RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 39, NUMBER 11 15 APRIL 1989-I

Negative magnetoresistance in the variable-range-hopping regime in *n*-type GaAs

F. Tremblay, M. Pepper, D. Ritchie, D. C. Peacock,*
J. E. F. Frost, and G. A. C. Jones
Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 OHE, United Kingdom

(Received 18 July 1988; revised manuscript received 27 September 1988)

Low-field magnetoresistance measurements for compensated, *n*-type three-dimensional GaAs with net donor concentration just below the metal-insulator transition show a quadratic field dependence for values of B less than 750 G. Temperature-dependent measurements in zero field show that transport is by variable-range hopping, and are consistent with the presence of a Coulomb gap which narrows close to the transition. It is found that the temperature dependence of the effective area in which the flux is enclosed is not related to the temperature dependence of the optimum hopping length. The relevant dephasing time appears not to be the hopping time. Rather, phase coherence may be lost after successive hops assisted by long-wavelength acoustic phonons or as a result of electron-electron interactions.

The low-field orbital negative magnetoresistance (NMR) observed in disordered conductors showing metallic conductivity has been widely studied and is understood in terms of a suppression of quantum-localization corrections in the Drude equation for the conductivity.[1,2] The available theory is not applicable to highly disordered systems where thermal-assisted hopping is the mode of conduction. Theories on quantum-interference effects in the variable-range-hopping (VRH) regime have recently been proposed.[3–5] In Refs. 3 and 4, interference among the paths associated with the hopping between two sites at a distance R_M apart, where R_M is the optimum hopping length, is considered and it is shown that this interference affects the hopping probability between these two sites. In the first case,[3] by averaging numerically the logarithm of the conductivity over many random impurity realizations, in the presence of a magnetic field, NMR, which is linear in the field, is found. In the second case,[4] the calculations are based on a critical path analysis and NMR, quadratic in the flux through an effective area $A = R_M^{3/2}\chi^{1/2}$, is predicted, where χ is the typical distance between impurities. This gives

$$\frac{\delta\rho}{\rho} \sim A^2B^2 , \tag{1}$$

where B is the applied magnetic field and where $R_M \sim T^{-1/(d+1)}$ in d dimensions[6] ($d=2,3$) for a noninteracting system. It is seen that the assumption that Eq. (1) is equally valid for a strongly interacting system where a Coulomb gap[7] is present and where $R_M \sim T^{-1/2}$, leads to the prediction of a different temperature dependence of the effective area in which the flux is enclosed. In both theories, the semiconductor is considered lightly doped and sufficiently far from the transition so that only the contribution of the forward-going paths is included, the contribution of closed loops being completely neglected. In Ref. 5, the case of impurity concentrations close to the critical concentration n_c is studied and an expression of the form (1) is also derived by considering coherent hops returning to the initial center and causing interference in a similar way to the metallic case. Here the temperature dependence of the proportionality factor A is a function of $R_M(T)$ and also of υ, the critical exponent of the conductivity and correlation length.

An orbital NMR quadratic in the magnetic field has been observed in the VRH in In_2O_{3-x} films.[8] Here we present the results of low-field magnetoresistance measurements performed on *n*-type three-dimensional GaAs samples where VRH is observed.

The samples used were molecular-beam-epitaxy (MBE) grown GaAs, on insulating substrates, all sufficiently lightly doped to lie below the metal-insulator transition for which the critical concentration is $n_c \sim 1.6\times10^{16}$ cm^{-3}. Their properties are listed in Table I. The values of N_D and N_A were determined from a single measurement of the Hall coefficient and resistivity at 77 K (Ref. 9). All samples were etched Hall bars with evaporated Au-Ni-Ge contacts for samples 4 and 5 and indium contacts for samples 1, 2, and 3. The temperature was measured with a resistance thermometer calibrated with an accuracy of ~10 mK and in good thermal contact with the sample via a copper block. The conductivity was measured using a four-terminal low-frequency technique and in the Ohmic regime. The uncertainty in the conductivity measurement was typically less than 0.05% due to the low noise level. The accuracy in the determination of the temperature and conductivity combined with the large number of experimental points taken (between 60 and 100 points) in the range 1.2–4.5 K, makes it possible to distinguish unambiguously between the exponent $x=\frac{1}{4}$ or $x=\frac{1}{2}$ in the law[6,7]

$$\rho = \rho_0 \exp(T_0/T)^x . \tag{2}$$

The resistivity ρ is plotted as a function of $T^{-1/2}$ and $T^{-1/4}$ for one representative sample (No. 5) in Fig. 1. To determine the best exponent, the value x was varied and then fit with the smallest square deviation obtained for each x value. These fits were sufficiently sharp at the best value of x for the sums of the square of the deviations to differ by a factor greater than 3 between the best value of x and $x \pm 0.13$, for all samples. The sum of the squares of

