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Brief Reports

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Fluctuations in granular superconductors

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By expanding the order parameter for an array of Josephson-coupled grains in powers of $1/z$, where z is the number of nearest neighbors, I systematically incorporate the effect of phase fluctuations. The correction of order $1/z$ vanishes when the mean-field solution is known to be exact, for $\alpha=zJ/U=\infty$ and $T^*=T/zJ=0$. For larger T^* and smaller α, the first-order correction increases until it diverges at the mean-field transition temperature.

Granular superconductors are commonly modeled as arrays of Josephson-coupled, superconducting grains with order parameters $\Delta_i=|\Delta|e^{i\phi_i}$. Below the bulk superconducting temperature, $|\Delta|\geq 0$ but phase fluctuations may keep the conductivity of the array finite. At the lower temperature T_c, the tunneling of Cooper pairs between neighboring grains becomes strong enough to support phase coherence across the whole array. For $T<T_c$, the expectation value $\langle e^{i\phi_i}\rangle$ is nonzero and the resistivity of the array vanishes.

Far below the bulk transition temperature, fluctuations of the magnitude $|\Delta|$ can be neglected and the phases ϕ_i become the only dynamical variables. The Hamiltonian for an array of N Josephson-coupled grains with z nearest neighbors can then be written

$$H=2U\sum_i n_i^2+J\sum_{\langle i,j\rangle}[1-\cos(\phi_i-\phi_j)]\,, \tag{1}$$

where J is proportional to the probability of Cooper-pair tunneling between neighboring grains, U is inversely proportional to the capacitance[1] of a grain, and $n_i=-id/d\phi_i$ is the operator for the number of excess Cooper pairs on the ith grain. Since n_i and ϕ_i are conjugate variables, the charging energy $2U\sum_i n_i^2$ disrupts the phase coherence across the array. When U/J exceeds a critical value, global phase coherence is impossible and $T_c=0$.

The Hamiltonian of Eq. (1) has been studied by a variety of methods.[2-11] Mean-field[3-5] (MF) and self-consistent harmonic[6] (SCH) methods agree that the order parameter $|\langle e^{i\phi_i}\rangle|$ is a monotonically decreasing function of temperature for $\alpha=zJ/U$ above the critical value α_c. Monte Carlo simulations,[11] however, indicate that for large but finite α the array undergoes a first-order phase transition at the temperature $T_{cq}\approx 0.03T_c$, when the helicity modulus jumps about 2% from its zero-temperature value.

Both analytic techniques have shortcomings that may explain their disagreement with the Monte Carlo simulations. The SCH approximation, which replaces the low-energy excitations of the array by a system of phase phonons, violates[12] the periodicity of the Hamiltonian, $H(\phi_i+2\pi)=H(\phi_i)$. MF theory, on the other hand, neglects the coupling of phase fluctuations on neighboring grains, which suppress the order parameter. Thus, both the SCH and MF methods become unreliable when phase fluctuations are large, such as near α_c and near T_c.

In this paper, I systematically include the effects of phase fluctuations by expanding the order parameter in powers of $1/z$. To zeroth order, $\langle e^{i\phi_i}\rangle$ is given by its MF value. Each higher-order correction involves a sum over an infinite number of diagrams which couple the phase fluctuations on neighboring grains. I explicitly calculate the $1/z$ correction to the order parameter. As expected, the coupling of phase fluctuations acts to decrease the order parameter. At $\alpha=\infty$ and $T^*=T/zJ=0$, where the MF solution $|\langle e^{i\phi_i}\rangle|=1$ is exact, the $1/z$ correction vanishes. For lower α and higher T^*, this correction increases until it diverges at T_c^*, signaling the breakdown of MF theory for any finite z. To order $1/z$, the order parameter is a single-valued function of α and T^*. Therefore, a first-order phase transition between two solutions, as suggested by Monte Carlo simulations, it not possible to this order.

An expansion in powers of $1/z$ is generated by separating the Hamiltonian into three parts:

$$H=H_{\text{eff}}+H_1+\lambda H_2\,, \tag{2}$$

where

$$H_{\text{eff}}=\sum_i H_{\text{MF}}^i\,, \tag{3}$$

$$H_{\text{MF}}^i=2Un_i^2-zJ\langle\cos\phi_1\rangle_{\text{MF}}\cos\phi_i\,, \tag{4}$$

$$H_2=-J\sum_{\langle i,j\rangle}R_{ij}\,, \tag{5}$$

$$R_{ij}=(\cos\phi_i-\langle\cos\phi_1\rangle_{\text{MF}})\times(\cos\phi_j-\langle\cos\phi_1\rangle_{\text{MF}})+\sin\phi_i\sin\phi_j\,, \tag{6}$$

and H_1 is a c number. The coefficient λ, which equals 1, is used to keep count of powers in H_2, which couples the phase fluctuations on neighboring grains.

