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Spatial dispersion effects on the optical properties of an insulator–excitonic-semiconductor superlattice

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A transfer-matrix formalism is developed to study the effects of spatial dispersion in the optical properties of a superlattice made up by alternating an excitonic semiconductor with an insulator. We obtain the dispersion relation of the electromagnetic normal modes of an infinite superlattice and the reflectance for *p*-polarized light incident on a semi-infinite superlattice in the vicinity of an excitonic transition. Calculations are presented for the semiconductor CdS, and the results obtained with different choices of the additional boundary conditions and with the classical local model are compared. They display a very rich structure which is interpreted in terms of the different guided and surface waves supported by the semiconductor layers.

I. INTRODUCTION

The study of electromagnetic wave propagation in periodic superlattices has received considerable attention. Yeh *et al.*[1] first employed a diagonalization of the unit-cell translation operator to obtain the electromagnetic band structure of periodically stratified media made up of dispersionless dielectrics. Later, Camley and Mills[2] studied the collective excitations of superlattices made up of conducting layers with a frequency-dependent dielectric function alternating with dispersionless dielectrics. Surface plasmons on adjacent interfaces were found to couple through their evanescent electric fields to form bands of collective excitations of the whole structure. These modes are capable of transporting energy in the direction normal to the interfaces at frequencies for which each conductor is opaque.

The collective excitations of spatially dispersive systems are richer due to the presence of further energy-transport mechanisms. The modes arising from the coupling among guided and evanescent plasmons in nonlocal conducting superlattices were studied in the nonretarded limit by Eliasson *et al.*[3] and Mochán *et al.*,[4] who have recently developed a transfer-matrix formalism for studying the normal modes and the reflectance of nonlocal conductor-insulator superlattices. Spatial dispersion and retardation were accounted for within a hydrodynamic model for the conducting layers. A variety of modes consisting of coupled transverse, longitudinal, and surface waves were obtained. The formalism was later generalized to local-nonlocal[5] and nonlocal-nonlocal[6] conductor superlattices and to incorporate the excitation of electron-hole pairs.[7] The transfer-matrix formalism to study nonlocal heterostructures within the hydrodynamic model has also been developed by Abraham *et al.*[8]

In this paper we focus our attention on superlattices made up of nonlocal excitonic semiconductor layers alternating with dispersionless insulators. When light is incident on the semiconductors, it may create electron-hole pairs. These may form either large (Wannier-Mott) or small (Frenkel) excitons, i.e., bound pairs that can propagate in the crystal. Since the pioneering works of Pekar,[9] Hopfield,[10] and Hopfield and Thomas,[11] spatial dispersion has been included in the study of the optical properties of semiconductors near an excitonic transition. This can be done using a single oscillator model for the wave-vector- and frequency-dependent dielectric function

$$\epsilon_s(\mathbf{q},\omega)=\epsilon_0+\frac{\omega_p^2}{\omega_T^2+D|q|^2-\omega^2-i\omega\nu}\ , \tag{1}$$

where ϵ_0 is the background dielectric constant, $\hbar\omega_T$ is the energy required to create the exciton, ω_p^2 is a measure of the oscillator strength of the excitonic transition, ν is a damping factor, and $D|q|^2$ relates to the kinetic energy of the exciton, with $D=\hbar\omega_T/(m_e+m_h)$, and m_e and m_h the masses of the electron and hole, respectively. Maxwell's equations, together with Eq. (1), yield longitudinal bulk normal modes and additional transverse modes, beyond those appearing in local theories. These modes propagate in the bulk unaffected by each other, but they may couple among themselves at surfaces, where translational symmetry is reduced. It is important to account for this coupling in superlattices due to their large interfacial area to volume ratio; the purpose of this paper is to develop a theory to calculate the optical properties of excitonic semiconductor superlattices taking this coupling into account.

