Hot-electron magnetophonon resonance in a two-dimensional electron gas

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When the broadening of the Landau levels is small, we find that with increasing electron velocity (or equivalently applied electric field) the magnetophonon resonance peak splits up into two peaks. Considering the separate contributions of LO-phonon absorption and emission processes to the nonlinear momentum balance equation, we propose a *new physical interpretation* for this splitting. A shift and no splitting of the magnetophonon resonant peaks is found when the broadening of the Landau levels is large. Several existing theoretical models are reconciled, and it turns out that the physical behavior for the two-dimensional (2D) case and the 3D case is qualitatively the same when the Landau-level broadening is small. We also study in detail the dependence of the position and amplitude of the nonlinear magnetophonon resonance peaks on lattice temperature, broadening parameter, and electron velocity.

I. INTRODUCTION

The magnetophonon effect is a powerful spectroscopic tool. The nonlinear magnetophonon effect has been investigated in considerable detail for three-dimensional (3D) systems[1-3] at low temperatures, where the LO-phonon emission process dominates. The review papers of Nicholas[4,5] cover recent experimental and theoretical results on the magnetophonon resonance effect in two-dimensional (2D) systems.

Several aspects of the magnetophonon resonance effect in the two-dimensional electron gas (2DEG) in a GaAs-$Al_xGa_{1-x}As$ single heterojunction remain unclear. For example, it is not clear which phonon modes are involved, nor is it understood how important interface phonons are. The cyclotron resonance measurements of Langerak *et al.*[6], which were performed at low temperature, clearly show coupling of the electrons to the bulk LO phonon. On the other hand, the phonon frequencies derived from magnetophonon resonance measurements[5] fall in between the bulk LO-phonon and TO-phonon frequencies. Coupling to TO phonons was suggested in Ref. 5, but no theoretical arguments can be found to justify this. A related problem concerns the role of the concentration of electrons. The experimental data[5] on the position and the damping of the magnetophonon resonance (MPR) peaks depend strongly on the electron concentration, which may indicate that the effective electron–polar-LO-phonon interaction is strongly electron-concentration dependent. The latter was confirmed by the recent low-temperature cyclotron resonance experiments of Langerak *et al.*[6] and agrees with the many-electron theory of Ref. 7.

The problems mentioned above relate to the normal as well as to the hot-electron magnetophonon resonance effect. The "normal" and "hot electron" refer to the linear-response regime (very small applied electric field) and to the nonlinear, non-Ohmic regime (large applied electric field, respectively. The study of the hot-electron magnetophonon effect in a 2DEG is still at an initial stage and only few works have been devoted to it so far. In view of the potentially useful information it contains (about the energy-loss processes in the 2DEG, the carrier-distribution function, and the damping of the oscillations), further study is necessary. As in Ref. 8, we will employ a drifted-Maxwellian form of the electron-momentum distribution function, which has the advantage of leading to tractable expressions. This ansatz is used as a starting point and, although its applicability to the full hot-electron regime is limited, we believe it leads to the correct physical trends (for a discussion, see, e.g., Refs. 1 and 8).

In Ref. 8 (hereafter referred to as I) we discussed the magnetophonon resonances in the linear and nonlinear resistivity of a GaAs single heterojunction in the framework of the momentum-balance equation, which in the linear limit reduces to the Kubo formula.[9] The shift of the positions of the maxima and minima was studied as a function of the average electron velocity (quasi-2DEG at $T=77$ and 200 K). For large velocities [$v/v_{LO}>0.1$ with $v_{LO}=(2\hbar\omega_{LO}/m^*)^{1/2}$] it was predicted that the magnetophonon resonance maxima in the resistivity convert into minima and vice versa. This conversion occurs at lower velocities for the lower-field oscillations in agreement with the findings of Vasilopoulos *et al.*[10] for GaAs quantum well. This prediction was confirmed by the experimental observations by Leadley *et al.*[11] of the hot-electron magnetophonon resonance effect on a low-carrier-density GaAs-$Al_xGa_{1-x}As$ single heterojunction at intermediate temperatures ($40<T<100$ K, where Shubnikov–de Haas oscillations can be excluded). They found the following interesting results. (1) At low electric fields (linear regime), the magnetoresistance exhibits a series of maxima at $N\hbar\omega_c=\hbar\omega_{LO}$, which are converted into minima (at the same magnetic field) in the hot-electron (nonlinear) regime. Lower electric fields are needed to induce this conversion from maximum into minimum for larger N. (2) With increasing electric field the MPR amplitude increases rapidly (approximately as E^2). A maximum in the MPR amplitude was observed around 60 K. (3) In samples with higher electron densities, additional oscillations occur which were attributed to interelectric subband scattering.

