# Field-dependent viscosity in partially polarized normal liquid $^3$He

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We calculate the field dependence of the low-temperature viscosity $\eta(H)$ in normal liquid $^3$He, viewed as a nearly magnetic itinerant-fermion system. In such a picture, we find that $\eta(H)$ should increase with $H$ in low fields and at low temperatures. This is in good agreement with the latest experimental data. Such a finding contradicts both an earlier experiment and the possibility for a metamagnetic transition in liquid $^3$He proposed by other authors.

In the interesting field of spin-polarized quantum systems, there has only been, so far, two experiments measuring the variation of the viscosity $\eta$ in partially polarized liquid $^3$He as a function of the polarization or the applied magnetic field $H$. One experiment[1] found that $\eta(H)$ initially *decreases* when $H$ increases, passes through a minimum, and then increases; the other one[2] found instead that $\eta(H)$ initially *increases* with $H$. The first experiment was understood[3] considering liquid $^3$He as a nearly metamagnetic,[4] nearly localized[5] system for which the low-field dependence of the spin susceptibility increases with $H$, i.e., $\delta\chi/\chi=[\chi(H)-\chi(0)]/\chi(0)>0$.

In the present paper we examine another approach where normal liquid $^3$He is viewed as an enhanced nearly ferromagnetic itinerant-fermion system containing strong spin fluctuations, the paramagnons.[6] In such a picture, the field-dependent magnetization was computed earlier[7] and is such that $\delta\chi/\chi<0$ at low fields and low temperatures. Using this result, we compute $\eta(H)$ and show that the inverse viscosity $\eta^{-1}(H)$ ought to follow the field dependence of $\chi(H)$. Therefore, we expect $\delta\eta/\eta=[\eta(H)-\eta(0)]/\eta(0)$ to be positive or negative depending on whether $\delta\chi/\chi$ is negative or positive. At this stage the result of Ref. 3 is consistent with the nearly localized picture of Refs. 4 and 5 where $\delta\chi/\chi>0$ close to the metamagnetic transition. Instead, since the paramagnon picture predicts $\delta\chi/\chi<0$, we indeed find that $\eta(H)$ ought to increase with $H$. We also show that, quantitatively, our result is in good agreement with the experimental one of Ref. 2. Further experiments should settle which one of the data sets in Refs. 1 or 2 is correct. If the results of Ref. 2 are confirmed, this would cast some doubts on the possibility for a metamagnetic transition to occur at some polarization in liquid $^3$He and would reinforce instead the role of the paramagnons in such a system.

The details of our calculation for $\eta(H)$ in liquid $^3$He will be given elsewhere[8] together with the calculation of other transport properties like the magnetoresistivity in exchange-enhanced metals. To compute $\eta(H)$, we suppose[9] that the fermion system of $^3$He spins is in a shear motion with a constant local velocity $v_x$ in the $x$ direction and a uniform velocity gradient $\delta v_x/\delta y=a$ in the $y$ direction. Then we write the Boltzmann equation in the presence of an applied magnetic field $H$. The fermion distribution functions $f_\pm$ for up(+) and down(−) spins are determined under the influence of the shear stress. In the steady state

$$(\delta f_\pm/\delta t)_d+(\delta f_\pm/\delta t)_c=0\,, \tag{1}$$

the "drift" term is

$$(\delta f_\pm/\delta t)_d=-ak_xk_y(\delta f_{0\pm}/\delta\varepsilon_\pm)\,, \tag{2}$$

with $f_{0\pm}$ the Fermi-distribution functions

$$f_{0\pm}(\varepsilon_\pm)=\{\exp[(\varepsilon_\pm-\varepsilon_F)/T]+1\}^{-1}\,,$$

$\varepsilon_F$ the Fermi energy, and $\varepsilon_\pm=k^2/2\pm B$, in atomic units; $B$ is a Zeeman energy modified[10] by the interaction $I$ between the fermions

$$B=-(H+IM/2)\,, \tag{3}$$

$M(T,H)$ is the magnetization of the system and has been computed in Ref. 7. [The sign in (3) follows from the definitions in Ref. 10 and will not matter as only $B^2$ will appear in low fields.] To write the collision term $(\delta f_\pm/\delta t)_c$, we suppose that the scattering processes of fermions on paramagnons may be described by the relaxation times $\tau_\pm$:

$$(\delta f_\pm/\delta t)_c=-(f_\pm-f_{0\pm})/\tau_\pm\,. \tag{4}$$

The viscosities for up and down spins are then given by

$$\eta_\pm(H)=(2\pi)^{-3}a^{-1}\int d^3\mathbf{k}\,k_xk_y(f_\pm-f_{0\pm})\,, \tag{5}$$