Due to the multitude of scattered waves at each semiconductor-insulator interface, the independent boundary conditions of electromagnetic origin, namely, the continuity of the tangential projections of the electric **E** and magnetic **H** fields,[12] are insufficient to completely determine the problem. Therefore, additional boundary conditions (ABC's) are required. In the past few years, several papers have addressed the ABC problem; Halevi and Fuchs[13] developed a generalized ABC formalism which includes surface scattering of the exciton through phenomenological parameters, and they applied it to calculations on semi-infinite semiconductors. Agranovich *et al.*[14] used a similar generalized ABC to study the reflectivity of semiconducting films deposited on metal substrates. Comparison between theory and experiment, pursuing the determination of the appropriate ABC, has been carried out by Halevi and Hernandez-Cocoletzi[15] and by Ruppin and Engleman.[16] In Ref. 15, the results favor the Pekar ABC (Ref. 9) and in Ref. 16 the results are ABC independent. It seems to us that the ABC problem is still open.

In this paper we develop a 2×2 transfer-matrix formulation similar to that reported in Ref. 4 to calculate the dispersion relation of the electromagnetic normal modes of infinite superlattices and the reflectance of semi-infinite superlattices made up of alternating insulator layers and excitonic semiconductor layers. Spatial dispersion effects are introduced through an excitonic dielectric function [Eq. (1)] and two well-known ABC's are employed, allowing for the coupling of the transverse normal modes of the insulator with the excitonic normal modes of the semiconductor. The organization of the paper is as follows: in Sec. II we construct the transfer matrix of the semiconductor layers, and we show how it can be employed to calculate the normal modes and the optical properties of the superlattice, in Sec. III we present results for an insulator-CdS superlattice, and we devote Sec. IV to conclusions.

II. THEORY

We consider here the superlattice shown in Fig. 1. The insulating layers I have dielectric constant ϵ_i and thickness d_i, while the excitonic semiconductor layers S have a wave-vector and frequency-dependent dielectric function $\epsilon_s(\mathbf{q},\omega)$ and thickness d_s. The I-S interfaces are parallel to the X-Y plane. We only take into account waves propagating along the X-Z plane whose electric fields lie on the same plane, that is, p-polarized and longitudinal waves. These are uncoupled to s-polarized waves, whose treatment is similar and somewhat simpler.

We start our calculation by constructing the transfer matrix of a semiconductor layer following a procedure similar to that of Refs. 4–6 and 8. As usual in optics, we confine our attention to waves with the same frequency ω and wave-vector projection $\mathbf{Q}=(Q,0,0)$ unto the X-Y plane. Given ω and Q, there are four p-polarized normal modes with wave vectors $(Q,0,\pm q_j)$; $j=1,2$, where $\pm q_j$ are the solutions of the transverse wave dispersion relation $q_j^2+Q^2=\epsilon_s(\mathbf{Q},q_j,\omega)\omega^2/c^2$. There are also two longitudinal modes with wave vector $(Q,0,\pm q_3)$, where q_3

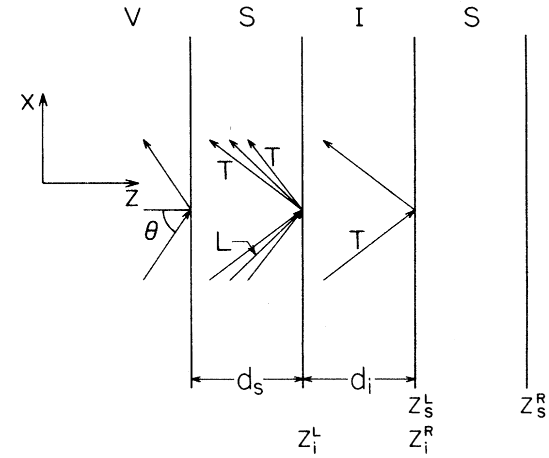

FIG. 1. Superlattice made up of excitonic semiconductor layers (S) of width d_s alternating with dispersionless insulator layers (I) of width d_i. The wave vectors of the two transverse (T) waves that may propagate in the insulators and those of the four transverse and two longitudinal (L) waves that may propagate in the semiconductors, as well as that of the light incident from vacuum (V) at the angle θ and that of the light reflected by the system, are shown schematically. The positions of the left and right boundaries of an insulator (z_i^L and z_i^R) and a semiconductor (z_s^L and z_s^R) layer are indicated as well as the coordinate system.