In the three-dimensional electron gas (3DEG) a similar conversion of the magnetophonon resonance maxima into minima with increasing electric field was established in the experiments of Eaves *et al.*[12] on an n^+-n^--n^+ GaAs structure. This case was modeled by Mori *et al.*[13] by using the Kubo formalism. He expanded the oscillatory part of the nonlinear conductivity to first order in the electric field and a splitting of the magnetophonon resonance peaks was found, in agreement with experiment. However, no physical interpretation was given for the splitting of the peaks, which, moreover, only occurred in the second derivative of the conductivity with respect to the magnetic field. In contrast to this splitting found by Mori *et al.* for a 3DEG, our theoretical results in I for a 2DEG, the model of Vasilopoulos *et al.*,[10] and the experimental results of Leadley *et al.*[11] give only evidence for a shift (not a splitting) of the magnetophonon resonance peaks in the 2DEG. Thus the 2D case appears to behave differently in this aspect from the 3D case. At first sight this looks unlikely, since up to now there is no evidence[4,5] for a qualitative difference of the magnetophonon effect between the 2D and 3D cases. This discrepancy motivated us to investigate this point in more detail for the 2D case. We found that the value of the Landau-level broadening can lead to different hot-electron MPR behavior. It turns out that in the case of small broadening of the Landau levels the magnetophonon resonance peaks split into two peaks with increasing average electron velocity, one of which is a consequence of the contribution of the LO-phonon absorption processes to the nonlinear magnetoresistance, while the other peak appears because of contributions from the emission processes.

The aim of the present paper is to analyze in detail the nonlinear magnetophonon effect in the resistance of a 2D electron system, for which the interaction with bulk LO phonons is the dominant scattering mechanism. We will focus on two aspects: (1) the basic physical mechanism behind the shift and splitting of the magnetophonon resonance peaks, and (2) detailed numerical results for the qualitative behavior of the nonlinear magnetophonon resonance effect as a function of all the relevant physical parameters. The momentum-balance equation approach will be adopted as a working frame. Numerical calculations will be performed for a 2DEG in a single-interface GaAs system.

The organization of the present paper is as follows. The shift of the magnetophonon resonance peak positions with average electron velocity is analyzed in Sec. II in terms of the separate contribution of LO-phonon emission and absorption processes to the momentum balance. Section III contains detailed numerical results for the dependence of the position and amplitude of the magnetophonon resonance extrema on temperature, broadening parameter, and average electron velocity. Most of these dependencies will turn out to be described by simple analytical expressions as derived from the momentum-balance equation. A comparison is made with other theories and available experimental data. Our conclusions are presented in Sec. IV.

II. INTERPRETATION OF THE MAGNETOPHONON RESONANCE PEAKS IN THE NONLINEAR REGIME

In I we studied linear and nonlinear magnetophonon resonance within a momentum-balance approach. For clarity we list the approximations which were made: (1) an effective-mass approximation for the electrons; (2) only the first electric subband is populated, which is valid for not-too-high electron densities; (3) screening is neglected; (4) Boltzmann statistics is assumed, which for $T>77$ K was found to give almost the same results as Fermi statistics; (5) the LO phonons are taken to be the bulk phonons (3D) of GaAs; and (6) the first-order Born approximation for weak electron–LO-phonon interaction is used, which is valid for GaAs with $\alpha \approx 0.06$. Within these approximations the nonlinear momentum-balance equation can be written as

$$e[\mathbf{E}+(\mathbf{v}\times\mathbf{B})]=\mathbf{F}(\mathbf{v}) , \tag{1}$$

where $\mathbf{E}$ is the total electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{v}$ is the average electron velocity, $-e$ is the electron charge, and $\mathbf{F}(\mathbf{v})$ is the average force exerted on the electron by the interaction with LO phonons,

$$\mathbf{F}(\mathbf{v})=\frac{2}{N_e}\sum_{\mathbf{q}}\frac{\mathbf{q}}{\hbar}|V_{\mathbf{q}}|^2[n(\omega_{\mathbf{q}})-n(\omega_{\mathbf{q}}-\mathbf{q}\cdot\mathbf{v})] \times \mathrm{Im}[D^r(\mathbf{q},\omega_{\mathbf{q}}-\mathbf{q}\cdot\mathbf{v})] , \tag{2}$$

with $\mathrm{Im}[D^r(\mathbf{q},\omega)]$ the imaginary part of the retarded density-density correlation function. $V_{\mathbf{q}}$ is the electron-phonon–interaction Fourier coefficient, N_e the electron density, $\omega_{\mathbf{q}}$ the phonon frequency with phonon wave vector $\mathbf{q}$, and $n(\omega_{\mathbf{q}})$ the phonon occupation number. Inserting the expression for the retarded density-density correlation function in Eq. (2) we find

$$\mathbf{f}(\mathbf{v})=\frac{e^{\beta\mu}}{\pi l^2 N_e}\sum_{\mathbf{q}}\frac{\mathbf{q}}{\hbar}|V_{\mathbf{q}}|^2\sum_{n,m=0}J_{n,m}(l^2q^2/2)I_E , \tag{3}$$

with μ the chemical potential, and where

$$I_E=\int_{-\infty}^{\infty}dE e^{-\beta E}n(\omega_{\mathrm{LO}})(1-e^{\beta\mathbf{q}\cdot\mathbf{v}}) \times \mathrm{Im}[G_n(E+\hbar\omega_{\mathrm{LO}}-\hbar\mathbf{q}\cdot\mathbf{v})]\mathrm{Im}[G_m(E)] , \tag{4}$$

where we introduced

$$J_{n,n+j}(x)=[n!/(n+j)!]x^j e^{-x}[L_n^j(x)]^2 ,$$

$L_n^j(x)$ is the associated Laguerre polynomial, $l=(\hbar c/eB)^{1/2}$ is the magnetic length, and $\mathrm{Im}[G_n(E)]$ is the imaginary part of the Green's function for Landau level n, which for Gaussian broadening of the electronic density of states equals $\sqrt{2/\pi}(1/\Gamma_G)\exp[-2(E-\varepsilon_n)^2/\Gamma_G^2]$ where ε_n is the energy of the nth Landau level and $\Gamma_G=(2\hbar\omega_c\Gamma_0/\pi)^{1/2}$.

