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Cooper-pair mass

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The Cooper-pair mass m^* is defined by use of the gauge-invariant momentum, $\mathbf{p}^* = m^*\mathbf{v} = [(h/2\pi)\nabla\phi - (e^*/c)\mathbf{A}]$, where ϕ is the phase of the macroscopically occupied wave function of Cooper pairs of charge e^* (twice the electron charge), $\mathbf{A}$ is the total magnetic vector potential, and we have taken the limit of $v \ll c$. We point out that for a general class of experiments in which the superconductor is in steady-state motion with respect to the laboratory (including uniform rotation), the physically observable Cooper-pair mass m' is measurable, where $m' = m^* + e^*\langle\Phi\rangle/c^2$ and $\langle\Phi\rangle$ is the expectation value of the total microscopic electrostatic potential in the bulk metal averaged over the single-electron states that contribute to the superconducting pair wave function. To lowest order $m' = m^* = 2m_e$, and to first order $m^*c^2 = \mu^* - e^*\langle\Phi\rangle$ and $m'c^2 = \mu^*$, where μ^* is the electrochemical potential (including the rest mass $2m_ec^2$ and the work function W). For niobium the intrinsic mass is $m^*/2m_e \approx 1.000\,18$, and the observable mass for experiments is $m'/2m_e \approx 0.999\,992$.

INTRODUCTION

Superconductivity provides two classic relations which involve the exact values of fundamental constants: the quantization condition for magnetic flux and the Josephson relation between voltage and frequency.[1] One might be tempted to add to this list a third relation, whose status, however, is somewhat less clear. London[2] argued that in the interior of a rotating superconductor there exists a uniform magnetic field which is proportional to the angular velocity of rotation ω by

$$\mathbf{B}_{\text{interior}} = -(2m_{\text{CP}}c/e^*)\boldsymbol{\omega} \ . \tag{1}$$

We now know that $e^* = -2|e|$ exactly and that the Cooper-pair mass parameter m_{CP} is given by $2m_e$ twice the electron mass, in the nonrelativistic limit of electron velocity.[3] Equation (1) follows easily by converting the uniform rotation to a uniform magnetic field using Larmor's theorem. In this paper, we clarify the nature of the first relativistic correction to m_{CP}. We show that m_{CP} receives corrections from the internal structure of the Cooper pair which depend on the microscopic properties of the superconductor, but which can be sensibly computed.

Currently, the size and sign of the corrections to m_{CP} are a matter of controversy in the literature. An approach to the problem from macroscopic thermodynamic arguments was begun by Anderson (with reference to Josephson),[4] and extended by Brady[5] and most recently by Anandan.[6] These authors have predicted $m_{\text{CP}} - 2m_e < 0$. A microscopic approach was followed by Cabrera, Gutfreund, and Little (CGL).[7] They obtained $m_{\text{CP}} - 2m_e > 0$, in clear disagreement. Our purpose here is to reconcile these two approaches. To do this, we need to take proper account of the atomic electrostatic potentials. We define two masses: m^* the *intrinsic* mass of the Cooper pair and m' the *observable* mass measured in experiments. As we shall see, m^* is greater than $2m_e$ as found in the CGL treatment, but the observable mass m' is less than $2m_e$, and m' is the quantity accessible to experiments.

As part of our argument, we show that the Cooper-pair mass parameter is observable not only in uniform rotation but also in a larger class of steady-state motions of superconductors. In particular, m^* appears when a local uniform translation at constant velocity is considered. A simpler and more transparent analysis of m^*, and later of m', can then be performed from the viewpoint of inertial frames rather than dealing with the more cumbersome mathematics of accelerated rotating frames.

COOPER-PAIR MOMENTUM

Let us begin by analyzing a piece of superconductor in uniform translation, and relating this system to a ring of superconductor in uniform rotation. That such a relation should exist is clear from Fig. 1. Figure 1(a) shows a thin (but still macroscopic) superconducting ribbon set into steady-state motion around two-rollers. As the roller diameter is decreased, a larger percentage of the path length around the ribbon moves at a fixed constant linear velocity with respect to the laboratory frame. We will argue that *the flux quantization condition receives equal contributions per unit length from each uniformly moving segment as well as each accelerated segment.* Then, in the limit of small rollers, the effects of the ends (where the superconductor clearly undergoes acceleration) become negligible. Figure 1(b) shows that a more complicated arrangement of rollers and uniformly moving segments can

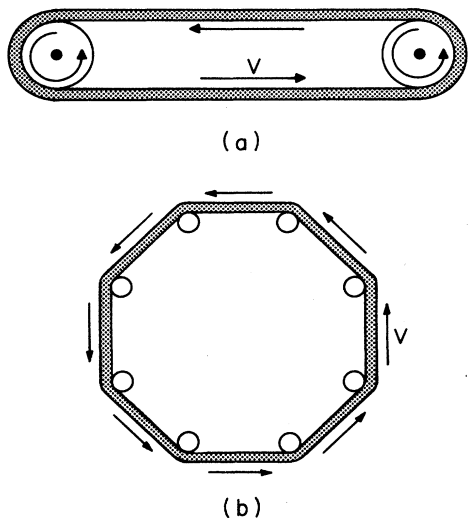

FIG. 1. (a) Schematic of flexible superconducting pulley with most of path length undergoing uniform translation, and (b) approximate uniform rotation by piecewise uniform translation.

approximate a uniform rotation.