obeys the longitudinal dispersion relation $\epsilon_s(\mathbf{Q},q_3,\omega)=0$. We assume that the electromagnetic field is given by a superposition of these six waves, even near the surfaces of the semiconducting layer (the inclusion of a dead layer at the surfaces will be discussed below). Then the field everywhere inside the layer is determined by six independent field quantities at one point. We chose as independent quantities the tangential components of the electric and magnetic field, E_x and H_y, the excitonic polarization field P_x and P_z, and its normal derivative $\partial_z P_x$ and $\partial_z P_z$. These are related to the amplitudes $E_n^{\pm}$ on the right- (+) and left- (−) moving transverse ($n=1,2$) and longitudinal ($n=3$) modes through

$$E_x(z)=\sum_{n=1}^{3}(E_n^+e^{iq_nz}+a_nE_n^-e^{-iq_nz}) ,$$

$$H_y(z)=\sum_{j=1}^{2}Y_j(E_j^+e^{iq_jz}+E_j^-e^{-iq_jz}) ,$$

$$P_x(z)=\frac{\omega_p^2}{4\pi D}\sum_{n=1}^{3}X_n(E_n^+e^{iq_nz}+a_nE_n^-e^{-iq_nz}) , \tag{2}$$

$$P_z(z)=\frac{\omega_p^2}{4\pi D}\sum_{n=1}^{3}X_nD_n(-E_n^+e^{iq_nz}+a_nE_n^-e^{-iq_nz}) ,$$

where we have used Eq. (28) of Ref. 13 and Maxwell's equations. Here $a_1=a_2=-a_3=-1$, $Y_j=1/Z_j$, $Z_j=(q_jc)/(\epsilon_j\omega)$ is the surface impedance for transverse wave propagating with wave vector q_j, $\epsilon_j=\epsilon_s(Q,q_j,\omega)$, $D_j=Q/q_j$, $D_3=-q_3/Q$, $X_n=(q_n^2-\Gamma^2)^{-1}$, $\Gamma^2=(\omega^2+i\omega\nu-\omega_T^2)/D-Q^2$, and we let j take the values 1,2 and $n=1,2,3$.

We write Eqs. (2) as the block equation

$$\begin{bmatrix} \mathsf{F} \\ \mathsf{P} \\ \partial_z \mathsf{P} \end{bmatrix}_z = \mathsf{G} \begin{bmatrix} \mathsf{A}_1 \\ \mathsf{A}_2 \\ \mathsf{A}_3 \end{bmatrix}_z , \tag{3}$$

where $\mathsf{F}=(E_x,H_y)^T$ (T denotes transpose), $\mathsf{P}=(P_x,P_z)^T$, $\mathsf{A}_n=(E_n^+e^{iq_nz},E_n^-e^{-iq_nz})^T$, and

$$\mathsf{G}=\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 1 \\ Y_1 & Y_1 & Y_2 & Y_2 & 0 & 0 \\ X_1 & -X_1 & X_2 & -X_2 & X_3 & X_3 \\ -D_1X_1 & -D_1X_1 & -D_2X_2 & -D_2X_2 & D_3X_3 & -D_3X_3 \\ iq_1X_1 & iq_1X_1 & iq_2X_2 & iq_2X_2 & iq_3X_3 & -iq_3X_3 \\ -iq_1D_1X_1 & iq_1D_1X_1 & -iq_2D_2X_2 & iq_2D_2X_2 & iq_3D_3X_3 & iq_3D_3X_3 \end{bmatrix} . \tag{4}$$

