Isobaric electrical resistance along the critical line in nickel: An experimental test of universality

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(Received 15 March 1988; revised manuscript received 28 September 1988)

High-precision measurement of the electrical resistance of nickel along its critical line, a first attempt of this kind, as a function of pressure to 47.5 kbar is reported. Our analysis yields the values of the critical exponents $\alpha=\alpha'=-0.115\pm0.005$ and the amplitude ratios $|A/A'|=1.17\pm0.07$ and $|D/D'|=1.2\pm0.1$. These values are in close agreement with those predicted by renormalization-group (RG) theory. Moreover, this investigation provides an unambiguous experimental verification to one of the key consequences of RG theory that the critical exponents and amplitude ratios are insensitive to pressure variation in nickel, a Heisenberg ferromagnet.

One of the major features of critical-point phenomena is the concept of universality, according to which the critical exponents, and certain amplitude ratios that describe the singularities of various properties near the critical point, depend only upon such generalities of the system as its spatial dimensionality d, spin dimensionality n, and the symmetry of the Hamiltonian.[1] If a field variable, for instance, pressure for the Heisenberg ferromagnet ($d=3$, $n=3$) does not alter the symmetry of the ordered state; all the quantities which characterize a given universality class remain unchanged by the variation of this field variable. Our motivation for this research is to test this idea in nickel. There is no such high-pressure study reported so far in magnetic materials to the best of our knowledge.

There is compelling theoretical and experimental evidence that the magnetic energy is proportional to the spin-dependent electrical resistivity.[2,3] Hence a study of the temperature derivative of the electrical resistivity will lead to the similar critical parameters as deduced from the heat-capacity measurements. Besides, electrical resistivity can be measured with a precision far better than that realized in heat-capacity experiments.

High-pressure, high-temperature studies were performed on 99.999% pure foils of nickel; the technique developed here enabled continuous *in situ* pressure and temperature calibration. The pressure cell, fully described elsewhere[4,5] consisted of an opposed anvil device and calibration was performed by following the phase dia-

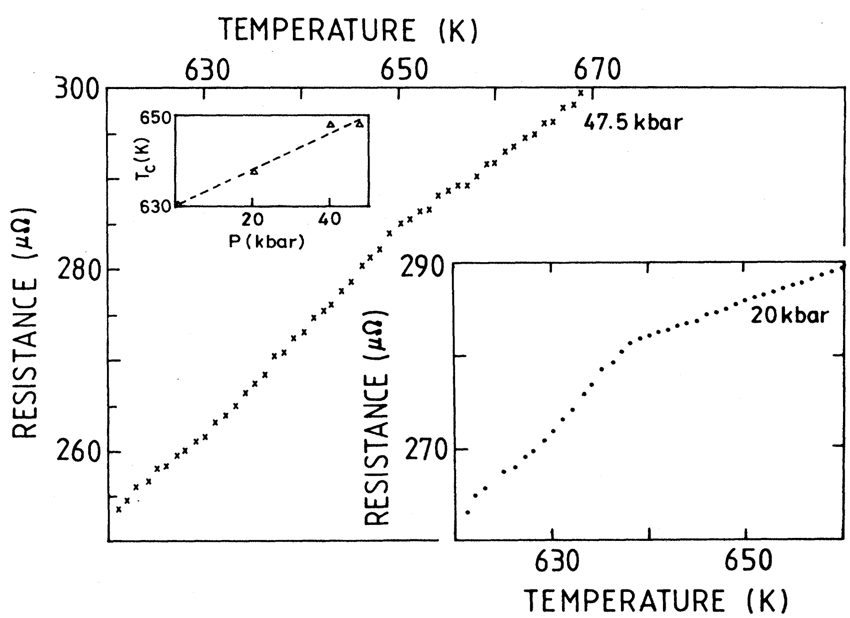

FIG. 1. Electrical resistance vs temperature at typical pressures of 20 and 47.5 kbar. The shape of transition seems changed under pressure because of the substantial change in the Fermi surface across the magnetic transition (see Ref. 5 for details). The inset exhibits the variation of T_c with pressure up to 47.5 kbar.

