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Critical current of an inhomogeneous superconductor as a percolation-breakdown phenomenon

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A percolation model for the critical current in inhomogeneous superconductors is introduced. The model is a network of randomly configured superconducting (concentration p) and normal (concentration $1-p$) bonds on a lattice. Each superconducting bond has a critical current i_c above which it becomes a normal Ohmic resistor. The current distribution in the superconducting regions is solved using the linearized Landau-Ginzburg equations for a network of wires as proposed by de Gennes. The current distribution in the normal regions is solved using Kirchoff's laws. The critical current and the voltage-current relations are studied numerically in two dimensions on a square lattice, and comparisons are made with recent voltage-current experimental data on high-T_c superconductors. The scaling concepts and statistics of extremes introduced by Duxbury, Leath, and Beale (DLB) for general breakdown behavior, based on the most critical defect (normal region) in the network, are tested and found to be accurate for the scale-size dependence of the critical current and for the predicted critical-current distribution of random samples. In particular, it appears that the critical current goes to zero logarithmically in the thermodynamic limit, as proposed by DLB.

I. INTRODUCTION

Defects are known to be a dominating factor limiting the electrical and mechanical strength of materials, and therefore an enormous effort has gone into the study of their effect on the breakdown of materials.[1–3] However, the problem of handling a random distribution of defects is sufficiently difficult that only recently have percolation models of breakdown and their computer simulation been successful in illuminating the general features of breakdown phenomena.[4–7] In this paper, we apply essentially the same percolation approach[6] to the problem of critical current in a superconducting network. We use the pioneering approach of de Gennes[8] and Alexander[9] to formulate the theory of a superconducting network as a model of an inhomogeneous superconducting system. A related approach has been used by others to better understand critical magnetic fields,[10,11] but this is the first such calculation for the critical supercurrent and for its sample-size dependence. The computer simulations reported here are limited to two-dimensional square lattices for simplicity, but also because of the recent interest in thin films of high-T_c superconductors. In fact, we show that the results here are consistent with recent experiments in superconducting thin films and make suggestions for future experiments to be done.

The results here show that only a few defects will have a dramatic effect on the critical current and thus defects probably limit the critical current in many real superconducting systems. Hence we believe it will soon be possible to understand the principal limiting factors on supercurrent density in many applications.

The model for our computer simulations is a square lattice network with a randomly distributed fraction p of superconducting bonds and a remaining fraction $(1-p)$

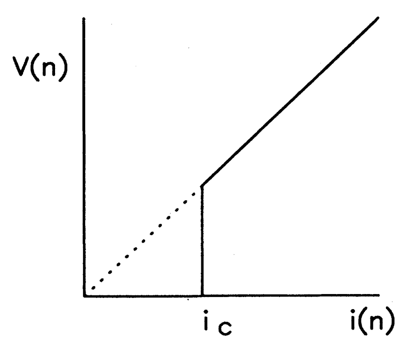

FIG. 1. The voltage-current relation of a superconducting bond n. For current $i(n)$ greater than the critical current i_c the bond is normal with a resistance $R=1$.

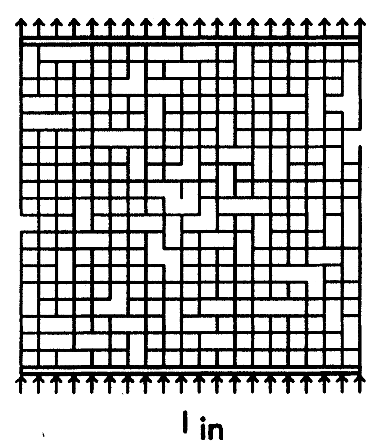

FIG. 2. A typical, random configuration of superconducting (solid lines) and normal (open) bonds on a 20×20 square lattice at $p=0.90$. The current source puts a current I_{in} into the lattice uniformly across the bottom bus bar and takes it out across the top bus bar.