Since $(\mathsf{A}_1,\mathsf{A}_2,\mathsf{A}_3)_z^T=\mathsf{T}(z-z')(\mathsf{A}_1,\mathsf{A}_2,\mathsf{A}_3)_{z'}^T$, where

$$\mathsf{T}(z)=\mathrm{diag}(e^{iq_1z},e^{-iq_1z},e^{iq_2z},e^{-iq_2z},e^{iq_3z},e^{-iq_3z})$$

and diag(. . .) is a diagonal matrix constructor, the fields at the left boundary of the layer z_s^L are linearly related to the fields at the right boundary $z_s^R=z_s^L+d_s$ through

$$\begin{bmatrix} \mathsf{F} \\ \mathsf{P} \\ \partial_z \mathsf{P} \end{bmatrix}_{z_s^R} = \mathsf{N}_s \begin{bmatrix} \mathsf{F} \\ \mathsf{P} \\ \partial_z \mathsf{P} \end{bmatrix}_{z_s^L} , \tag{5}$$

where we introduced the semiconductors's 6×6 transfer matrix N_s given by

$$\mathsf{N}_s=\mathsf{G}\mathsf{T}(d_s)\mathsf{G}^{-1} . \tag{6}$$

The field components E_x, H_y, P_x, P_z, ∂_zP_x, and ∂_zP_z, constitute a set of six independent field components in an infinite semiconductor. However, they are related among themselves by the ABC's imposed at the surfaces of a thin film. These ABC's may be employed to collapse the 6×6 transfer matrix N_s to a 2×2 matrix such as those appearing in classical optics, but which has embedded information on the multitude of modes which exist in the semiconductor near the excitonic transition due to the spatial dispersion of the system.[9] Following Agranovich *et al.*,[14] we write the two ABC's required at each interface as

$$\alpha\mathsf{P}+\partial_n\mathsf{P}=0 , \tag{7}$$

where α is a 2×2 matrix with complex components in general and ∂_n is the derivative along the outward normal direction $\hat{n}=\pm\hat{z}$. Equation (7) can be cast into the so-called generalized ABC form[13] by letting $\alpha_{\kappa\kappa}=-i\Gamma(1-U_\kappa)/(1+U_\kappa)$, $\kappa=x,z$, and $\alpha_{xz}=\alpha_{zx}=0$, where U_κ is the surface scattering parameter.

Writing the transfer matrix N_s as

$$\mathsf{N}_s=\begin{bmatrix} \mathsf{N}_{11} & \mathsf{N}_{12} & \mathsf{N}_{13} \\ \mathsf{N}_{21} & \mathsf{N}_{22} & \mathsf{N}_{23} \\ \mathsf{N}_{31} & \mathsf{N}_{32} & \mathsf{N}_{33} \end{bmatrix} , \tag{8}$$

where N_{nm} are 2×2 blocks, Eqs. (5) and (7) yield

$$\begin{aligned} \mathsf{P}(z_s^R)&=\mathsf{N}_{21}\mathsf{F}(z_s^L)+\mathsf{N}_{22}\mathsf{P}(z_s^L)+\mathsf{N}_{23}\alpha\mathsf{P}(z_s^L) , \\ -\alpha\mathsf{P}(z_s^R)&=\mathsf{N}_{31}\mathsf{F}(z_s^L)+\mathsf{N}_{32}\mathsf{P}(z_s^L)+\mathsf{N}_{33}\alpha\mathsf{P}(z_s^L) , \end{aligned} \tag{9}$$

which may be solved for $\mathsf{P}(z_s^L)$ in terms of $\mathsf{F}(z_s^L)$, and substituted back together with $\partial_z\mathsf{P}(z_s^L)=\alpha\mathsf{P}(z_s^L)$ at the r.h.s. of Eq. (5) to obtain a relation between $\mathsf{F}(z_s^R)$ and $\mathsf{F}(z_s^L)$. We write this relation as

$$\mathsf{F}(z_s^R)=\mathsf{M}_s\mathsf{F}(z_s^L) , \tag{10}$$

where M_s is the 2×2 transfer matrix of the semiconductor and it is given by

$$\mathsf{M}_s=\mathsf{N}_{11}-(\mathsf{N}_{12}+\mathsf{N}_{13}\alpha)\mathsf{S}^{-1}(\alpha\mathsf{N}_{21}+\mathsf{N}_{31}) , \tag{11}$$

where

$$\mathsf{S}=\alpha\mathsf{N}_{22}+\alpha\mathsf{N}_{23}\alpha+\mathsf{N}_{32}+\mathsf{N}_{33}\alpha . \tag{12}$$

Notice that in order to calculate the 2×2 transfer matrix of the semiconductor we only need to invert the 2×2 matrix S, which is simply made up of submatrices of the full 6×6 transfer matrix. The information on the ABC's is incorporated in M_s through the parameters α. From here on the calculation proceeds as in the local case.