TABLE I. Comparison of the results for $\alpha=\alpha'$, A/A', and D/D' derived from Eq. (2) at atmospheric pressure with those predicted by the renormalization-group (RG) theory (Refs. 6–8) and the high-temperature (HT) series expansion (Ref. 9). Also listed are the values obtained by Källbäck *et al.* (Ref. 3) which should be compared with our analysis [using Eq. (2)] of their data. The rms error (m_r) is insensitive to the uncertainties indicated into the best-fit parameters. See Appendix A regarding the change in sign for the amplitude ratios.

Quantities of interest	RG theory	HT series expansion	Values reported by Källbäck *et al.* (Ref. 3)	Data of Källbäck *et al.* (Ref. 3) analyzed using our program [Eq. (2)]
$\alpha=\alpha'$	-0.115 ± 0.009	-0.14 ± 0.06 -0.09 ± 0.04	-0.095 ± 0.005	-0.097 ± 0.005 [a]
A/A'	1.24	1.52	-1.52 ± 0.05	-1.59 ± 0.05
D/D'	1.13	···	-0.8 ± 0.2	-1.1 ± 0.1
m_r	···	···	3.828×10^{-7}	2.423×10^{-7}

[a] $T_c=630.284\pm0.003$ K, $\sigma=0.1$ K.

grams of bismuth, antimony, and iron. Appropriate corrections for the effect of pressure on thermal emf's were made and all runs in which signals produced by temperature gradients exceeded 0.1% of the sample voltage were rejected. The electrical resistance R could be measured to a sensitivity[5] of 1 in 10^6.

At pressures of 0.001, 20, 40, and 47.5 kbar electrical resistance was measured as a function of temperature T. The temperature range scanned $1\times10^{-3}\leq|t|\leq2\times10^{-2}$ for $T\rightarrow T_c^-$ and $1\times10^{-3}\leq t\leq1.9\times10^{-2}$ for $T\rightarrow T_c^+$, where $t=T/T_c-1$. Figure 1 displays a typical run for the pressure of 47.5 kbar. For comparison, we have given the data obtained on the sample at 20 kbar. Shown in the inset is the critical line in the pressure range encompassed by our study. The data points at each pressure and for $T\rightarrow T_c^+$ are fitted to the expression

$$R=C_0+C_1t+C_2t^2+At^{1-\alpha}(1+Dt^{\Delta}) . \quad (1)$$

Here C_0, C_1, and C_2 refer to coefficients pertaining to the normal variation of resistance with temperature, A and D are the critical amplitude and the correction-to-scaling term amplitude, while α denotes the critical exponent for specific heat, and Δ is the correction-to-scaling exponent. Equation (1) with prime coefficients and exponents is used for $T\rightarrow T_c^-$ (see Appendix A).

By plotting the singular part of dR/dT as a function of T, an initial estimate of the T_c is obtained, e.g., at 1 bar, $T_c\simeq630$ K. While fitting the experimental data to Eqs. (1) and (2), T_c was varied around this value in steps of 0.01 K. The precise magnitude of T_c was deduced from the intersection of curves of α and α' vs T_c, where the scaling law $\alpha=\alpha'$ holds. At 1 bar, $\alpha=\alpha'=-0.115\pm0.005$ and $T_c=630.268\pm0.003$ K. A similar procedure was followed at higher pressures. For a given iteration, T_c and α were held fixed and the value of $\Delta=\Delta'$ was always taken[1,3] as 0.57.

Characteristic to solids is the presence of imperfections and lattice strains leading to a distribution in T_c's. In order to account for the above, we assume a Gaussian distribution of T_c's and the data points are fitted to the equation

$$R^*(T,T_c,\sigma)=\int R(T,T_c-x)g_\sigma(x)\,dx , \quad (2)$$

where $g_\sigma(x)$ is Gaussian in x of width σ. The integration of Eq. (2) is performed numerically using a 15-point Hermite integration and σ is also a parameter in the fit. The best rms error is obtained with $\sigma=0.1$ K.

To test efficiency of our nonlinear least-squares fit program, using Eq. (2), we first analyzed the reported data of

TABLE II. Best-fit values for $\alpha=\alpha'$, $|A/A'|$, and $|D/D'|$ and the associated rms errors (m_r) deduced from Eqs. (1) and (2) using our data (Ref. 5).