of normal Ohmic conducting resistors. We assume that each superconducting bond has a critical current i_c. For current i greater than i_c the bond becomes normal with a resistance R. Thus the voltage $V(n)$ across bond n is as shown in Fig. 1. In reality, the voltage-current curve for a single bond should be rounded in some way at the transition, but this rounding is ignored here. The square lattice of bonds (see Fig. 2) is connected to bus bars at the top and bottom and free boundary conditions applied to the left and right sides of the lattice. The external applied current I_{in} flows into the network through the bottom bus bar and out at the top. The network displays qualitatively different behavior at different concentrations p and at different applied currents I_{in}. For $p<p_c$, the critical percolation threshold where the superconducting path first appears across the sample, the sample is normal with only isolated islands of superconductivity; we shall only consider $p>p_c$ in this paper. Thus, for I_{in} sufficiently small, none of the superconducting regions will have gone normal so that we will have an ordinary percolation problem for an inhomogeneous superconductor. But as the current I_{in} is raised to the first critical value I_1, the current carried through some (the most critical) bond of the sample exceeds i_c and that bond goes normal. As the current is further increased, bond after bond goes normal until at $I_{\text{in}}=I_b$ breakdown occurs, there is no longer a superconducting path across the sample, and for the first time there appears a voltage across the sample. This region of $I_{\text{in}}<I_b$ we shall call region I. As the current $I_{\text{in}}>I_b$ is further increased, the voltage increases more rapidly than Ohmically as the remaining isolated superconducting regions go normal until, for $I_{\text{in}}>I_2$, the entire sample is normal and the voltage-current relation becomes Ohmic. The region $I_b<I<I_2$ shall be called region II, and $I>I_2$ we shall call region III.

II. THE NETWORK EQUATIONS

We consider the Landau-Ginzburg free energy on a network of thin wires.[8-10] Following Alexander,[9] we find that this leads to the Landau-Ginzburg (LG) equation on a wire

$$\left[-\frac{1}{\xi_s}+\left[i\frac{\partial}{\partial s}-\kappa\right]^2+b|\Delta|^2\right]\Delta=0\ , \tag{1}$$

where Δ is the coherent wave function or superconducting order parameter, $s=(\hat{\mathbf{u}}\cdot\hat{\mathbf{r}})$, $\hat{u}$ is a unit vector along the direction of the wire, where $\kappa=(2e/hc)(\hat{u}\cdot\mathbf{A})$ with $\mathbf{A}$ being the vector potential, and ξ_s is the coherence length. This equation is then solved for each bond of the lattice with appropriate boundary conditions for continuity of Δ and current conservation at each node of the lattice. We assume that the penetration depth is very large, and following Alexander[9] linearize the LG equation by assuming $b=0$ in Eq. (1). Alexander[9] has shown that these assumptions lead to the coupled set of linear equations for the regular square lattice which are

$$-\Delta_i\sum_j(\text{cotan}\theta)+\sum_j\left[\Delta_j\frac{e^{i\gamma_{ij}}}{\sin\theta}\right]=0\ , \tag{2}$$

where Δ_i is the order parameter at node i of the network, and the summations are over nearest neighbors. The angle θ is determined by

$$\theta=a/\xi_s\ , \tag{3}$$

where a is the lattice spacing. The coherence length depends upon temperature according to

$$1/\xi_s^2\propto(T_c-T)/T_c\ , \tag{4}$$

and γ_{ij} is given by

$$\gamma_{ij}=\int_i^j\left[\frac{2e}{hc}\right](\hat{u}_{ij}\cdot\mathbf{A})ds\ . \tag{5}$$

The supercurrent I_{ij} between nodes i and j on the lattice is given by[10]

$$I_{ij}\propto\text{Im}(\Delta_i^*\Delta_j)\ . \tag{6}$$

Therefore, current conservation (Kirchoff rule) at each node gives the boundary condition of the superconducting phase:

$$\alpha\sum_j\text{Im}(\Delta_i^*\Delta_j e^{i\gamma_{ij}})=I_i^{\text{ext}}\ , \tag{7}$$

where the sum is over nearest neighbors j, α is a constant, and I_i^{ext} is the external current applied at site i. Since α is a constant its precise value is unimportant as it simply scales the applied current I^{ext}. Generally, I_i^{ext} will be zero at all sites on the interior of the superconducting regions, will equal the applied external current on the boundaries of the sample, and will equal the boundary current on the interfaces between superconducting and normal regions of the sample. For the purposes of this paper, we shall focus on $T<T_c$ and zero magnetic field,