Since an insulating layer can only sustain two p-polarized waves and has no longitudinal modes, it can be characterized by the usual 2×2 transfer matrix

$$\mathsf{M}_i=\begin{bmatrix} \cos(q_id_i) & iZ_i\sin(q_id_i) \\ iY_i\sin(q_id_i) & \cos(q_id_i) \end{bmatrix} , \tag{13}$$

defined so that

$$\mathsf{F}(z_i^R)=\mathsf{M}_i\mathsf{F}(z_i^L)\ , \tag{14}$$

where z_i^L is the position of the layer's left boundary, and $z_i^R=z_i^L+d_i$ that of its right boundary, $q_i^2=\epsilon_i\omega^2/c^2-Q^2$, $Z_i=(q_ic)/(\epsilon_i\omega)$, and $Y_i=1/Z_i$.

Finally, since E_x and H_y are continuous, the transfer matrix for one full superlattice period is[4]

$$\mathsf{M}=\mathsf{M}_i\mathsf{M}_s\ . \tag{15}$$

The presence of a dead layer can be easily incorporated by multiplying its insulatorlike transfer matrix on both sides of M_s before substituting into Eq. (15). The normal modes of the infinite superlattice are Bloch-like waves[17] such that $\mathsf{F}(z+d)=e^{ipd}\mathsf{F}(z)$, where $d=d_i+d_s$ is the period and

$$p=\arccos\left[\frac{1}{2d}\mathrm{Sp}\mathsf{M}\right] \tag{16}$$

is Bloch's (possibly complex) wave vector. The surface impedance of the semi-infinite superlattice is

$$Z_p=-\frac{M_{12}}{M_{11}-e^{ipd}}=-\frac{M_{22}-e^{ipd}}{M_{21}}\ , \tag{17}$$

in terms of which the reflectance is simply given by[18]

$$R_p=|r_p|^2=\left|\frac{Z_v-Z_p}{Z_v+Z_p}\right|^2\ , \tag{18}$$

with $Z_v=\cos\theta$ the surface impedance of vacuum and θ the angle of incidence.

III. RESULTS

In this section we present results calculated for superlattices made up of dispersionless insulator layers alternating with the semiconductors CdS, for frequencies in the vicinity of its $A_{n=1}$ excitonic transition. The dielectric function employed for the semiconductor is given by Eq. (1) with the following parameters:[19] $\epsilon_0=9.1$, $\hbar\omega_T=2.552$ eV, $\omega_p=0.115\,17\omega_T$, $D=5.3147\times10^{-6}c^2$, and $\nu=4.857\times10^{-5}\omega_T$. Being quite unimportant and for simplicity, we set $\epsilon_i=1$. We also disregard for the present the presence of dead layers.

In Fig. 2 we show the dispersion relation ω versus p of the normal modes for an infinite superlattice with $d_i=d_s=c/\omega_T=773$ Å, calculated using the Pekar[9] boundary conditions $\alpha=\infty$. For each ω we gave the value $Q=(\omega/c)\sin 60°$ to the parallel wave vector, so that the dispersion relation corresponds to those modes which may couple to light incident from vacuum at an angle $\theta=60°$. Although the figure contains a very rich structure in the neighborhood of the excitonic transition, this can be readily understood. For this purpose, we display in Fig. 3 the dispersion relation ω versus q of the transverse and longitudinal electromagnetic modes of an infinite CdS crystal.

