, yields the following integral

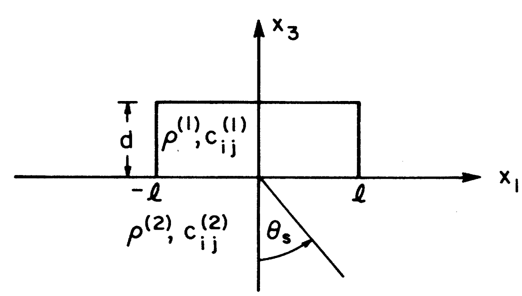

FIG. 1. The structure studied in this paper.

equation for the determination of the scattering amplitude $R(k,\omega)$:

$$R(k,\omega)=R^{(0)}(k,\omega)+\int\frac{dq}{2\pi}K(k|q)R(q,\omega)\ , \tag{1}$$

where

$$R^{(0)}(k,\omega)=\frac{l}{d}\frac{c_{44}^{(1)}}{c_{44}^{(2)}}\frac{1}{\alpha(k,\omega)}\times\sum_{n=0}^{\infty}\epsilon_n(\beta_n(\omega)d)\tan[\beta_n(\omega)d]\times S_n(k)S_n^*(\omega/c_2)\ , \tag{2a}$$

$$K(k|q)=\frac{l}{d}\frac{c_{44}^{(1)}}{c_{44}^{(2)}}\frac{1}{\alpha(k,\omega)}\sum_{n=0}^{\infty}\epsilon_n(\beta_n(\omega)d)\tan[\beta_n(\omega)d]\times S_n(k)S_n^*(q)\ . \tag{2b}$$

The functions ϵ_n, $\beta_n(\omega)$, $\alpha(k,\omega)$, and $S_n(k)$ have been defined in Eqs. (I2.9), (I2.10), (I2.13), and (I2.17), respectively.

Equation (1) is readily solved if we make use of the separable form of the kernel $K(k|q)$. Thus, if we define

$$c_n(\omega)=\epsilon_n(\beta_n(\omega)d)\tan[\beta_n(\omega)d]\ ,$$
$$\phi_n(k)=S_n(k)/\alpha(k,\omega)\ , \tag{3}$$
$$\psi_n(k)=S_n(k)\ ,$$

we can express $K(k|q)$ as

$$K(k|q)=\frac{l}{d}\frac{c_{44}^{(1)}}{c_{44}^{(2)}}\sum_{n=0}^{\infty}c_n(\omega)\phi_n(k)\psi_n^*(q)\ . \tag{4}$$

The solution of Eq. (1) can then be written as

$$R(k,\omega)=R^{(0)}(k,\omega)+\frac{l}{d}\frac{c_{44}^{(1)}}{c_{44}^{(2)}}\sum_{n=0}^{\infty}c_n(\omega)D_n\phi_n(k)\ , \tag{5}$$

where the coefficients $\{D_n\}$ are the solutions of the (infinite-order) matrix equation

$$\sum_{n=0}^{\infty}\left[\delta_{mn}-\frac{l}{d}\frac{c_{44}^{(1)}}{c_{44}^{(2)}}M_{mn}(\omega)c_n(\omega)\right]D_n=D_m^{(0)}\ , \tag{6}$$

with

$$D_m^{(0)}=\frac{l}{d}\frac{c_{44}^{(1)}}{c_{44}^{(2)}}\sum_{n=0}^{\infty}M_{mn}(\omega)c_n(\omega)S_n^*(\omega/c_2)\ , \tag{7a}$$

$$M_{mn}(\omega)=\int\frac{dk}{2\pi}\frac{S_m^*(k)S_n(k)}{\alpha(k,\omega)}\ . \tag{7b}$$

In practice, the infinite-order matrix equation (6) is replaced by an $N\times N$ matrix equation by restricting m and n to assume the values $0,1,2,\dots,N-1$, and increasing N until convergent results have been obtained.