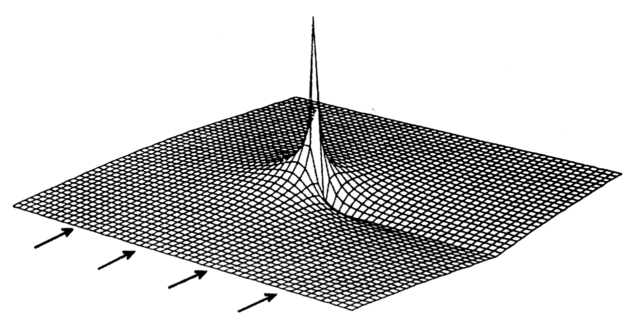

FIG. 3. The supercurrent component in the y direction, $I_y(n)$, plotted vertically at the site n of a square lattice where the bus bars are along the x direction (so that the current I_{in} is input from left to right in the y direction). A single slit (defect) has been cut in the lattice from its center along the x direction to the center of the right-hand edge. No current can flow across the slit, but instead must go around the left tip of the defect, causing the large current spike just to the left of the tip of the defect. This figure was taken from Ref. 12.

where the coherence length ξ_s is much larger than the lattice spacing a so that $\theta \simeq 0$ and $\gamma_{ij} \simeq 0$. In this case both the LG equation (2) and the boundary condition (7) are linear in the imaginary part of Δ_i and the problem is numerically equivalent to a resistor network problem with a constant current source applied across the top and bottom boundaries. The real part of Δ_i can be considered a constant throughout the superconducting regions and hence does not affect these equations except for an overall scale factor.

These equations have been solved recently[12] for the case of a perfect superconductor (no defects) but with a single horizontal slit of length n sites cut into the lattice from the side to simulate a single large defect or weak link. Then the LG equations give the current distribution in the lattice with a single defect. The results are that the supercurrent peaks strongly (see Fig. 3, which is taken from Ref. 12) at the edge of the slit as the current is forced to go around the defect. The numerical results[12] indicate that the peak current at the tip of the defect is proportional, for large n, to

$$I_{\text{tip}} \propto n^{1/2} , \tag{8}$$

and the current decays back to the background current as one goes away from the defect tip as $1/r^2$ at large distance.

III. RANDOMLY DISORDERED NETWORKS

Our solution here is for a randomly disordered lattice with $p > p_c$ and we start at small I_{in} (region I), where there is still a superconducting path across the sample. We choose I_i^{ext} to be zero except along the top and bottom bus bars where the external current $I_i^{\text{ext}} = (I_{\text{in}}/L)$ is applied to each site (see Fig. 2). For a given value of I_{in} and a given random configuration of the fraction p of superconducting bonds we can find that superconducting bond in the network which carries the largest supercurrent (the most critical bond). This bond will be the first to go through the superconducting-to-normal transition when I_{in} is increased to $I_{\text{in}} = I_1$, which can be found easily since the equations are linear. This bond is then transformed to a normal bond and the new distribution solved to see if one or more superconducting bonds have currents at or above i_c and they are transformed to normal bonds. This process is repeated until there are no remaining bonds carrying a current greater than i_c. This procedure gives the equilibrium state for a given value of I_{in}. Then the external current is increased and the process repeated until the last superconducting path across the sample is broken, which will eventually happen at $I_{\text{in}} = I_b$. An example of the steps of a typical procedure of this breakdown is shown in Fig. 4. In the case shown, the current required to break the first bond was the highest so that the cascade which broke the network followed the breaking of the first bond and $I_1 = I_b$. Such was often case. Only rarely did more than a few bonds seem to be broken before I_b was reached.

The dynamics of the breakdown procedure used here was like that used by Takayasu[2] (in the resistor network problem) and indeed, just as in Ref. 2, produced fractal cascades of breakdown path about a critical defect rather than the more linear paths or cracks seen in Refs. 4 and 5. When only that superconducting bond carrying the maximum current is removed in each time step (a kind of adiabatic process), a linear crack results. When all superconducting bonds carrying current $i > i_c$ are removed in each time step (a more sudden process), fractal cascades develop. Nevertheless, as far as we could determine from our numerical data, the precise form of the breakdown process did not essentially change the breakdown current I_b or the way it scaled with sample size since the breakdown current I_b in both procedures followed closely that current I_1 required to break the first bond. Thus, we believe that the breakdown currents I_b quoted here are essentially independent of the dynamics used, but would like to see further studies of the various dynamical models. The single-defect case is discussed in more detail in Ref. 12.