An exact expression[13] for $\langle\cos\phi_1\rangle$ is given by

$$\langle\cos\phi_1\rangle = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H_{\rm eff}}T_\tau \exp\left(-\lambda\int_0^\beta \hat{H}_2(\tau)d\tau\right)\times\cos\hat{\phi}_i(0)\right], \quad (7)$$

$$Z = \mathrm{Tr}\left[e^{-\beta H_{\rm eff}}T_\tau \exp\left(-\lambda\int_0^\beta \hat{H}_2(\tau)d\tau\right)\right], \quad (8)$$

where $\beta = 1/T$, T_τ is the time-ordering operator, and operators in the interaction representation are defined by

$$\hat{A}(\tau) = e^{\tau H_{\rm eff}}Ae^{-\tau H_{\rm eff}}. \quad (9)$$

Both $\langle\cos\phi_1\rangle$ and the partition function Z can be formally expanded in powers of the fluctuation energy λH_2. The lowest-order term in the expansion of $\langle\cos\phi_1\rangle$ can be written as $Z_0\langle\cos\phi_1\rangle_{\rm MF}/Z$, where the MF expectation value is defined by

$$\langle A\rangle_{\rm MF} = \frac{1}{Z_0}\mathrm{Tr}(e^{-\beta H_{\rm eff}}A) \quad (10)$$

and Z_0 is the lowest-order term in an expansion of Z:

$$Z_0 = \mathrm{Tr}(e^{-\beta H_{\rm eff}}). \quad (11)$$

With the choice of Eqs. (3)–(6), $\langle\sin\phi_1\rangle = \langle\sin\phi_1\rangle_{\rm MF} = 0$.

The MF solutions are obtained[3-5] by solving for the eigenstates and eigenvalues of $H^1_{\rm MF}$. These solutions are plotted in Fig. 1: The critical value of α is $\alpha_c = 2$ and the $\alpha = \infty$ transition occurs at $T_c^* = 0.5$. It is important to realize that, written as a function of α and T^*, $\langle\cos\phi_1\rangle_{\rm MF}$ is independent of the number of nearest neighbors z.

In terms of these dimensionless variables, an expansion of $\langle\cos\phi_1\rangle$ in powers of $1/z$ is given by

$$\langle\cos\phi_1\rangle = \sum_{n=0}^{\infty}\frac{1}{z^n}M_n(\alpha, T^*). \quad (12)$$

The lowest-order term in this expansion, which depends on z only through α and T^*, is the MF solution $M_0 = \langle\cos\phi_1\rangle_{\rm MF}$. Each coefficient M_n can itself be expanded in powers of the fluctuation energy λH_2:

$$M_n(\alpha, T^*) = \sum_{m=0}^{\infty}\lambda^m M_{nm}(\alpha, T^*). \quad (13)$$

From the definitions of $H_{\rm eff}$ and H_2, it follows that $M_{0,m>0} = 0$.

To derive the first-order coefficient $M_1(\alpha, T^*)$, I consider the first two terms in the expansion of $\langle\cos\phi_1\rangle$ in powers of λH_2:

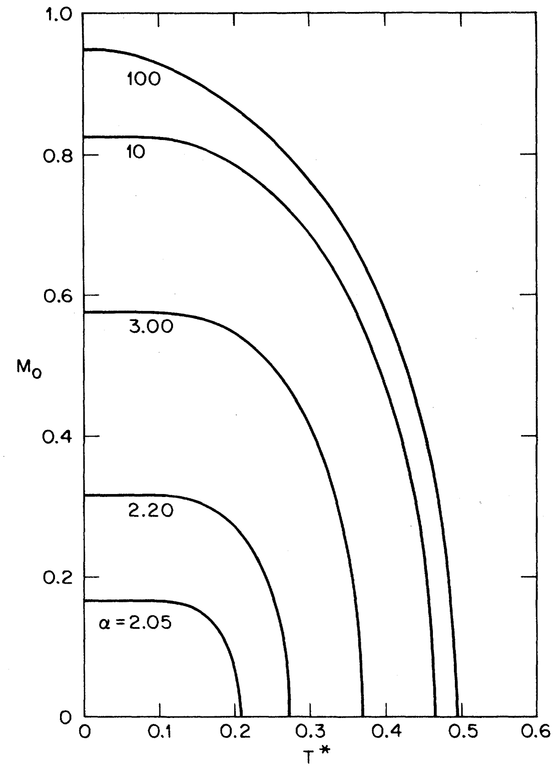

FIG. 1. The MF order parameter $M_0 = \langle\cos\phi_1\rangle_{\rm MF}$ vs T^* for various α.