To present our results in a fashion that reveals the excitation of acoustic surface shape resonances by the incident surface-skimming bulk transverse wave, we calculate the differential scattering cross section for the process considered here in the following way. The total energy in the incident wave crossing the plane $x_1=\text{const}$ (<0) per unit time is

$$P_{\text{inc}}=\int_{-L_2/2}^{L_2/2}dx_2\int_{-L_3}^{0}dx_3\,\text{Re}(P_1^c)_{\text{inc}}=\tfrac{1}{2}\rho^{(2)}c_2\omega^2L_2L_3\ , \tag{8}$$

where $\mathbf{P}^c$ is the complex, elastic Poynting vector,[3] L_2 is the width of the sample in the x_2 direction, and L_3 is its thickness along the x_3 axis. The total energy in the scat-

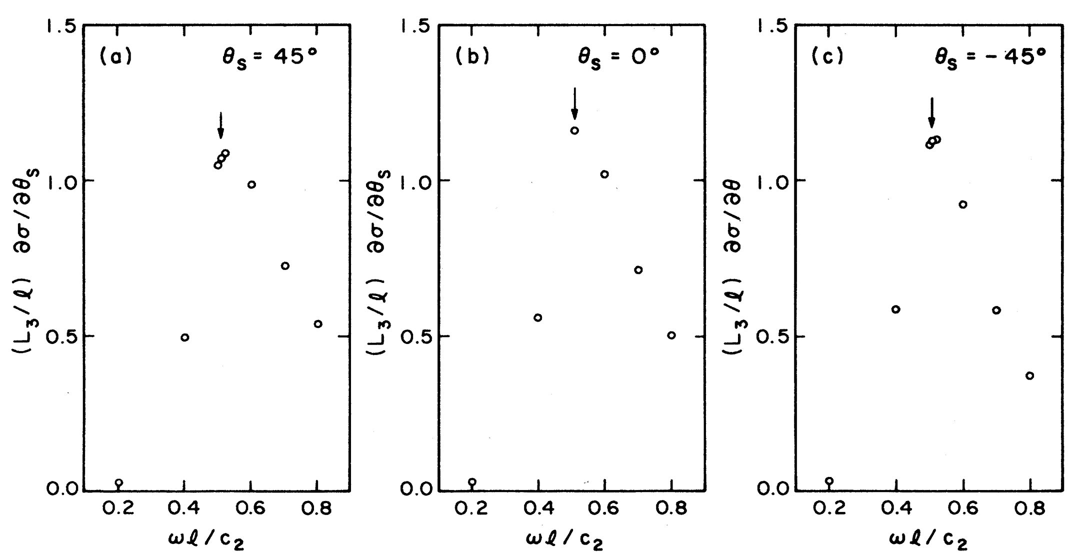

FIG. 2. The differential scattering cross section $d\sigma/d\theta_s$ as a function of the dimensionless frequency $\Omega=\omega l/c_2$. Results for scattering from the lowest-frequency mode of even symmetry for $\theta_s=$(a) 45°, (b) $\theta_s=0°$, and (c) $\theta_s=-45°$ are shown. In this case $\rho^{(1)}=\rho^{(2)}$, $c_{44}^{(1)}=c_{44}^{(2)}$, and $d=2l$.

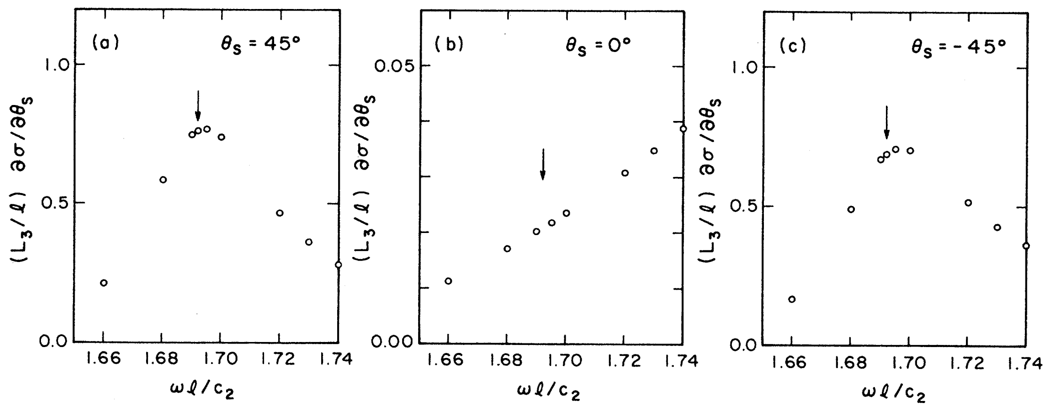

FIG. 3. The same as in Fig. 2 but for scattering from the lowest-frequency mode of odd symmetry. Notice that no resonance is observed in (b).