At frequencies $\omega<\omega_T$ there are no propagating longitudinal waves in the semiconductor, and there is only one propagating long-wavelength transverse wave, similar to that appearing in local optics. Above ω_T its wavelength decreases rapidly so that the resonance condition $q_1=n\pi/d_s$ with n an integer is repeatedly met. These resonances are similar to guided transverse modes whose wavelength fits a half-integer number of times in the semiconductor layers' thickness. However, they can leak out of the semiconductor and into the insulator layers since we took $Q<(\epsilon_i)^{1/2}\omega/c$, and therefore they are not truly guided waves. The coupling among the transverse resonances of nearby semiconductor layers through the

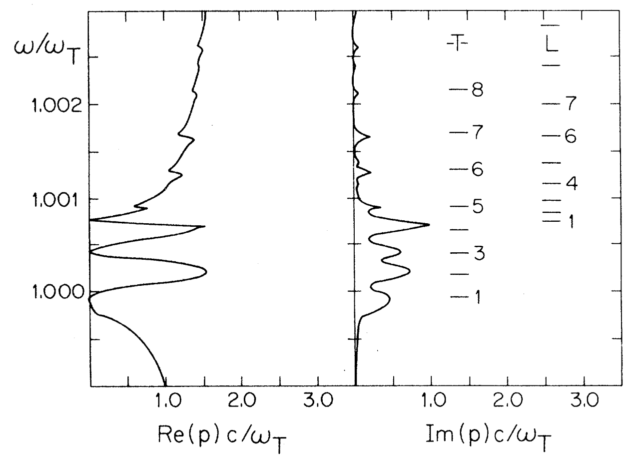

FIG. 2. Dispersion relation ω versus $p=\mathrm{Re}(p)+i\,\mathrm{Im}(p)$ of the electromagnetic normal modes of a periodic superlattice made up of alternating CdS and vacuum layers of width $d_s=d_i=773$ Å. The parallel component of the wave vector Q is chosen so that coupling with light incident at an angle of $\theta=60°$ is possible. The frequencies of the transverse (T) and longitudinal (L) resonances are indicated.

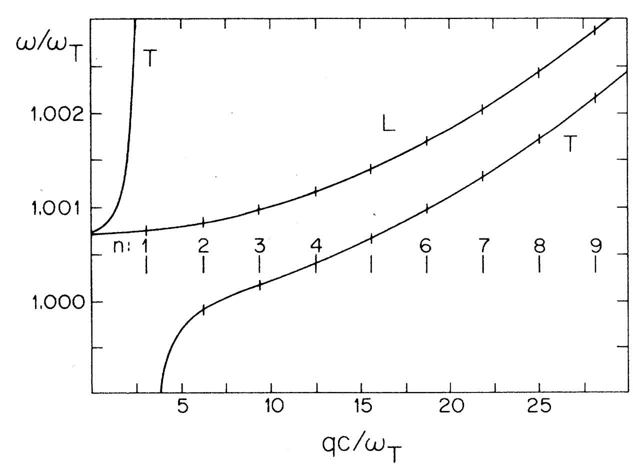

FIG. 3. Dispersion relation ω vs q of the longitudinal (L) and the two transverse (T) waves which propagate in CdS and may couple to light incident at an angle of $\theta=60°$. The wave vectors $n\pi/d_s$ which lead to resonant behavior in a semiconducting film of width $d_s=773$ Å are indicated.

fields induced in the insulator gives rise to most of the structure in Fig. 2: they correspond to maxima and in the imaginary part of Bloch's wave vector Im(p) and to maxima and minima in its real part Re(p).

Above $\omega_L = (\omega_T^2 + \omega_p^2/\epsilon_0)^{1/2}$ a propagating longitudinal mode appears. Its wave vector grows rapidly with ω originating longitudinal resonances whenever $q_3 = n\pi/d_s$. The longitudinal field is confined within the semiconductor layers since there is no longitudinal propagation in local media, so the resonances may be regarded as guided longitudinal modes. However, they couple at the layers' surface to transverse waves in the insulators, originating leakage and hopping between adjacent semiconductors in the superlattice. The longitudinal resonances are responsible for some features such as maxima, minima, and inflection points in Fig. 2. This structure is less prominent than that due to the transverse resonances since the transverse-transverse coupling is stronger than the longitudinal-transverse one.

Finally, above the critical frequency $\omega_c = [\omega_T^2 + \omega_p^2/(\epsilon_0 - \sin^2\theta)]^{1/2}$ a second transverse wave can propagate in the semiconductor; it has a long wavelength and its dispersion relation follows closely that obtained with the local model. Recall that in the latter there is a forbidden gap between ω_T and ω_c. This transverse wave constitutes another propagation mechanism whose consequences are a global decrease in Im(p), a steady increase in Re(p), and a reduction of the structure due to the transverse and longitudinal resonances above ω_c.