$$\langle\cos\phi_1\rangle = M_0 + \frac{1}{Z_0}\lambda^2 zJ^2\mathrm{Tr}\left(e^{-\beta H_{\rm eff}}\int_0^\beta d\tau_1\int_0^{\tau_1}d\tau_2\hat{R}_{12}(\tau_1)\hat{R}_{12}(\tau_2)[\cos\hat{\phi}_1(0) - M_0]\right) + \cdots. \quad (14)$$

The factor of z arises from the z different orientations that can be assumed by the "loop" $\hat{R}_{1j}(\tau_1)\hat{R}_{1j}(\tau_2)$ with one point fixed at grain 1. Expressed in terms of α and T^*, the λ^2 correction to M_0 is of order $1/z$. With the definition

$$\hat{U}_i(\tau) = \cos\hat{\phi}_i(\tau) - M_0, \quad (15)$$

this lowest-order correction is

$$M_{12} = \frac{1}{Z_0}(zJ)^2\mathrm{Tr}\left(e^{-\beta H_{\rm eff}}\int_0^\beta d\tau_1\int_0^{\tau_1}d\tau_2\hat{R}_{12}(\tau_1)\hat{R}_{12}(\tau_2)\hat{U}_1(0)\right), \quad (16)$$

which is a function only of α and T^*. It is straightforward to compute M_{12} in terms of the complex matrices

$$G^{\pm}_{m_1m_2} = \langle m_1|e^{\mp i\phi}|m_2\rangle = \sum_n\langle m_1|n\rangle\langle n\pm 1|m_2\rangle, \quad (17)$$

where $|m\rangle$ is the mth MF eigenfunction and n is the Cooper-pair number. These matrices are easily obtained from the solutions of Mathieu's equation.

The higher-order terms $M_{1,m>2}$ are identified graphically in the inset of Fig. 2. Each diagram begins at grain 1 and ends in a loop. Since each line or loop can be oriented in z directions, the graph of order m is proportional to $z^{m-1}J^m=(zJ)^m/z$. I exclude diagrams with closed paths, which contribute to higher order in $1/z$. The contribution of the mth order diagram is

$$M_{1m}=\frac{1}{Z_0}(zJ)^m\int_0^\beta d\tau_1\int_0^{\tau_1}d\tau_2\ldots\int_0^{\tau_{m-1}}d\tau_m\,\mathrm{Tr}[e^{-\beta H_{\mathrm{eff}}}P(\hat R_{i_1j_1}(\tau_1)\hat R_{i_2j_2}(\tau_2)\ldots\hat R_{i_mj_m}(\tau_m))\hat U_1(0)]\,, \tag{18}$$

where the indices $i_1,j_1;i_2,j_2;\ldots;i_m,j_m$ lie along the graph of order m with the origin at grain 1. The operator P sums all distinct permutations of pairs of indices. For example, if $m=3$

$$P(\hat R_{i_1j_1}(\tau_1)\hat R_{i_2j_2}(\tau_2)\hat R_{i_3j_3}(\tau_3))=\hat R_{12}(\tau_1)\hat R_{23}(\tau_2)\hat R_{23}(\tau_3)+\hat R_{23}(\tau_1)\hat R_{12}(\tau_2)\hat R_{23}(\tau_3)+\hat R_{23}(\tau_1)\hat R_{23}(\tau_2)\hat R_{12}(\tau_3)\,. \tag{19}$$

Notice that P respects the time ordering of the operators.

I now show that the higher-order contributions $M_{1,m>2}$ can be expressed in terms of M_{12}. Using the cyclic property of the trace, $M_{1,m>2}$ can be rewritten

$$M_{1m}=\frac{1}{Z_0}(zJ)^m\sum_{n=1}^{m}\int_0^\beta d\tau_1\int_0^{\tau_1}d\tau_2\ldots\int_0^{\tau_{m-1}}d\tau_m\,\mathrm{Tr}[e^{-\beta H_{\mathrm{eff}}}P(\hat R_{i_nj_n}(\tau_{n+1}-\tau_n+\beta)\ldots\hat R_{i_{m-1}j_{m-1}}(\tau_m-\tau_n+\beta)$$
$$\times\hat R_{i_1j_1}(\tau_1-\tau_n)\ldots\hat R_{i_{n-1}j_{n-1}}(\tau_{n-1}-\tau_n))\hat U_2(0)]$$
$$\times\frac{1}{Z_0}\mathrm{Tr}[e^{-\beta H_{\mathrm{eff}}}\hat U_1(\tau_n)\hat U_1(0)]\,, \tag{20}$$