P (kbar)	T_c (K)	$\alpha=\alpha'$	$\lvert A/A'\rvert$	$\lvert D/D'\rvert$	m_r	Eq. used
0.001	630.64	-0.115 ± 0.006	1.27 ± 0.02	1.2 ± 0.1	3.8×10^{-5}	(1)
	630.284	-0.115 ± 0.005	1.13 ± 0.07	1.2 ± 0.1	1.1×10^{-6}	(2)
20	638.26	-0.118 ± 0.008	1.23 ± 0.02	1.2 ± 0.1	6.3×10^{-4}	(1)
	638.369	-0.117 ± 0.007	1.17 ± 0.07	1.2 ± 0.1	9.7×10^{-5}	(2)
40	648.33	-0.114 ± 0.008	1.25 ± 0.02	1.2 ± 0.1	9.6×10^{-4}	(1)
	648.663	-0.117 ± 0.008	1.16 ± 0.02	1.2 ± 0.1	3.0×10^{-5}	(2)
47.5	648.6	-0.115 ± 0.005	1.24 ± 0.02	1.13 ± 0.05	4×10^{-5}	(2)

Källbäck, Humble, and Malmstrom[3] on Ni at 1 bar. We give the results of our analysis in Table I along with the results obtained by them and find good agreement. Furthermore, it should be noted that our root-mean-square error (m_r) is better.

Turning now to the analysis of our data[5] on Ni, the same approach that yielded the numbers in Table I is followed. The results obtained through Eqs. (1) and (2) are listed in Table II. The use of Eq. (2), in contrast to Eq. (1), improves the m_r by at least an order of magnitude at all pressures. The pressure independence of $\alpha=\alpha'$, $|A/A'|$ and $|D/D'|$ is articulated more convincingly by Eq. (2) than Eq. (1). Figure 2 depicts the results obtained using Eq. (2). The insensitivity of relevant parameters with pressure is evident. Also shown in Fig. 2 are the reanalyzed data of Källbäck *et al.*[3] (Table I) as well as renormalization-group (RG) theory predictions.[6-8] Figure 3 depicts the critical contribution to measured resistance at 47.5 kbar using Eq. (2).

Our investigations quantitatively verify the crucial inference of the RG theory that a field variable that does not alter the symmetry of the ordered state should not change the universality class to which the system belongs. For instance, the values of α, α', $|A/A'|$, and $|D/D'|$ should remain invariant. This finding is a reflection of the smoothness of the critical line for nickel (inset, Fig. 1). An earlier investigation along the λ line of ^{4}He($d=3$, $n=2$) led to a similar conclusion,[10] but our findings are the first ones in a magnetic system and also the first ones at such high pressures. In case the critical line has an extremum, the quantities which characterize a universality class shall be influenced by pressure or any other appropriate field variable in the neighborhood of that extremum. Such a situation can, indeed, occur near a double critical point[11] or near the Néel temperature of an antiferromagnet.[12]

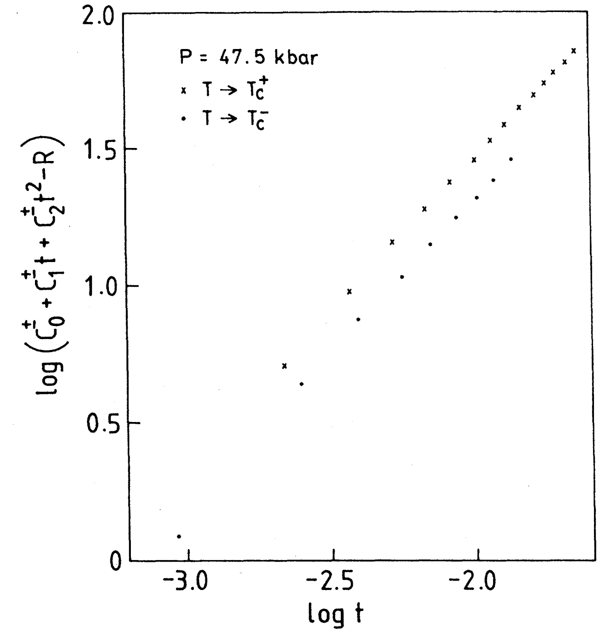

FIG. 3. A log-log plot of the critical (singular) contribution to resistance ($C_0^\pm+C_1^\pm t+C_2^\pm t^2-R$) vs t. Values of C_0, C_1, and C_2 are obtained from Eq. (2). Here + and − signify the data points for $T\to T_c^+$ and $T\to T_c^-$, respectively, and $t=(T-T_c)/T_c$. This figure is drawn essentially to bring out the quality of fit to the data at 47.5 kbar, since the change in slope of these data in Fig. 1 is not as pronounced as that at 20 kbar.