and the total viscosity $\eta(H)$ is

$$\eta(H)=\eta_+(H)+\eta_-(H)\,. \tag{6}$$

The collision term is expressed using the golden rule and introducing the transition probabilities $P(\mathbf{k}\alpha;\mathbf{k}'\beta)$ that a fermion of momentum $\mathbf{k}$ and spin $\alpha=\pm\frac{1}{2}$, after scattering on a paramagnon, will be in a state $\mathbf{k}'$ with a spin $\beta=\alpha$ (for processes without spin flip), or $\beta=-\alpha$ (for processes with spin flips). In zero field $H=0$, the transport properties of a paramagnon system have been computed long ago,[11] in particular, the zero-field viscosity $\eta(0)$ in normal liquid $^3$He. The scattering processes on paramagnons are extensively described in Ref. 11. In the presence of a field, the calculation is slightly more complicated because one has to write separate equations for up and down spins. As in Ref. 11, we suppose that the same parabolic band is responsible for the scattered fermions and for the scattering paramagnons. Moreover, the interaction $I$ will enter both in the paramagnon formation and in the fermion-paramagnon scattering interaction. The useful ingredients will be $\tau_\pm$, which, with (4), (5), (1), and (2), will allow computation of $\eta(H)$ in (6). We obtain

$$\frac{1}{\tau_\pm} \propto \int\int d\theta\, d\varepsilon'_\pm k'_\pm \frac{\sin\theta F(k,k')}{1-f(\varepsilon)[1-e^{\omega/T}]_{\omega=\varepsilon'_\pm-\varepsilon_\pm}}[1+n(\omega)]\,\mathrm{Im}[\chi^{\mp\mp}(q,\omega)] + \int\int d\theta\, d\varepsilon'_\mp k'_\mp \frac{\sin\theta G(k,k')}{1-f(\varepsilon)[1-e^{\omega/T}]_{\omega=\varepsilon'_\mp-\varepsilon_\pm}}[1+n(\omega)]\,\mathrm{Im}[\chi^{\mp\pm}(q,\omega)]\,, \quad (7)$$

where $n(\omega)$ is the Bose factor, $n(\omega)=[\exp(\omega/T)-1]^{-1}$, $\theta$ the scattering angle between $\mathbf{k}$ and $\mathbf{k}'$,$\mathbf{q}$ the momentum transfer $\mathbf{q}=\mathbf{k}'-\mathbf{k}$. The functions $F$ and $G$ are such that

$$\frac{k'_x k'_y}{k_x k_y}=1-F=\frac{\tau_\pm}{\tau_\mp}(1-G)=1-\tfrac{3}{2}\sin^2\theta\,, \quad (8)$$

and

$$q=|\mathbf{q}|=2k_F\sin(\theta/2)\,. \quad (9)$$

The paramagnon propagators enter via their imaginary parts in (7). They are the longitudinal $\chi^{\mp\mp}$ and the transverse $\chi^{\mp\pm}$:[7,10]

$$\chi^{\mp\mp}(q,\omega)\propto(I/2)[1-I^2\chi^{0\mp}\chi^{0\pm}]\,,$$
$$\chi^{\mp\pm}(q,\omega)\propto I/[1-I\chi^{0\mp\pm}]\,, \quad (10)$$

where $I$ is the strong Hubbard-type contact repulsion between opposite spins of the order of the Fermi energy $\varepsilon_F$ in the paramagnon picture: $\bar{I}\equiv IN(\varepsilon_F)\sim I/\varepsilon_F\sim 1$ with $N(\varepsilon_F)$ the density of states at the Fermi level. The $\chi^0$'s are the spin-spin correlation functions in absence of interaction. Their field dependences are given in Ref. 7. Putting all the ingredients together and using (3) with $M$ given by formula (10) of Ref. 7, we finally find, for $T\to 0$,

$$\eta(0)/\eta(H)\simeq\begin{cases}[\chi(H)/\chi(0)]^{1/2}, & 0.75<\bar{I}\lesssim 1, \quad (11a)\\ [\chi(H)/\chi(0)]^{2}, & \bar{I}\lesssim 0.75. \quad (11b)\end{cases}$$