In I, Eq. (1) was solved for $\mathbf{v}$ for rather large broadening ($\Gamma_0=3.7$ meV). Here we will be concerned with the opposite limit of very small broadening, in the hope of obtaining evidence either for a shift or for a splitting of the MPR peaks as was found theoretically for the 3DEG by Mori *et al.*[13] Figure 1 shows the nonlinear resistivity

ρ_{xx} (top) and its second derivative with respect to the magnetic field (bottom) as a function of the cyclotron frequency for $T=140$ K and $\Gamma_0=0.4$ meV. Results are displayed in arbitrary units for different values of the average electron velocity as indicated on the left of the figure. Peaks are found in the resistivity for $\omega_c=\omega_{\rm LO}/N$, where $N=n-m$. Let us concentrate on the $N=2$ peak, which occurs at $\omega_c/\omega_{\rm LO}\approx 0.5$. With increasing velocity we found that for $v/v_{\rm LO}\leq 0.05$ this peak splits into two peaks. This is in contrast with the large-broadening case, discussed in I, where we found that no splitting occurs but only a shift of the peak to smaller ω_c values (smaller magnetic fields). This splitting is more pronounced in the second derivative, as shown in the bottom part of Fig. 1. Notice that already for $v/v_{\rm LO}\geq 0.03$ the second derivative of ρ_{xx} shows indications of a splitting. We will call the down-shifted peaks (shifted to *smaller* magnetic fields) the N^- peaks and the upshifted ones the N^+ peaks. Around $\omega_c/\omega_{\rm LO}\approx 1$ ($N=1$) the splitting occurs for larger velocities, i.e., $v/v_{\rm LO}\geq 0.05$.

These results seem to reconcile the 2D and 3D cases, at least in the sense that in both cases there is in principle a splitting of the peaks (not a shift of the single peak). Extending the model of Mori *et al.*[13] to two dimensions we can estimate the MPR peak positions:

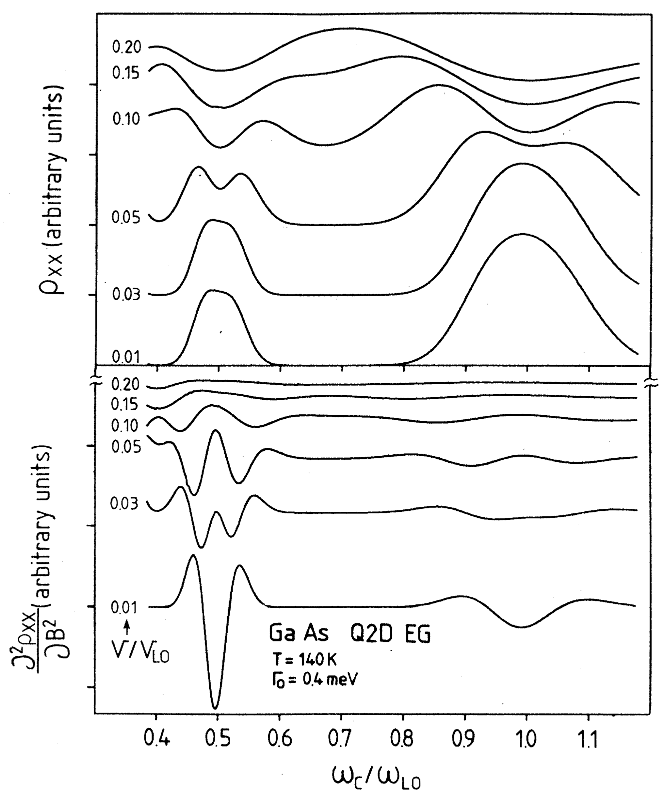

FIG. 1. The nonlinear resistivity ρ_{xx} (top) and its second derivative with respect to the magnetic field (bottom) as a function of the cyclotron frequency at $T=140$ K and $\Gamma_0=0.4$ meV. Results are displayed in arbitrary units for different values of the average electron velocity as indicated on the left.

$$\frac{\omega_{\rm LO}}{\omega_c}\pm\frac{\sqrt{3}}{2}A\left[\frac{\omega_{\rm LO}}{\omega_c}+\frac{3}{2}\right]^{1/2}=N\ , \tag{5a}$$

where $A=\sqrt{2}eEl/\hbar\omega_c$. For comparison we also display the result [Eq. (16) of Ref. 13] for the 3DEG:

$$\frac{\omega_{\rm LO}}{\omega_c}\pm\frac{\sqrt{3}}{2}A\left[\frac{\omega_{\rm LO}}{\omega_c}+1\right]^{1/2}=N\ . \tag{5b}$$

It is clear from Eqs. (5a) and (5b) that the qualitative behavior of the ideal 2DEG and the 3DEG is the same.