where the indices $i_1,j_1;i_2,j_2;\ldots;i_{m-1},j_{m-1}$ now lie along the graph of order $m-1$ with the origin at grain 2. New variables are used to rewrite the sum in Eq. (20) as

$$\sum_{n=1}^{m}\int_0^\beta dx_1\int_{x_1}^\beta dx_2\int_{x_1}^{x_2}dx_3\ldots\int_{x_1}^{x_{m-n}}dx_{m-n+1}\int_0^{x_1}dx_{m-n+2}\int_0^{x_{m-n+2}}dx_{m-n+3}\ldots\int_0^{x_{m-1}}dx_mB(\{x_i\})$$
$$=\int_0^\beta dx_1\int_0^\beta dx_2\int_0^{x_2}dx_3\ldots\int_0^{x_{m-1}}dx_mB(\{x_i\})\,, \tag{21}$$

where

$$B(\{x_i\})=\mathrm{Tr}[e^{-\beta H_{\mathrm{eff}}}P(\hat R_{i_1j_1}(x_2)\hat R_{i_2j_2}(x_3)\ldots\hat R_{i_{m-1}j_{m-1}}(x_m))\hat U_2(0)]\mathrm{Tr}[e^{-\beta H_{\mathrm{eff}}}\hat U_1(\beta-x_1)\hat U_1(0)]\,. \tag{22}$$

It follows that, for $m>2$,

$$M_{1m}(\alpha,T^*)=M_{1,m-1}(\alpha,T^*)f(\alpha,T^*)\,, \tag{23}$$

$$f=\frac{1}{Z_0}zJ\int_0^\beta dx\,\mathrm{Tr}[e^{-\beta H_{\mathrm{eff}}}\hat U_1(x)\hat U_1(0)]\,. \tag{24}$$

Thus, the series for M_1 can be summed:

$$M_1=M_{12}\sum_{n=0}^{\infty}f^n=M_{12}\frac{1}{1-f}\,, \tag{25}$$

which is the central result of this work.

The functions $f(\alpha,T^*)$ and $M_1(\alpha,T^*)$ are plotted in Figs. 2 and 3. The scaling function $f(\alpha,T^*)$ vanishes at $\alpha=\infty$, $T^*=0$, where the MF solution is exact. Therefore, the dominant $1/z$ correction very close to this point is given by M_{12}. The scaling function reaches the value 1 at T_c^*, signaling the breakdown of the MF solution for any finite z. For $\alpha<\alpha_c=2$ and $T^*=0$, $f=\alpha/2$. As shown, f remains finite for $T^*>T_c^*$, although M_{12} vanishes above the MF transition temperature. Correspondingly, the $1/z$ correction $M_1(\alpha,T^*)$ diverges to $-\infty$ at T_c^* for any α and at α_c for $T^*=0$. As expected $-M_1$ is very small near $\alpha=\infty$, $T^*=0$, and grows with decreasing α and increasing T^*.

It can be shown diagrammatically for $n\geq 1$ that $M_{n+1}/M_n\to 0$ as $\alpha\to\infty$ and $T^*\to 0$. Nonetheless, the sum of higher-order corrections $\sum_{n=2}^{\infty}M_n/z^n$ may be the same order as the $n=1$ correction M_1/z, even for low temperatures and large α. Although no limits can be placed on the sum of higher-order terms, it seems likely that the

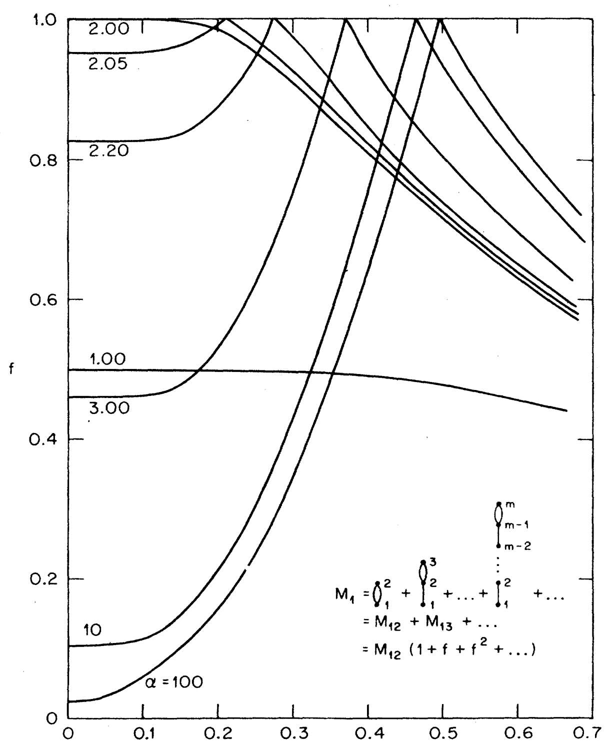

FIG. 2. The scaling function f vs T^* for various α. Inset: The graphs that contribute to the $1/z$ correction M_1.