At the breakdown current I_b a normal resistive region has grown (percolated) across the sample from left to right, cutting the last vertical superconducting path between the bus bars. At this point a voltage first appears across the sample and is supported by the percolating normal region (or regions).

It is important at this point to describe our handling of the normal regions. We assume each resistor in the normal region has a constant resistance $R = 1$. All nodes of a normal cluster connected to a single superconducting cluster are at the same voltage, that of the superconducting cluster. The current flowing in the normal cluster must obey Kirchoff's laws, and current must be conserved at each node on the boundary connecting normal

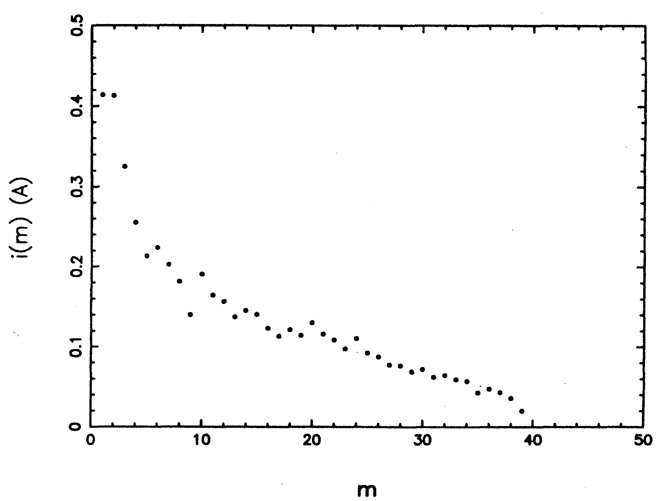

FIG. 4. An example of a typical breakdown process at $p = 0.90$ on a 50×50 lattice is shown. The current required to make the mth bond go normal is plotted vs m. After the 39th bond went normal the superconducting path across the sample was broken. Since the highest point occurs at $m = 1$, this example is one where $i_1 = I_1/L = I_b/L = 0.415$, i.e., the current required to destroy the first superconducting bond is sufficient to break the superconducting network.

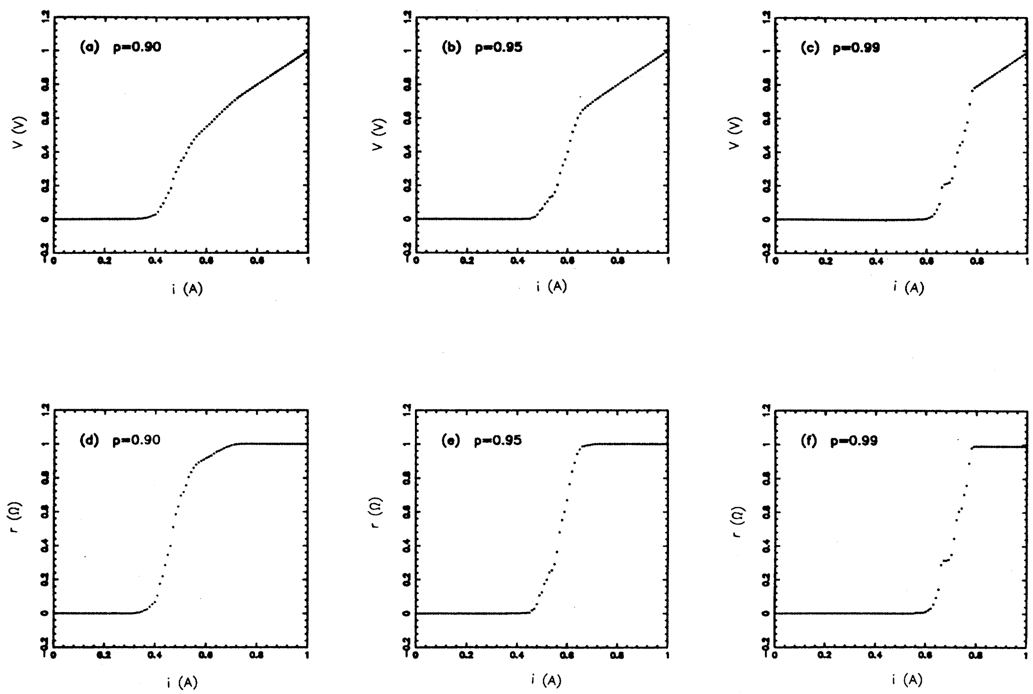

FIG. 5. (a)–(c) Voltage-current relations averaged over N 20×20 samples at $p=0.90$ ($N=100$ samples), $p=0.95$ ($N=200$ samples), and $p=0.99$ ($N=200$ samples). (d)–(f) Resistance-current relations obtained from the voltage-current curves above by dividing by the input current.