Next we will discuss the physical origin of the splitting of the MPR peaks. To this end we write Eq. (1) such that the contributions form LO-phonon emission and absorption processes to the force $\mathbf{F}(\mathbf{v})$ are separated. This separation is achieved first by making the transformation $E\to E'=E+\hbar\omega_{\rm LO}-\hbar\mathbf{q}\cdot\mathbf{v}$ and second by interchanging the Landau-level indices n and m. This results in

$$\begin{aligned}I_E=\int_{-\infty}^{\infty}dEe^{-\beta E}&{\rm Im}[G_m(E)]\\&\times\{n(\omega_{\rm LO}){\rm Im}[G_n(E+\hbar\omega_{\rm LO}-\hbar\mathbf{q}\cdot\mathbf{v})]\\&\quad-[1+n(\omega_{\rm LO})]{\rm Im}[G_n(E-\hbar\omega_{\rm LO}+\hbar\mathbf{q}\cdot\mathbf{v})]\}\ ,\end{aligned} \tag{6}$$

where the first term between curly brackets represents the process of absorption of an LO phonon by the electron, while the second term represents the corresponding emission process.

The numerical results for the magnetoresistivity as obtained from Eqs. (1), (3), and (6) are displayed in Fig. 2. The top part of this figure depicts the separate contribution of emission and absorption processes to the nonlinear resistivity ρ_{xx} as a function of cyclotron frequency at $T=140$ K and $\Gamma_0=0.4$ meV. The total nonlinear resistivity $\rho_{xx}=\rho_{xx}^{\rm em}+\rho_{xx}^{\rm abs}$ is also displayed, and at the bottom of the figure is displayed its second derivative with respect to the magnetic field. Results are displayed for a fixed value of the average electron velocity $v/v_{\rm LO}=0.05$. From these data the following physical picture emerges: the N^- peaks (in Figs. 1 and 2 with $N=1,2$) can be associated mainly with absorption processes, while the N^+ peaks result from the contribution of processes in which LO phonons are emitted. Note that if the MPR peaks are split then the $N=1^+$ peak should be observable experimentally. On the other hand, if the peaks are down shifted in magnetic field as the electric field is increased, then there should be no observable $N=1^+$ peak. This is true even for rather large values of the broadening parameter [for which the N^+ and the $(N-1)^-$ peaks are seen as only one peak] since for the $N=1^+$ peak there is no corresponding $(N-1)^-=0^-$ peak. In practice this requires measurements in high magnetic fields: e.g., for GaAs systems $20<B<30$ T.

Note that a similar behavior will also be observed in the magnetic-field dependence of the warm-electron coefficient β_v, which is an experimentally measurable quantity[14] [see Figs. 9(a) and 9(b) of I with $\Gamma_0=0.9$ meV and $X_{BG}=0$]. Note also that in Refs. 10 and 13 equipartition is assumed for the LO phonons. This assumption is

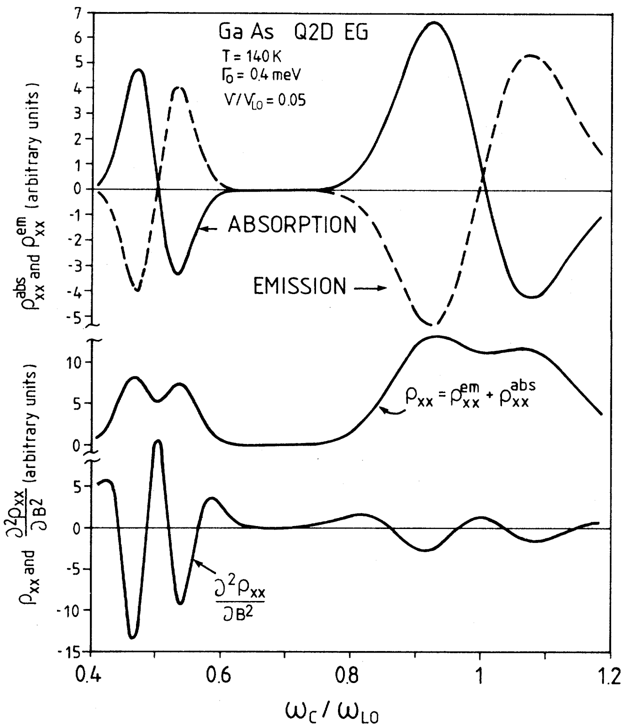

FIG. 2. The separate contribution to the nonlinear resistivity ρ_{xx} of emission and absorption processes (top) as a function of cyclotron frequency at $T=140$ K and $\Gamma_0=0.4$ meV. The total nonlinear resistivity $\rho_{xx}=\rho_{xx}^{\text{em}}+\rho_{xx}^{\text{abs}}$ and its second derivative with respect to the magnetic field (bottom) are also displayed. Results are given in arbitrary units for a fixed value of the average electron velocity $v/v_{\text{LO}}=0.05$.

valid in the high-temperature limit $k_BT \gg \hbar\omega_{\text{LO}}$. In the case of GaAs this requires $T \gg 400$ K, while in the experiments of Ref. 11 the temperature is below 100 K.