TABLE I. Thickness T, electronic concentration N, mobility $\mu(B=4\text{ kG})$, compensation, best exponent x [Eq. (2)], and best values of the parameters defined by Eq. (3).

Sample	T^a (μm)	$N_{300\text{ K}}$ (10^{16} cm^{-3})	$N_{77\text{ K}}$ (10^{16} cm^{-3})	$\mu_{300\text{ K}}$ (cm$^2V^{-1}$s^{-1})	$\mu_{77\text{ K}}$ (cm$^2V^{-1}$s^{-1})	N_A/N_D	x	α	$yB^2(K^\alpha)$ ($\alpha=1.22$)
1	3.9	1.4	1.2	6930	19000	0.20	0.26	1.23 ± 0.03	1.640
2	2.6	1.6	1.3	5840	16600	0.22	0.26	1.25 ± 0.03	1.210
3	6.25	0.79	0.60	6250	17700	0.43	0.62	1.19 ± 0.04	1.191
4	2.4		0.72		20000	0.31	0.51	1.22 ± 0.05	1.277
5	4	0.44	0.28	7100	39300	0.29	0.45	1.15 ± 0.07	1.043

[a] Assumed thickness taking depletion into account.

the deviations as a function of x is shown in Fig. 2 for samples 1 and 5 and the best value of x in each case is listed in Table I. It is found that the two samples with a doping level close to the critical concentration obey the Mott hopping law while the three most insulating samples obey the hopping law for an interacting system with a Coulomb gap in the density of states at the Fermi level. The value $x=\frac{1}{4}$ found for samples 1 and 2 can be explained by a narrowing of the minimum in the density of states near the transition. As the transition point is approached, the width of the energy band providing the maximum in conduction is expected to become greater than the width of the Coulomb gap and the system becomes noninteracting from the transport point of view. The $T^{-1/2}$ law in n-type GaAs has been observed earlier.[10,11] However, Benzaquen and co-workers[12,13] support the $T^{-1/4}$ law. In another system, evaluation of x for a compensated sample of InP with $n \sim n_c/3$ gave $x=\frac{1}{2}$ (Ref. 14) while a value of $x=\frac{1}{4}$ was found for a sample just on the insulating side of the metal-insulator transition.[15] This result is similar to the above result and also suggests a narrowing of the Coulomb gap near the transition.

Low-field magnetoresistance measurements show a variation of the resistance quadratic in the field and isotropic for all samples (Fig. 3). Results on the change in the resistance

$$\frac{\delta\rho}{\rho}=\frac{\rho(B)-\rho(0)}{\rho(0)}=yT^{-\alpha}B^2 \tag{3}$$

as a function of temperature for a chosen field of 720 G are presented in Fig. 4 and the best values of α are listed in Table I. α is constant (~ 1.22) within experimental error, that is the dephasing length $L_\phi \sim T^{-0.4}$ (from Refs. 1 and 4). This is unexpected as α should depend on the temperature dependence of the optimum hopping length, according to the above theories. This indicates that the relevant dephasing time in this system is not the expectation time for a hop.