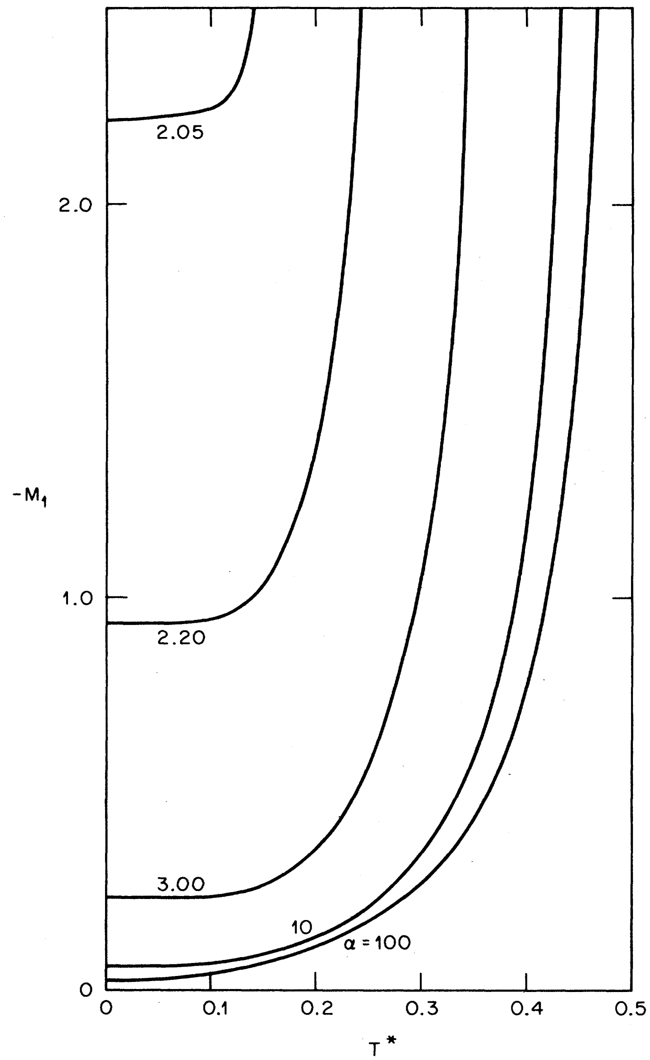

FIG. 3. The $1/z$ correction $-M_1$ vs T^* for various α.

$1/z$ correction is the dominant one at low T^* and large α.

The divergence of M_1 at T_c^* indicates that higher-order terms in the expansion of $\langle \cos\phi_1 \rangle$ in powers of $1/z$ must be included in the critical region. Although these higher-order terms also diverge at the MF transition temperature, the summation of these divergent terms is well defined and shifts the transition temperature from its MF value.

To order $1/z$, the dimension of the array does not affect the solution for the order parameter. However, the contributions of diagrams with closed paths will involve the connectivity of the system. Therefore, the higher-order coefficients $M_{n>1}$ do depend on the dimension of the array.

At least through order $1/z$, the expansion of $\langle \cos\phi_1 \rangle$ has only one solution as a function of α and T^*. Hence, a first-order phase transition is not possible to order $1/z$ but may be possible at some higher order. Jacobs and coworkers[11] have suggested that the first-order phase transition at T_{cq} involves the nucleation of vortex pairs in two dimensions. If so, then only higher-order corrections in $1/z$, which depend on the dimension of the array, can explain such a transition. The small size of the jump in the helicity modulus at T_{cq} may be a consequence of the high order in $1/z$ required to obtain this transition. Although a first-order phase transition is still possible, this work has shown that such a phase transition cannot be explained from the lowest-order effect of phase fluctuations.

Note added in proof. By expanding[14] α_c in powers of $1/z$, Ferrell and Mirhashem find that $M_0M_1 = -0.4$ at $\alpha = 2$ and $T^* = 0$. This result agrees with my numerical evaluation of M_0 and M_1.

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