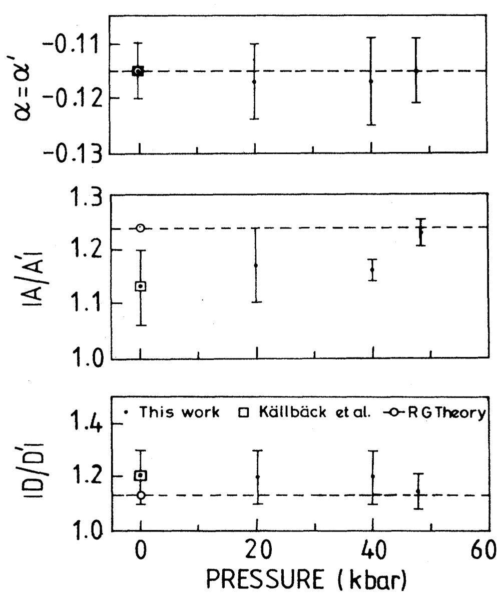

FIG. 2. Results for $\alpha=\alpha'$, $|A/A'|$, and $|D/D'|$ vs pressure. For comparison, the reanalyzed data of Källbäck *et al.* (Ref. 3) (at 1 bar) and the RG theoretical results are also displayed. The small error bars at 40 and 47.5 kbar for $|A/A'|$ may be an artifact of the data analysis, that does not reflect the systematic or other errors which may become important at such pressures.

In summary, this work on the isobaric electrical resistance near T_c of nickel has quantitatively tested the following RG theory predictions: (1) the values of the critical exponents, amplitude ratio, and the ratio of the correction to scaling term are close to the theoretical values, and (2) these parameters are invariant along the critical line.

We gratefully acknowledge stimulating discussions with Dr. G. Ananthakrishna, Dr. E. S. R. Gopal, Dr. K. Govinda Rajan, and Dr. G. Venkataraman. One of us (A.K.) is thankful to the Department of Atomic Energy for financial support of this work.

APPENDIX A

The Eq. (1) signifies change in resistance in the critical region such that

$$R=C_0+C_1t+C_2t^2+At^{1-\alpha}(1+Dt^{0.57}) \quad \text{for } T>T_c\,, \qquad \text{(A1a)}$$

and

$$R'=C_0'+C_1't+C_2't^2-A'|t|^{1-\alpha'}(1+D'|t|^{0.57}) \quad \text{for } T<T_c\,. \qquad \text{(A1b)}$$

It can be shown that their derivatives with respect to t are

$$\frac{\partial R}{\partial t}=C_1+2C_2t+At^{-\alpha}[(1-\alpha)+(1-\alpha+0.57)Dt^{0.57}]\,,$$

$$\frac{\partial R'}{\partial t}=C_1'+2C_2't+A'|t|^{-\alpha'}[(1-\alpha')+(1-\alpha'+0.57)D'|t|^{0.57}]\,,$$

and hence the temperature derivatives $\partial R/\partial T$, $\partial R'/\partial T$ are

$$\frac{\partial R}{\partial T}=\frac{1}{T_c}\frac{\partial R}{\partial t} \quad \text{and} \quad \frac{\partial R'}{\partial T}=\frac{1}{T_c}\frac{\partial R'}{\partial t}\,. \tag{1'}$$

From the above equations, it is evident that when $R\rightarrow \partial R/\partial T$ and $R'\rightarrow \partial R'/\partial T$, the coefficients transform by

$$A\rightarrow \frac{A(1-\alpha)}{T_c} \quad \text{and} \quad -A'\rightarrow A'\frac{(1-\alpha')}{T_c}\,,$$

$$D\rightarrow \frac{D(1-\alpha+0.57)}{(1-\alpha)} \quad \text{and} \quad -D'\rightarrow \frac{D'(1-\alpha'+0.57)}{(1-\alpha')}\,.$$

From the scaling relation $\alpha=\alpha'$, it is obvious that the quantities $|A/A'|$ and $|D/D'|$ are the same irrespective of the fact of whether they are derived from Eqs. (1) or (1').

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