and superconducting regions. In the mixed region II, as soon as the superconducting path is broken we search the lattice to trace out the various normal and superconducting regions of the sample. Constant values of the potential are assigned to the top and bottom surfaces of a normal percolating cluster which horizontally spans the sample. The uniform external potential of the top and bottom is kept fixed until the relative interior voltage at each node within the normal cluster can be calculated by Kirchoff's law. Then the potentials on top and bottom are adjusted until the total current flowing across the normal region is equal to I_{in}. By this means the total voltage across the sample is determined. If there are isolated superconducting regions surrounded by a single normal region, that superconducting region is shrunk to a single virtual node for this purpose since it must be at constant potential. These special virtual nodes are treated as having as many neighbors as there were bonds connecting the isolated superconducting region.

Once the current distributions in the normal-phase regions have been calculated, we then have the appropriate current inputs and outputs to each node of the superconducting region so that the current distribution within these regions can then be calculated via the LG equation, as was done above for the superconducting phases in region I.

IV. COMPUTER-SIMULATION RESULTS

We ran computer simulations on 20×20 square lattices for several values of the initial superconducting bond concentration p. The results were averaged over 100 randomly configured samples at each concentration. The results for the voltage and resistance versus concentration are shown in Fig. 5 for the entire range of input current I_{in} as the system goes from superconducting (region I) to mixed (region II), to entirely normal (region III). For these samples $i_c=1$ so that in the perfect lattice ($p=1.0$) the superconducting-to-normal transition would be sharp

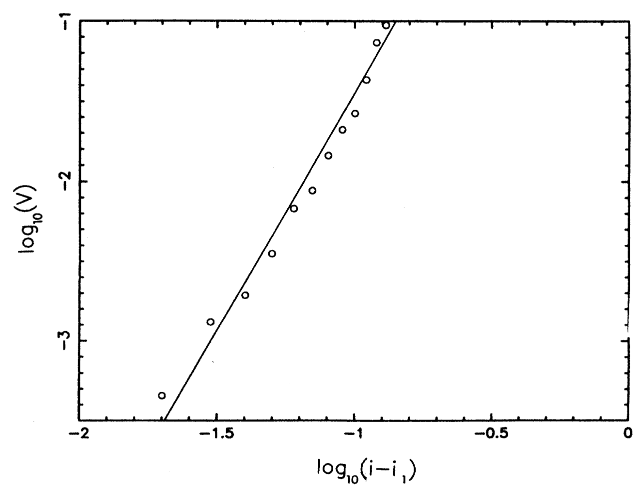

FIG. 6. The data of Fig. 5(a) for $0.32<i=I_1/L<0.50$ replotted as $\log_{10}(v)$ vs $\log_{10}(i-i_1)$, for $i_1=0.30$. The solid line has a slope of 3.0 indicating $x=3$ in Eq. (9).

at $I_{in}/L = i_c = 1$. For all cases shown here, where $p < 1$, the transitions I_1/L and I_2/L occur at values less than 1 and increase as p tends to 1. Also, we notice that the width of the transition $\Delta I = I_2 - I_1$ shrinks toward zero as $p \to 1$, but nevertheless is quite large. For a single vertical defect in an otherwise perfect infinite lattice, one would expect a sharp transition from region I to region III at a value of I_{in}/L presumably equal to $(\pi/4)i_c = 0.79 i_c$ (see the discussion of a single defect in Ref. 4). One can see here that at $p = 0.99$ this limit is being approached but that there is still a considerable width to the transition. This width at $p = 0.99$ is statistical and due to the finite sample size (20×20) since defects near the edge of the sample will tend to break at lower values of I_{in} than those in the center, and since we are seeing here an average over a distribution of random configurations. As p decreases further for the 20×20 samples, the larger defect clusters and neighboring defect clusters produce still-lower values of I_1 and I_2 and larger values of the transition width ΔI.