Other theoretical[16,17] and experimental[18] results suggest that phase memory is not completely destroyed in hopping processes involving long-wavelength acoustic phonons where the energy change of the electron is very small, as is the case for variable-range hopping. Accord-

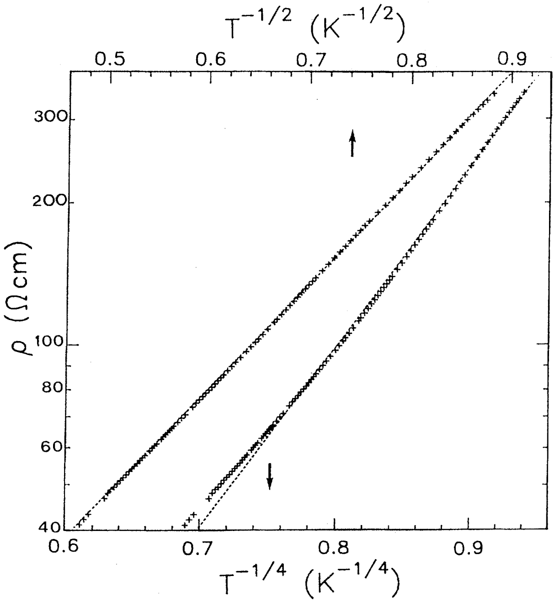

FIG. 1. Resistivity as a function of temperature for sample 5. The lines are least-squares fits to the data.

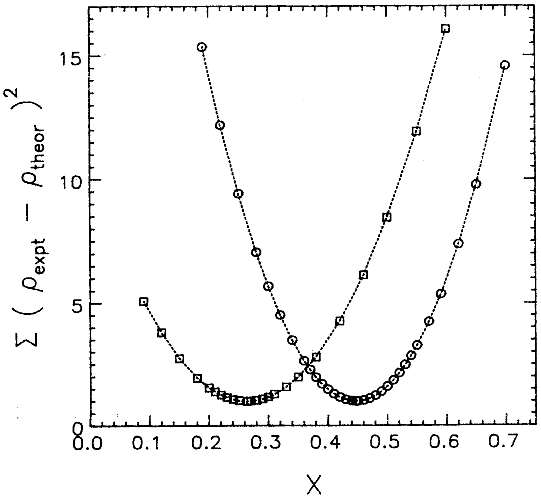

FIG. 2. $\Sigma(\rho_{\text{expt}}-\rho_{\text{theor}})^2$ as a function of x for sample 1 (□) and sample 5 (○). The minimum of the square of the deviations is normalized to 1. $[\Sigma(\rho_{\text{expt}}-\rho_{\text{theor}})^2]_{\text{min}}$ is equal to 2.95×10^{-5} (Ω cm)^2 and 50.3 (Ω cm)^2 for samples 1 and 5, respectively.

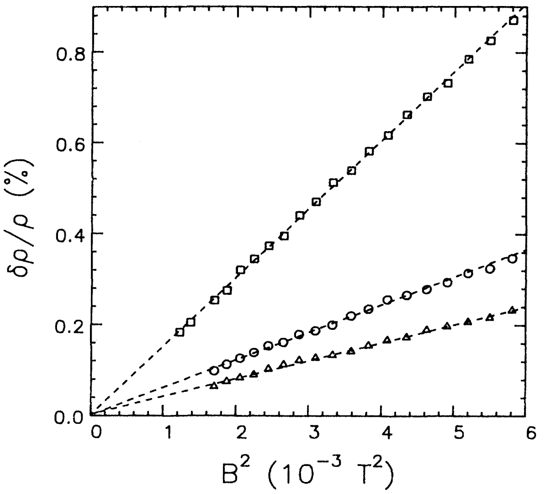

FIG. 3. $\delta\rho/\rho$ as a function of magnetic field $B < 775$ G for sample 3. The temperatures are 4.2 K (△), 3.1 K (○), 1.4 K (□), and the lines are least-squares fits to the data.