tered wave crossing the plane $x_3 = \text{const}\ (<0)$, i.e., the power carried away from the surface, per unit time is

$$P_{sc} = \int dx_1 \int_{-L_2/2}^{L_2/2} dx_2\, |\mathrm{Re}(P_3^c)_{sc}|$$
$$= \tfrac{1}{2}\rho^{(2)} c_2^2 \omega L_2 \int_{-\omega/c_2}^{\omega/c_2} dk \frac{i}{2\pi} \alpha(k,\omega) |R(k,\omega)|^2 \ . \quad (9)$$

If we introduce the scattering angle θ_s (Fig. 1), then we have that $k = (\omega/c_2)\sin\theta_s$, $\alpha(k\omega) = (\omega/c_2)\cos\theta_s$, $dk = (\omega/c_2)\cos\theta_s\, d\theta_s$. Equation (9) can then be rewritten in the form

$$P_{sc} = \frac{1}{4\pi}\rho^{(2)}\omega^3 L_2 \int_{-\pi/2}^{\pi/2} d\theta_s \cos^2\theta_s \times |R((\omega/c_2)\sin\theta_s, \omega)|^2 \quad (10a)$$

$$\equiv \int_{-\pi/2}^{\pi/2} d\theta_s\, P_{sc}(\theta_s) \ . \quad (10b)$$

The differential scattering cross section is defined by

$$\frac{\partial\sigma}{\partial\theta_s} = \frac{P_{sc}(\theta_s)}{P_{\mathrm{inc}}} = \frac{1}{L_3}\frac{\omega}{2\pi c_2}\cos^2\theta_s |R((\omega/c_2)\sin\theta_s,\omega)|^2$$
$$-\frac{\pi}{2} < \theta_s < \frac{\pi}{2} \ . \quad (11)$$

The proportionality of this cross section to L_3^{-1} is due to the fact that the incident wave is not a surface acoustic wave, so that the interaction volume is an ever-decreasing fraction of the total volume of the system as L_3 is increased.

In Figs. 2 and 3 we have plotted $\partial\sigma/\partial\theta_s$ as a function of the dimensionless frequency $\Omega = \omega l/c_2$ for scattering from modes of even and odd symmetry, respectively, for three scattering angles, in the case that $\rho^{(1)} = \rho^{(2)}$, $c_{44}^{(1)} = c_{44}^{(2)}$, i.e., for the case in which the ridge and substrate are made from the same material, and $d = 2l$. Peaks in the scattering cross sections corresponding to a scattered angle $\theta_s = \pm 45°$ are clearly visible when the frequency of the incident wave matches the real part of the frequencies of the lowest-frequency acoustic surface shape resonances of even and odd symmetry supported by this structure.[1] The latter are indicated by arrows in these figures. The small shift of the position of each peak in the scattering cross section from the real part of the frequency of the corresponding surface shape resonance is due to the frequency dependence of the factors multiplying the reciprocal of the determinant of the matrix of coefficients in Eq. (6) that enters the solution (5) through the coefficients $\{D_n\}$. The half-width of each peak at half maximum is essentially the imaginary part of the frequency of the surface shape resonance. The resonance is absent in the $\theta_s = 0°$ scattering from the mode of odd symmetry. In general it is found that for modes whose displacement fields are odd functions of x_1 no resonance is observed for $\theta_s = 0°$ scattering. It is seen that the amplitudes, and the detailed forms of the scattering cross sections, depend on the scattering angle, but the positions of the peaks they display are insensitive to changes in this variable.

The results obtained here suggest that the acoustic surface shape resonances studied in Ref. 1 can be observed in an experiment in which a surface-skimming bulk transverse wave, incident normally, is scattered from the ridge supporting these excitations.

A more favorable structure in which to observe acoustic surface shape resonances is likely to be a ridge on the surface of a thin plate that is bonded to a semi-infinite substrate of a material whose speed of transverse sound waves is larger than that of the film. Such a system supports a surface acoustic wave of shear horizontal polarization, viz., a Love wave,[4] which can be scattered from the ridge. The resulting differential cross section will be independent of any length, such as L_3, characterizing the size of the substrate, with a consequent enhancement of the structure associated with the acoustic surface shape resonances it possesses. The theory of such resonances and of the scattering of Love waves from the structure supporting them is now being worked out.

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[1]A. A. Maradudin, P. Ryan, and A. R. McGurn, Phys. Rev. B **38**, 3068 (1988). Equations from this paper will be prefixed by I.

[2]See, for example, A. A. Maradudin, in *Nonequilibrium Phonon Dynamics,* edited by W. E. Bron (Plenum, New York, 1985), p. 406.

[3]Ref. 2, p. 494.

[4]Ref. 2, p. 431.