For large values of Γ_0, e.g., in Fig. 10 of I we took $\Gamma_0=3.7$ meV, we found in I that the MPR peaks in the resistivity shift, but there is no splitting. This is in contrast with Fig. 1, where for $\Gamma_0=0.4$ meV the MPR maxima are found to split into two peaks with increasing average electron velocity. We checked that the second derivative of Fig. 10 of I shows a trace of the splitting, but due to the large broadening this is not found back in the resistivity itself. This leads us to the conclusion that the peaks in the resistivity of I are split even for very large broadening, but the N^+ and $(N+1)^-$ peaks are not resolved (they have merged for all velocities), while for extremely small broadening the splitting is very clearly resolved for small average velocities.

It is not clear what are realistic values for the broadening parameter. An estimation of Γ_0 for $\omega_c/\omega_{\text{LO}} \gg 1$ from the contribution of electron–LO-phonon scattering along the lines of Ref. 10 leads for GaAs to the value $\Gamma_0=1.0$ meV at $T=100$ K and for a quantum-well width of $L_z=100$ Å. For higher temperatures this estimation leads to $\Gamma_0 \sim \sqrt{T}$. This is in contrast to the experimental cyclotron resonance data of Brummell,[15] which give $\Gamma_0 \approx 0.2$ meV at $T=100$ K and the temperature dependence is $T^{1.7}$. Turberfield[16] deduced a $\Gamma_0 \approx 16.0$ meV from measurements of the energy relaxation in a magnetic field. Brummell *et al.*[17] and Gregoris *et al.*[18] find that the damping of the MPR oscillations has a significant dependence on electron concentration.

We extracted a Γ_0-versus-N_e relation by fitting the experimental electron-density dependence of the $N=3$ MPR amplitude at $T=180$ K of Ref. 17 to our theoretical Γ_0 dependence of the MPR amplitude. The experimental data show that the MPR amplitude decreases monotonically with increasing electron density for $N_e > 6\times10^{10}$ cm^{-2}. For this range of electron densities the fit results in a linear relation between Γ_0 and N_e, in qualitative agreement with recent findings of Hamaguchi *et al.*,[19] who included Thomas-Fermi static screening of the remote impurities.

In the following a simple argument based on Eqs. (3) and (6) will be given which contains most of the above results. In the case of zero broadening the density of states $\text{Im}[G_n(E)]$ are Dirac δ functions. After performing the energy integral in this case, Eq. (6) takes the very simple form

$$I_E \sim \delta(\varepsilon_m-(\varepsilon_n \mp \hbar\omega_{\text{LO}}+\hbar\mathbf{q}\cdot\mathbf{v}))\ , \tag{7}$$

where the minus sign corresponds to the absorption and the plus sign to the emission process. Equation (7) expresses the conservation of energy for the process of absorption or emission of a LO phonon. Equation (2) contains a weighted average of the δ function over $\mathbf{q}$. This averaging is described approximately here by taking $\langle \mathbf{q}\cdot\mathbf{v}\rangle \approx q_{\text{LO}}v$, with $\hbar q_{\text{LO}}=m^*v_{\text{LO}}$. Equation (7) then leads to the following MPR resonance condition:

$$\omega_c=\frac{\omega_{\text{LO}}}{N}\pm\frac{q_{\text{LO}}v}{N}\ , \tag{8}$$

where $N=n-m$. In this way the N^- peaks are again associated with absorption and the N^+ peaks with emission processes. The relative and absolute amplitudes of the N^- and N^+ peaks depend strongly on temperature via the phonon occupation number. All the results of Vasilopoulos *et al.* in Ref. 10 concerning electron–LO-phonon scattering are reproduced here on the basis of the energy-conserving δ function. Indeed, it is straightforward to generalize this argument to the case of occupation of more than one electric subband, which leads to additional oscillations in the magnetoresistivity characterized by the energy separation between the bottom of each pair of subbands.

In our momentum-balance model it was assumed so far that the effect of an electron temperature T_e different from the lattice temperature T is small on the MPR effect. We checked this point and it appeared that this does not lead to an additional shift of the peak positions. Only the MPR amplitudes are effected. For example, at $T=77$ K and for $\Gamma_0=0.9$ meV the $N=1$ peak is located at $\omega_0/\omega_{\text{LO}}=0.855$, 0.860, 0.870, 0.870, and 0.870 for $T_e=77$, 100, 140, 200, and 300 K, respectively.

III. NUMERICAL RESULTS FOR THE AMPLITUDE AND POSITION OF THE NONLINEAR MPR PEAKS

The nonlinear momentum-balance equation (1) will be solved here for the case of Boltzmann statistics in order to obtain detailed numerical results for the dependence of the position and amplitude of the MPR peaks on the relevant physical parameters: temperature, harmonic index N, broadening parameter, and average electron velocity. From our recent analysis of the effect of the exclusion principle on the magnetophonon resonances in the energy relaxation rate we know that this effect is small for filling factor smaller than 2. For an experimental situation with electron densities $N_e = 3.4\times 10^{11}$ cm^{-2} at $T = 77$ K this corresponds to magnetic fields $B > 8$ T.