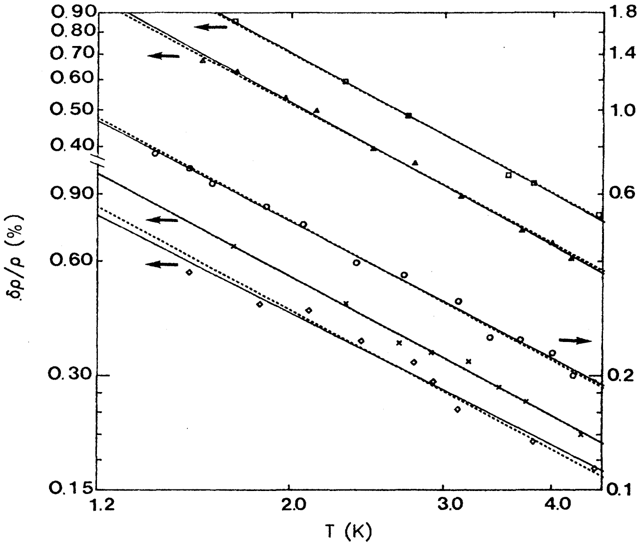

FIG. 4. $\delta\rho/\rho$ as a function of temperature for samples 1 (□), 2 (△), 3 (○), 4 (×), and 5 (◇). The applied magnetic field is equal to 720 G. The solid lines are the least-squares fits to the data, and the dotted lines are the least-squares fits to the data for $\alpha = 1.22$ in $\delta\rho/\rho = yT^{-\alpha}B^2$.

ing to Refs. 1 and 4, the deviation of the magnetoresistance data from Eq. (3) should appear at a field $B_c(T) \sim \hbar/eA(T)$. The expected deviation is observed and results indicate that $L_\phi \sim 1000$ Å and is of the same order of magnitude as the hopping length. This shows that the typical number of hops necessary to destroy phase memory must be small. It is also possible that the interaction of an electron with the electromagnetic field produced by all the other electrons can become dominant for the determination of the phase-breaking time. This could explain the single temperature dependence of the effective area found in all the samples investigated.

In conclusion, the temperature dependence of conductivity for n-type GaAs in the VRH regime is consistent with the presence of a Coulomb gap narrowing close to the transition. Low-field magnetoresistance measurements show a quadratic dependence on the field. The results of the temperature dependence of the magnetoresistance show that $L_\phi \sim T^{-0.4}$ and that the relevant dephasing time in this system is not the expected hopping time. The phase coherence may be lost after a few successive hops involving very low energy acoustic phonons or as a result of electron-electron interactions.

We are grateful for discussions with Y. Imry. F.T. acknowledges partial financial support from the Natural Science and Engineering Research Council of Canada. This work was supported by the Science and Engineering Research Council of the United Kingdom.

---

*Also at GEC Hirst Research Centre, Wembley, Middlesex HA9 7PP, United Kingdom.

[1] B. L. Altshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmelnitskii, Zh. Eksp. Teor. Fiz. **81**, 768 (1981) [Sov. Phys. JETP **54**, 411 (1981)].

[2] L. P. Gorkov, A. I. Larkin, and D. E. Khmelnitskii, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].

[3] V. L. Nguen, B. Z. Spivak, and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **89**, 1770 (1985) [Sov. Phys. JETP **62**, 1021 (1985)].

[4] U. Sivan, O. Entin-Wohlman, and Y. Imry, Phys. Rev. Lett. **60**, 1566 (1988).

[5] B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitskii, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 157 (1982) [JETP Lett. **36**, 195 (1982)].

[6] N. F. Mott, J. Non-Cryst. Solids **1**, 1 (1968).

[7] A. L. Efros and B. I. Shklovskii, J. Phys. C **8**, L49 (1975).

[8] Z. Ovadyahu, Phys. Rev. B **33**, 6552 (1986).

[9] G. E. Stillman and C. M. Wolfe, Thin Solid Films **31**, 69 (1976).

[10] R. Rentzoch *et al.*, Phys. Status Solidi (b) **137**, 691 (1986).

[11] D. Redfield, Phys. Rev. Lett. **30**, 1319 (1973).

[12] M. Benzaquen and D. Walsh, Phys. Rev. B **30**, 7287 (1984).

[13] M. Benzaquen, D. Walsh, and K. Mazuruk, Phys. Rev. B **36**, 4748 (1987).

[14] D. M. Finlayson and P. J. Mason, J. Phys. C **19**, L299 (1986).

[15] D. M. Finlayson, P. J. Mason, and I. F. Mohammed, J. Phys. C **20**, L607 (1987).

[16] X. C. Xie and S. Das Sarma, Phys. Rev. B **36**, 9326 (1987).

[17] P. Vasilopoulos and C. M. van Vliet, Phys. Rev. B **34**, 5217 (1986).

[18] Ya. B. Poyarkov, V. Ya. Kontarev, I. P. Krylov, and Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 291 (1986) [JETP Lett. **44**, 373 (1986)].