Many experimental voltage-current measurement[13–15] and calculations[16] in granular superconductors have been made over the last several years. These authors report a voltage-current relation just above I_1, which behaves according to the empirical formula

$$V/L \propto (I_{in} - I_1)^x , \tag{9}$$

where, in two dimensions, x approaches 3.0 as $T \to T_c$. The linearization of the GL equations here corresponds to neglecting the Δ^3 term which is valid near the critical point. We have plotted our data in Fig. 6 to test this behavior. We treated I_1 as a parameter to find the best straight-line fit to the data at $p = 0.90$; the resulting $I_1/L = (0.30 \pm 0.03) i_c$ gives $x = 3 \pm 0.5$ in close agreement with our expectation.

In order to study the sample-size dependence of the critical current we note that Duxbury, Leath, and Beale[6] (DLB) have predicted for a variety of breakdown processes that the scaling behavior of the breakdown current would be given by

$$I_1/L \sim 1/[1 + K(\log_{10} L)^{\alpha}] , \tag{10}$$

where K is a concentration-dependent constant, and where α is the exponent in the current enhancement induced by the most critical defect at its tip versus the defect length (or other appropriate dimension). Their formula is

$$i_{tip} \sim (I_{in}/L)(1 + k n^{\alpha}) , \tag{11}$$

where k is a constant. For the case of linear defects perpendicular to the current flow in two dimensions, Eq. (8) suggests that $\alpha = \frac{1}{2}$ although other critical defect configurations may give $\alpha = 1$. In order to test the DLB size scaling theory[6] we have carried out measurements of I_1 versus sample size, averaged over an ensemble of 50 $L \times L$ samples for each sample size L and each concentration shown. [In fact we did not actually measure I_b, the breakdown current for the superconducting path, but simply the current I_1 required to make the first superconducting bond go normal, which, we find, scales in the same manner (see Ref. 6) and indeed in the superconducting case we often found (as in Fig. 4) that the current required to make normal the first superconducting bond produced a cascade which then broke the superconducting path.] The results are shown in Fig. 7(a), where the value of I_1 is seen to continuously decrease with increasing sample size. According to Eq. (10), this decrease should be linear if $\alpha = 1$, and if it is plotted as L/I_1 versus $\log_{10} L$. This linear behavior is seen in Fig. 7(b). The data obtained are over a sufficiently small range (I_1 varies only by, say, a factor of 2) that we cannot distin-

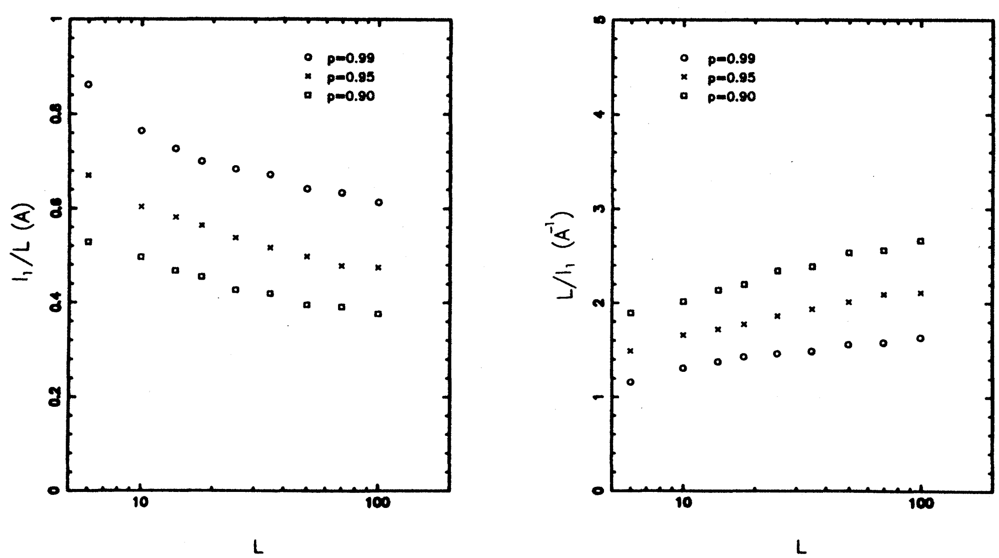

FIG. 7. (a) The breakdown current I_1/L vs sample size ($\log_{10} L$) averaged over 50 samples at each value of L for $p = 0.90$, 0.95, 0.99. (b) The data of (a) replotted as L/I_1 vs $\log_{10} L$ to test whether there is a straight line as predicted by Eq. (10), for $\alpha = 1$.