We introduce here a critical average electron velocity $v_{\rm crit}$, defined as the velocity for which the Nth MPR maximum in the resistivity occurs at the magnetic field for which a minimum occurs in the linear regime ($v = 0$). Numerical values for $v_{\rm crit}/v_{\rm LO}$ are shown in Fig. 3 as a function of harmonic number N for different values of temperature and broadening parameter Γ_0 as indicated in the inset. The error bars cover the range of velocities for which the difference between the position of the corresponding maximum and the position of the maximum at $v = v_{\rm crit}$ is less than 40%. Within our simple model [Eq. (8)] it follows that $v_{\rm crit} \sim 1/N$, which agrees roughly with the qualitative trend of the numerical results given in Fig. 3. But from Fig. 3 it is clear that there is an additional dependence on the broadening, especially for the half-integer values of the harmonic number which correspond to minima in the linear regime. Temperature apparently does not affect the value of the critical velocity significantly.

The position of the $1^+,1^-$ and $2^+,2^-$ maxima are displayed in Fig. 4 in units $\omega_{\rm LO}/\omega_c$ as a function of the average electron velocity at $T = 140$ K and $\Gamma_0 = 0.4$ meV. The qualitative trend of these results can be explained on the basis of our simple model [Eq. (8)]. However, the shift of the position of the maxima is linear in the electron velocity only for small velocities. For larger velocities (e.g., $v/v_{\rm LO} > 0.15$ for $N = 2$) a saturation of this shift sets in and the N^+ and $(N+1)^-$ peaks start to merge.

Figure 5 displays the position of maxima and minima in units $\omega_{\rm LO}/\omega_c$ as a function of the broadening parameter Γ_0 for two different values of the average electron velocity at $T = 200$ K. All the extremal positions are shifted to smaller magnetic fields with increasing Γ_0. This effect combines with the effect of temperature through the factor $\exp(-\beta^2\Gamma_G^2/16)$.

The amplitude of the magnetophonon resonance peaks is shown in Fig. 6 as a function of temperature for two

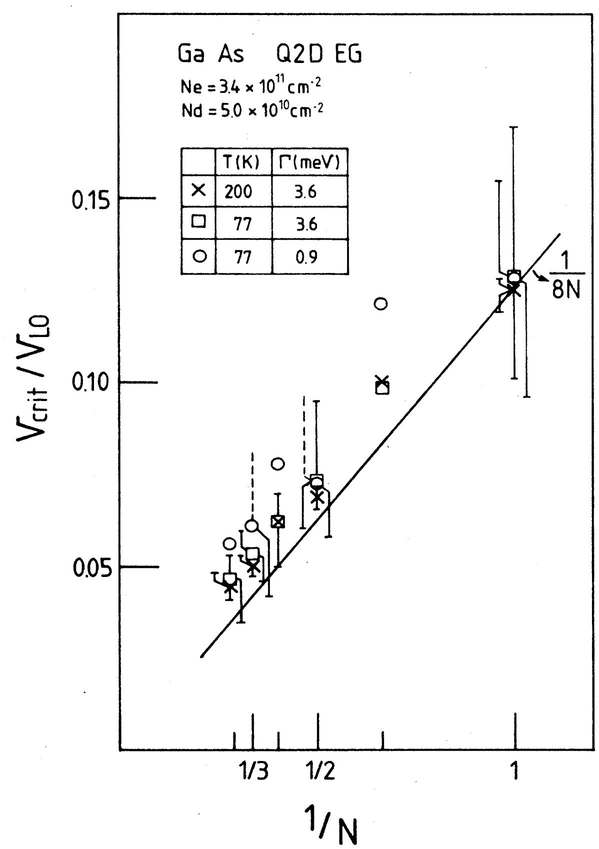

FIG. 3. Critical average electron velocity $v_{\rm crit}/v_{\rm LO}$ as a function of harmonic number N for different values of the temperature and broadening parameter Γ_0 (see inset table). The error bars are a measure of the width of the transition region.

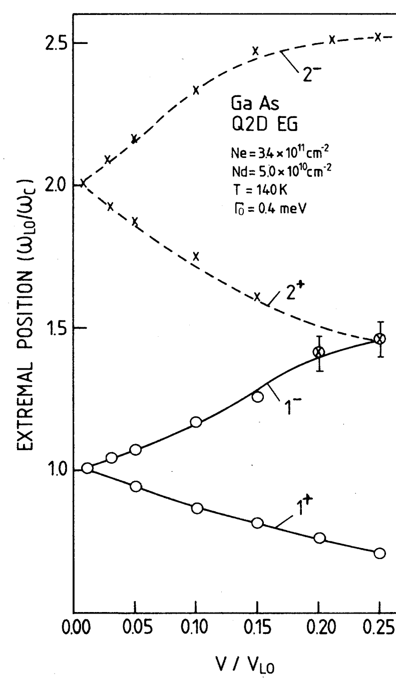

FIG. 4. The position of the $N = 1$ and 2 maxima in units of $\omega_{\rm LO}/\omega_c$ as a function of the average electron velocity at $T = 140$ K and $\Gamma_0 = 0.4$ meV. The curves are a guide to the eye.