The description given above for the dispersion relation of the superlattice's normal modes remains qualitatively correct for other choices of parameters, requiring only minor changes. If the layers' widths are reduced, the distance between consecutive resonances opens out; conversely, for wider layers, larger ϵ_i, or higher ω there might be transverse resonances in the insulators and new resonances in the semiconductors due to the second, long-wavelength, transverse mode. Increasing θ leads to a stronger longitudinal-transverse coupling, and if Q grows beyond $(\epsilon_i)^{1/2}\omega/c$ the modes of a single semiconducting layer become true guided waves, although there is still coupling among adjacent layers through the transverse evanescent fields induced in the insulators.

In Fig. 4 we show the reflectance calculated for a semi-infinite superlattice upon which p-polarized light is incident at the angle $\theta = 60°$, using the same parameters as in Fig. 2. The structure displayed by the reflectance is in close correspondence to that shown by the dispersion relation: there is a series of peaks near the transverse resonance frequencies and smaller features near the longitudinal resonances. For comparison, we also plot in Fig. 4 the reflectance of a single semiconductor film. For $\omega > \omega_c$ the resonant structure is more prominent in the superlattice than in the film as could be expected. However, in the region $\omega_T < \omega < \omega_c$ the results for the film and for the superlattice are quite similar, since in this region each semiconductor layer has a large absorbance and the reflectance of the heterostructure is dominated by its first few layers.

In order to exhibit the consequences of different choices of ABC's, in Fig. 5 we show the reflectance of the

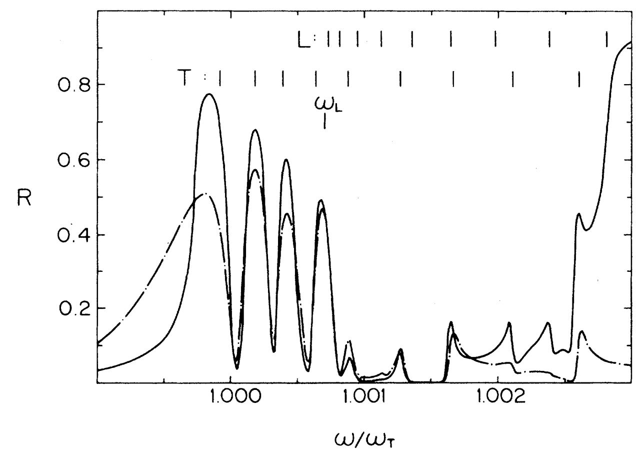

FIG. 4. Reflectance spectra of a semi-infinite superlattice (solid) and of a single CdS film (dashed), calculated with the Pekar ABC's. The frequencies of the transverse (T) and longitudinal (L) resonances are indicated, as well as $\omega_L \simeq \omega_c$.

same superlattice as in Fig. 4 using both the Pekar[9] ($\alpha = \infty$) and the Ting *et al.*[20] ($\alpha = 0$) boundary conditions. We also show the results of a local calculation ($D = 0$). While the Pekar results show maxima whenever the transverse resonance condition is met, the Ting *et al.* results show minima below ω_c and almost no structure at all above ω_c. On the other hand, the structure due to the transverse resonances is completely shifted to the region $\omega < \omega_T$ in the local calculation, since in the local case q_1 has a pole at ω_T. In this case there is also a new feature which can be seen as an inflection in the reflectance around $\omega/\omega_T \simeq 1.007$. This feature is unrelated to the resonances above; it has been discussed before in connection with conducting superlattices[4] and can be understood as follows.