The powers involved in (11) depend on the approximations made in the $q$ integrals [in (7) with (9)]; these approximations, in turn, depend on whether $\bar{I}q^2/(12k_F^2)$ is negligible or not compared to $(1-\bar{I})$ when $O\lesssim q/(2k_F)\lesssim 1$, or, in other words, whether $\bar{I}\lesssim 0.75$ or $0.75<\bar{I}\lesssim 1$. [Note that (11b) is not valid if $\bar{I}\to 0$. All the calculations are done for a strong enhancement $S=(1-\bar{I})^{-1}>1$ (say $\bar{I}>0.5$) and only the leading terms, when $(1-\bar{I})^{-1}>1$, have been retained.] For the sake of completeness, we recall the low-$T$ and low-$H$ expansion of $\chi$ of Ref. 7:

$$\chi(T,H)/\chi(0,0)=1-\alpha_1 S^2\frac{T^2}{T_F^2}-\beta_0 S^3\frac{H^2}{T_F^2}+(\beta_1+4\alpha_1\beta_0)S^2\frac{T^2}{T_F^2}S^3\frac{H^2}{T_F^2}+\cdots\,, \quad (12)$$

where $\alpha_1$, $\beta_0$, and $\beta_1$ are pure numbers slightly different whether $H>T$ or $H<T$. Note that in zero field, the low-temperature viscosity $\eta(0)$ was given in Ref. 11 to be

$$\eta^{-1}(0)\propto(1-\bar{I})^{-1/2}T^2\,, \quad (13)$$

and thus $\eta^{-1}(H)\propto\chi^{1/2}(T=0,H=0)$, as we find in (11a). [We point out that in (11), we only compute the change with $H$ in the coefficient of the $T^2$ term found in $\eta(0)$, so that the numerical estimates that we give below underestimate the effect.] $\chi(0,0)$ is the mean-field Stoner susceptibility; $\chi(T,0)$, including paramagnons effects, has been computed in Ref. 10 where the paramagnons affect only, but by a huge amount, the $T$ dependence. Finally, $\chi(T,H)$ (found in Ref. 7 and recalled above) includes, respectively, in the second, third, and last term of (12), the paramagnons effects on the $T$ dependence of $\chi$ at $H=0$, the $T=0$ field dependence of the Stoner susceptibility, and the combined field dependence of the temperature-dependent paramagnon effect on $\chi$.

We now put numbers in formulas (11) and (12) to compare our result for $\eta$ with the experimental one of Ref. 2(a). We use the same Fermi-liquid parameters[12] that are in Ref. 2(a), then, at fixed temperature $T=45$ mK, for a pressure of 30 bars and at a frequency of 317.528 MH, i.e., for a polarization $\bar{m}=3.96\times10^{-2}$, we obtain

$$1.1\times10^{-3}<(\delta\eta/\eta)_{\mathrm{theor}}<4.6\times10^{-3} \quad (14)$$

to be compared with the experimental result of Ref. 2(a):

$$(\delta\eta/\eta)_{\mathrm{expt}}=(3\pm1.5)\times10^{-3}\,. \quad (15)$$

The theoretical result depends whether formula (11a) or (11b) is used. Clearly, the experimental result (15) lies in the range of theoretical values. Therefore, the agreement is quite good, as far as the sign of the effect $(\delta\eta/\eta)>0$ and its magnitude are concerned. As mentioned earlier, such an *increase* of $\eta$ is in complete contradiction with the initial *decrease* of Ref. 1 and with the nearly metamagnetic picture of Refs. 3–5.

The next step in our comparison with experiments would be to plot $\eta(H)$ vs $\bar{m}(H)$, or $\eta(H)$ vs $\chi(H)$, to check which one of the power laws in the right-hand side of (11) is correct and should be used. This, in turn, would give more precise information on the strength of the phenomenological interaction $\bar{I}$ which could then be consistently compared with the $\chi(0,0)$ value used experimentally. For now, however, the experiments cannot provide enough points to do such a plot in a meaningful way.

In conclusion, we find that the paramagnon picture applied to normal liquid $^3$He provides a satisfactory estimate of the field dependence of the viscosity when compared with the presently available data of Ref. 2(a). It is hoped that future experiments will settle the experimental controversy between Refs. 1 and 2 and decide by comparison with either this paper or Refs. 3–5, which one of these theories better account for the spin properties of normal liquid $^3$He. It is clear at the present time the paramagnon theory better describes the spin-dependent properties, and that the nearly localized theory better describes the spin-independent properties. As we have already stressed,[13] we believe that liquid $^3$He is, as well, nearly ferromagnetic *and* nearly solid, but a unified theory including both tendencies is yet to be elaborated.

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