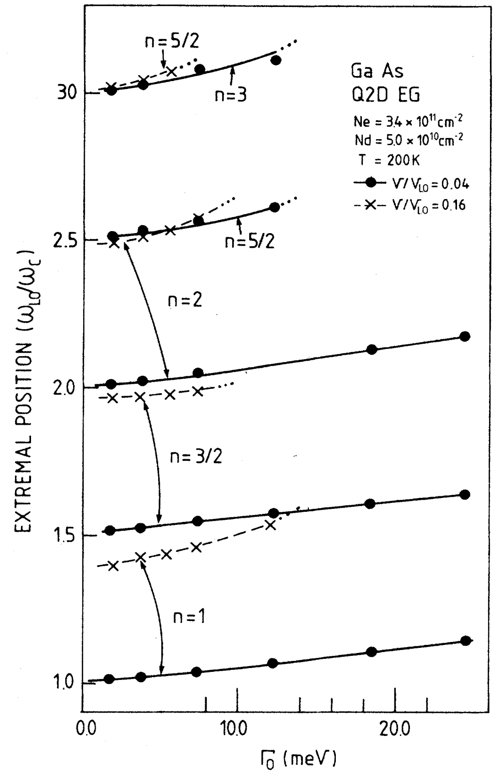

FIG. 5. The position of maxima and minima in units $\omega_{\rm LO}/\omega_c$ as a function of the broadening parameter Γ_0 for two different values of the average electron velocity at $T=200$ K. The curves are a guide to the eye.

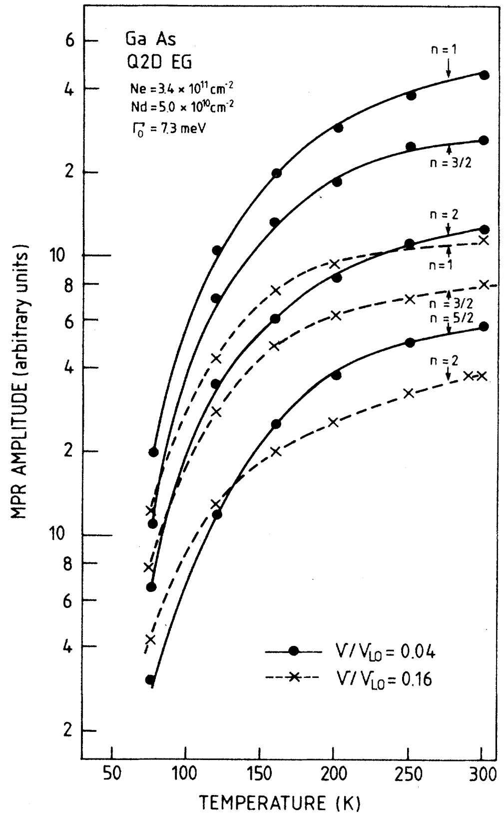

FIG. 6. The amplitude of the magnetophonon resonance peaks as function of temperature for two different values of the average electron velocity and for fixed $\Gamma_0=7.3$ meV. The curves are a guide to the eye.

different values of the average electron velocity and for $\Gamma_0=7.3$ meV. The MPR amplitude increases exponentially up to temperatures of about 140 K. For higher temperatures this amplitude starts to level off towards a value which is different for different N. This behavior of the MPR amplitude for $v/v_{\rm LO}=0$ can be described to a good approximation by the following expressions:

$$\rho_{xx}\sim\frac{\omega_c^{3/2}}{\Gamma_G}\frac{n(\omega_{\rm LO})}{T}e^{-\beta\hbar\omega_{\rm LO}/2}(1-e^{-\beta\hbar\omega_c})$$
$$\sim\frac{n(\omega_{\rm LO})}{N^2T^2}e^{-\beta\hbar\omega_{\rm LO}/2}, \tag{9}$$

valid for $\omega_c/\omega_{\rm LO}\sim 1/N$, and consequently

$$\rho_{xx}\sim\begin{cases}\dfrac{1}{N^2T^2}e^{-(3/2)\beta\hbar\omega_{\rm LO}} & \text{for } k_BT\ll\hbar\omega_{\rm LO}\,, \quad (10a)\\[2ex] \dfrac{1}{N^2T} & \text{for } k_BT\gg\hbar\omega_{\rm LO}\,. \quad (10b)\end{cases}$$

We note that in Fig. 6 we found additional dependence of the MPR amplitude on the electron velocity. No simple analytic expression for this velocity dependence was found.

Figure 7 displays the MPR amplitude as a function of the broadening parameter Γ_0 for $T=200$ K. Apparently the MPR amplitude decreases exponentially with increasing broadening parameter Γ_0. More specifically, the amplitude behaves as $\exp(-a\sqrt{N}\,\Gamma_0/\hbar\omega_{\rm LO})$ where $a\approx 10$ for $v/v_{\rm LO}=0.04$. The MPR amplitude is displayed in Fig. 8 as a function of the average electron velocity for $\Gamma_0=3.7$ meV and $T=200$ K. The MPR amplitude first decreases with increasing velocity. Subsequently it shows a small maximum and then decreases monotonically. This behavior mirrors the conversion of maxima into minima.