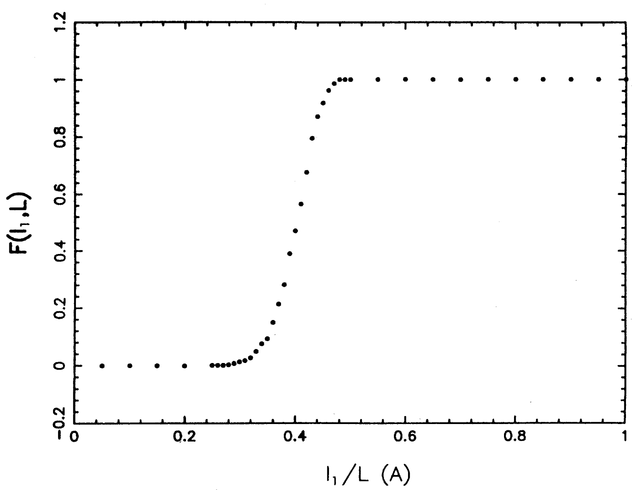

FIG. 8. The probability $F(I_1,L)$ of failure at current I_1 on an $L\times L$ lattice vs I_1/L for $L=50$, and $p=0.90$. This distribution was made from an ensemble of 500 random configurations.

guish clearly between values of α. Much larger samples than the 100×100 maximum here would be needed to accurately determine α. Nevertheless, log-log plots of $\log(L/I_1)$ versus $\log_{10}(\log_{10}L)$ give varying values of α in the range $\frac{1}{2}<\alpha<1$.

Finally, we report results for the failure distribution rate $F(I_1,L)$, the probability that a sample of size L will fail (i.e., the superconducting path will be broken, or the critical supercurrent exceeded) versus applied current I_{in} for 50×50 samples. This distribution was obtained from an ensemble of 500 samples. The numerical results for $F(I_1,L)$ are shown in Fig. 8. Previously, Duxbury and Leath[5] have shown, from the statistics of extremes, that for cases where the probability of large critical clusters decays exponentially with cluster size (which should apply here), that the failure distribution is given in d dimensions by the formula

$$F_{\text{DL}}(I_1,L)=1-\exp[-cL^d\exp(-kL/I_1)] , \tag{12}$$

where c and k are constants. They argue that this distribution F_{DL} provides a better fit to the data in general than the Weibull form (which indeed should apply at $p=p_c$ or wherever the probability of a large critical cluster only decays algebraically with cluster size). The Weibull form is

$$F_W(I_1,L)=1-\exp[-cL^d(I_1/L)^m] . \tag{13}$$

In order to test the usefulness of these two formula we plot data for $F(I_1,L)$ by forming the quantity

$$A(I_1/L)=-\ln\{-\ln[1-F(I_1,L)]/L^d\} , \tag{14}$$

which according to Eq. (12) should behave as

$$A_{\text{DL}}(I_1/L)=-\ln C+kL/I_1 , \tag{15}$$

and according to Eq. (13) should behave as

$$A_W(I_1/L)=-\ln C+m\ln(L/I_1) . \tag{16}$$

The same data for $F(I_1,L)$ as in Fig. 8 are potted versus L/I_1 in Fig. 9(a) to test the Duxbury-Leath form [Eq. (12)], and versus $\log_{10}(L/I_1)$ in Fig. 9(b) to test the Weibull form [Eq. (13)]. The results clearly indicate that the Duxbury-Leath double-exponential form [Eq. (12)] is better for the data seen here. The deviation from the straight line in Fig. 9(b) is at the top of the graph which corresponds to the low-current toe (bottom of the S-shaped curve) in Fig. 8, where the samples are generally reliable, i.e., seldom break down. This is precisely the most difficult region to measure experimentally because the statistics of failure are so low, but also is the most im-