In a local calculation there is always a gap between ω_T

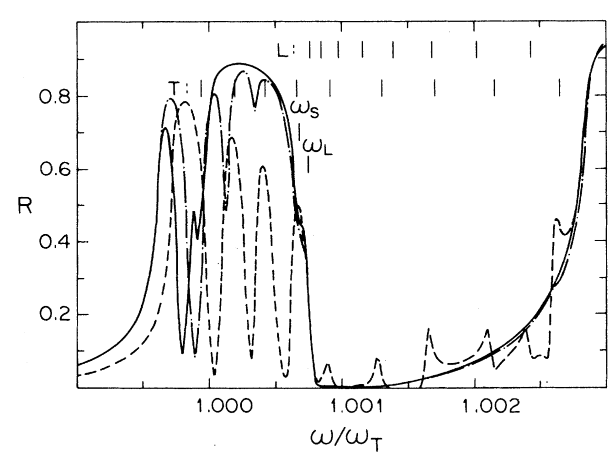

FIG. 5. Reflectance spectra of a semi-infinite superlattice calculated with the local model (solid), with the Pekar ABC's (dashed), and with the Ting *et al.* ABC's (dashed-dotted). The frequencies of the transverse (T) and longitudinal (L) resonances are indicated, as well as ω_L, ω_c, and ω_s.

and ω_L for which there are no propagating waves in the semiconductor. If Q is large enough, there are no propagating waves in the insulators either. In this case there might be surface electromagnetic waves (SEW's) moving along each semiconductor-insulator interface and decaying exponentially away from the surface. Their frequency in the nonretarded limit is $\omega_s = [\omega_T^2 + \omega_p^2/(\epsilon_0+\epsilon_i)]^{1/2} = 1.000\,66\omega_T$. In a superlattice, the SEW's of nearby interfaces couple among themselves through their evanescent fields, giving rise to two bulk bands since there are two interfaces per unit supercell. These bulk modes constitute a mechanism which can transport energy from interface to interface towards the bulk, thereby reducing the reflectance of the superlattice. We remark that although their physical origin consists of surface waves, these modes may exist even in the region $Q < (\epsilon_i)^{1/2}\omega/c$ for which the field is no longer evanescent in the insulators. However, in this retarded region the lower-frequency mode lies so close to ω_T that its effect is erased by the high dissipation and only the high-frequency mode is apparent in the reflectance. We also remark that, as shown in Fig. 3, in the nonlocal models there is no gap between ω_T and ω_L, so the generation of a well-defined SEW is precluded by the coupling of the evanescent fields to the short-wavelength propagating transverse wave.

IV. CONCLUSIONS

In the present paper we have developed a transfer-matrix formalism which allows the inclusion of spatial dispersion in the study of the optical properties of local-nonlocal layered heterostructures, taking into account the manifold electromagnetic waves that may exist in spatially dispersive systems and the coupling among themselves at interfaces. We developed the theory for longitudinal and p-polarized transverse waves in superlattices made up by alternating excitonic semiconductors with dispersionless insulators, although it is presented in a readily generalizable form. Our approach consists of building an $N\times N$ transfer matrix for a nonlocal layer, where N is the number of waves which may propagate in it, by identifying N independent field components ($N=6$ in the present case). Use of additional boundary conditions at its two interfaces allows us to reduce the size of the matrix to a simple 2×2 matrix such as that appearing in classical local optics, but with information on the spatial dispersion embedded in its components. Multiplying this matrix with the 2×2 matrices of adjacent layers and following well-known procedures, the optical properties of the system can be calculated.

We have applied the procedure above to calculate the 6×6 transfer matrix of an excitonic semiconductor in which two longitudinal and four p-polarized transverse waves may propagate uncoupled to another four s-polarized waves. The corresponding 2×2 matrix was obtained using a general parametrized expression for the ABC's. The theory was applied to the calculation of the dispersion relation of the normal modes of an infinite periodic superlattice made up of layers of the excitonic semiconductor CdS. We also calculated the reflectance of a semi-infinite superlattice. The results show a lot of structure around the excitonic transition frequency, which is easily explained in terms of guided-wave-like transverse and longitudinal resonances. A comparison with a single film showed that the reflectance spectrum of a superlattice has a more prominent structure outside, but is quite similar inside the region $\omega_T < \omega < \omega_c$, and that different choices of ABC's lead to very different results. Finally, a local calculation yielded shifted transverse resonances, no longitudinal ones, and absorbance due to excitation of coupled surface electromagnetic waves which are absent in spatially dispersive models.

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