IV. CONCLUSIONS

In this paper the nonlinear magnetophonon effect in the resistance of a 2D electron system was investigated when subjected to crossed electric and magnetic fields. The interaction of electrons with polar LO phonons was taken to be the dominant scattering mechanism. The momentum-balance–equation approach, which we formulated in Ref. 8, was adopted as a working frame.

We looked for the basic mechanism behind the shift of the magnetophonon resonance peaks. Contrary to the

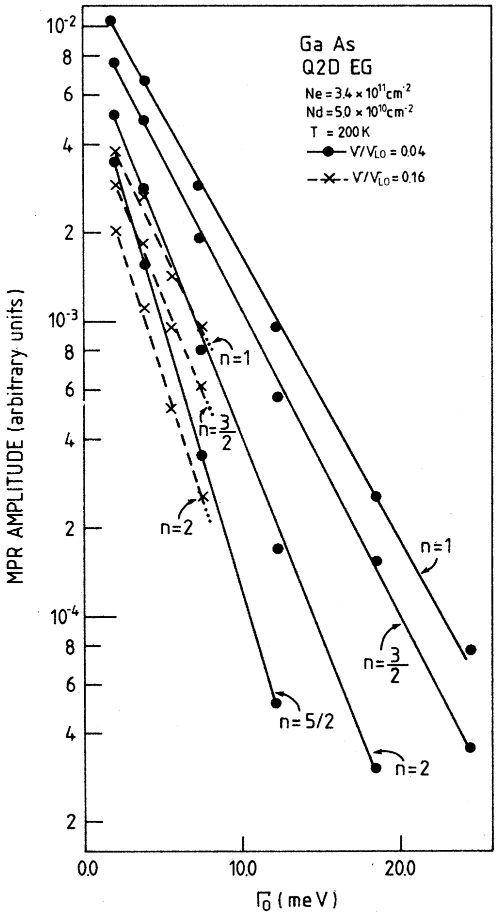

FIG. 7. The amplitude of the magnetophonon resonance peaks as a function of the broadening parameter Γ_0 for two different values of the average electron velocity at $T=200$ K.

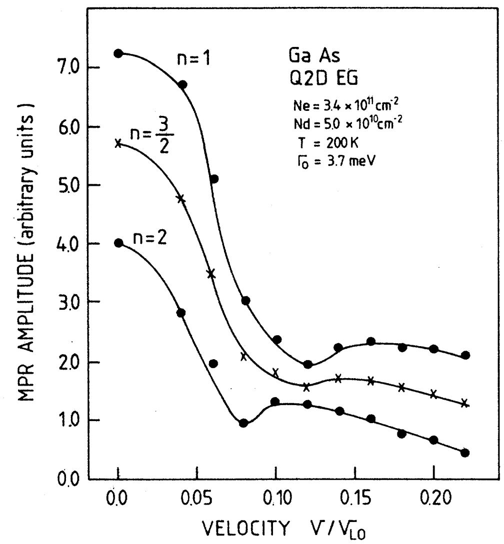

FIG. 8. The amplitude of the magnetophonon resonance peaks as a function of the average electron velocity for $\Gamma_0=3.7$ meV at $T=200$ K.

3D case, which was modeled recently by Mori *et al.*[13] within the Kubo formalism and which shows a splitting of the magnetophonon resonance peaks, our theoretical results in I, the model of Ref. 10, and the experimental results of Ref. 11 give evidence only for a shift of the magnetophonon resonance peaks in the 2DEG. In this paper we found that for small broadening of the Landau levels, magnetophonon resonance peaks split into two peaks with increasing velocity (or, equivalently, with applied electric field). In order to understand this physically, the separate contribution of LO-phonon absorption and emission processes to the nonlinear resistance was analyzed both within a simple model and by solving the nonlinear momentum-balance equation. In doing so, several existing theoretical models are reconciled and it turns out that the physical behavior for the 2D and 3D cases is qualitatively the same for small broadening. It is concluded that for realistic values of the broadening parameter, the splitting of the magnetophonon resonance peaks is obscured by the fact that every split-off N^+ peak merges with the neighboring $(N+1)^-$ peak. This implies that the splitting is only experimentally observable when the broadening of the Landau levels is small, except for the $N=1^+$ peak, which should always be observable if the MPR peaks are split even for relatively large broadening. A simple model was introduced which is based on the energy-conservation law. The qualitative behavior of this model agrees with the above results for small average electron velocities. Moreover, it reproduces all the results of Ref. 10 concerning electron–LO-phonon interaction, including intersubband processes.

Motivated by the interesting experimental observations of Leadley *et al.*[11] on the two-dimensional electron gas in a single-interface GaAs-$Al_xGa_{1-x}As$ system, detailed numerical results are presented for the dependence of the position and amplitude of the nonlinear magnetophonon resonance on lattice temperature, broadening parameter, and electron velocity. Where possible, these numerical results were interpreted with simple analytical expressions describing the general trends.

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