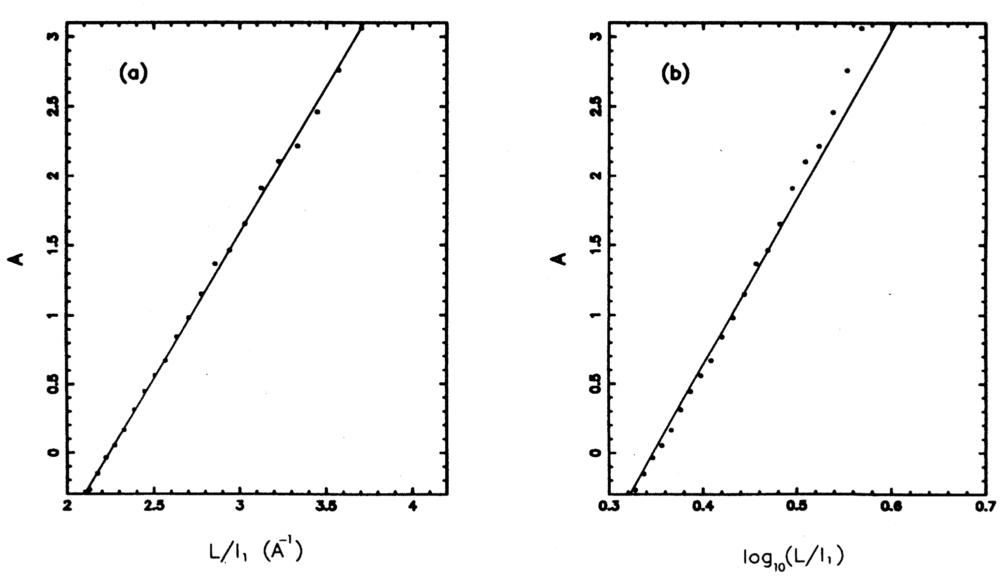

FIG. 9. A replot of the data in Fig. 8 in terms of $A(I_1/L)$ of Eq. (14). (a) Plotted vs L/I_1 to test Eq. (15). (b) Plotted vs $\log_{10}(L/I_1)$ to test Eq. (16).

portant region for commercial applications since one generally tries to build reliable systems. The difference in failure rates between the Duxbury-Leath and Weibull form can be a substantial factor in this high-reliability region. It seems likely that fitting experimental data to formula (12) will give much more accurate results when extrapolated to high-reliability conditions. It would be nice to have real experimental verification of these results.

These computer simulations were run on the VAX 8600 at Rutgers and on the Cyber 205 at the John von Neumann Computer Center. The conjugate gradient method was used to efficiently converge to the proper current distribution for each sample configuration. Small (up to 35×35) samples were run on a VAX 8600, where 35×35 samples typically took 350 sec each, and the large (up to 200×200) samples were run on the Cyber 205, where 200×200 samples typically took 32 sec each.

V. CONCLUSIONS

We have studied the critical current and voltage-current relations in a mixed superconducting-normal network of wires in two dimensions and found a strong dependence upon the distribution of defects or normal regions in the superconductor. Our model was to use the linearized Landau-Ginzburg equations of de Gennes[8] and Alexander[9] for a superconducting network and to treat it as a breakdown problem on the percolating network by assuming a critical current i_c for each superconducting bond. The results are consistent with the critical current going to zero logarithmically in the thermodynamic limit (as the sample size goes to infinity). The Duxbury-Leath formula for the failure distribution is found to accurately describe the data, and the voltage-current relations are in agreement with recent experimental results[13–16] on real, thin-film high-temperature superconductors. In fact, it was somewhat disappointing to us that everything fit so nicely with the existing breakdown theories which have been used for brittle fracture, dielectric breakdown, and electrical network failure, since there are no really new breakdown phenomena seen here. Nevertheless, this is the first breakdown approach to answer the question of critical current in superconductors, which is of great importance in applications. Thus, we look forward to experimental verification of the numerical results and theories outlined here. It would be particularly nice to see experimental results on sample-size dependence of the critical current and the failure-distribution curve.

On the other hand, there are many approximations in the model used here which will surely be possible to avoid and which indeed may lead to interesting new breakdown phenomena. First, the effect of varying the coherence length ξ_s and the temperature T should be explored. Second, the nonlinear terms in the Landau-Ginzburg equation should be studied. Even the de Gennes method[8,9] of discretizing the LG equation to a lattice could be tested here. And, of course, we have neglected Ohmic heating in the normal regions, which should make the superconducting-to-normal transition even more precipitous. Then, there remains the study of three-dimensional systems, where the most critical defects are probably disks (or, "penny-shaped" regions) perpendicular to the current flow and for which the DLB theory of breakdown has specific predictions.[6] In addition, the concentration region near percolation threshold should show critical exponents and a crossover to the Weibull distribution of failure rates. And finally, one should study the dynamic process as the network fails or goes normal, where there surely are interesting, measurable effects.

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