A simply-connected quiescent superconductor at rest in the laboratory has a superconducting order parameter $\psi=|\psi|e^{i\phi}$ with uniform spatial phase ϕ, if we choose a gauge in which the macroscopic $\mathbf{A}$ field is zero. If this piece of superconductor is set into uniform motion, we should expect to see a spatial variation of the order-parameter phase. We can interpret the order parameter ψ as the macroscopically occupied wave function of Cooper pairs and we expect this function to have the form $\psi_o e^{2\pi i \mathbf{p}^* \cdot \mathbf{x}/h}$ when the assemblage of Cooper pairs is in motion. In principle, we can obtain an expression for $\mathbf{p}^*$ by Lorentz boosting the superconductor from rest; however, we must be careful always to refer the boost to the same gauge condition that we had originally imposed. The easiest way to keep track of this condition is to work, with the gauge-invariant momentum $[(h/2\pi)\nabla\phi-(e^*/c)\mathbf{A}]$. For a superconductor in uniform motion with velocity $\mathbf{v}$, we should expect that this vector be uniform in space (at the macroscopic level) and that it point in the direction of $\mathbf{v}$. Thus we may define m^* by the relation

$$m^*\mathbf{v}=(h/2\pi)\nabla\phi-(e^*/c)\mathbf{A} \tag{2}$$

for $\mathbf{v} \ll c$. In the nonrelativistic approximation, the phase variation described in Eq. (2) reflects the momentum of the Cooper-pair electrons; thus $m^* \approx 2m_e$.

Equation (2) can be connected to more standard equations of superconductivity by using it in conjunction with the assumptions of Ginzburg-Landau theory.[8] In this mean-field approximation of marcroscopic variables, observables are local functions of the order parameter ψ. Actually, it is conventional to write the current-density equation of this theory in terms of a wave function Ψ for superconducting electrons as

$$\mathbf{j}=(e^*/m^*)\Psi^*\Psi[(h/2\pi)\nabla\phi-(e^*/c)\mathbf{A}] \ . \tag{3}$$

The phase of Ψ is identical with that of ψ and $\Psi^*\Psi \approx (E_F/\Delta)\psi^*\psi$, where E_F is the Fermi energy and Δ is the superconducting energy gap ($E_F/\Delta \approx 10^3$ to 10^4). If we indentify the superconducting electron charge density $\rho=e^*\Psi^*\Psi$, we find that $\mathbf{j}=\rho\mathbf{v}$. Note that $\nabla\phi$ can be thought of as the wave vector $\mathbf{k}$ for the coherent state.

Defining the penetration depth $\lambda=(m^*c^2/4\pi e^{*2}\Psi^*\Psi)^{1/2}$, we can obtain several classic results[8] by rewriting Eq. (3) as

$$(4\pi e^*\lambda^2/c^2)\mathbf{j}=(h/2\pi)\nabla\phi-(e^*/c)\mathbf{A} \ , \tag{4}$$

where now m^* appears only in λ. If we assume that the only steady-state current within the superconductor is the supercurrent $\mathbf{j}$, then taking the curl of Eq. (4) and using Maxwell's equation curl $\mathbf{B}=(4\pi/c)\mathbf{j}$ leads to curl(curl $\mathbf{B}$)$=-(1/\lambda^2)\mathbf{B}$. Taking the curl once again gives the identical equation for $\mathbf{j}$, curl (curl $\mathbf{j}$)$=-(1/\lambda^2)\mathbf{j}$. The solutions to these equations always result in $\mathbf{B}$ and $\mathbf{j}$ exponentially attenuated to zero, within a characteristic length λ, as a function of the distance to the nearest surface of the metal. Thus the left-hand side of Eq. (4) is negligible in the interior of the superconductor.

By taking the line integral of Eq. (4) around a closed path Γ contained within the superconductor which bounds the surface S, we obtain

$$\int_\Gamma (4\pi\lambda^2/c)\mathbf{j}\cdot d\mathbf{l}=n(hc/e^*)-\int_S \mathbf{B}\cdot d\mathbf{S} \ , \tag{5}$$

where we have used the single-valued condition to set $\int_\Gamma \nabla\phi\cdot d\mathbf{l}=n2\pi$ (n an integer). Equation (5) is the fluxoid quantization condition of London.[2] For the path Γ everywhere many λ away from any surface, we obtain exact quantization of the magnetic flux in quanta of hc/e^*, and the result does not depend on an exact knowledge of λ. The discovery of flux quantization by Deaver and Fairbank and by Doll and Nabauer[9] in 1961 determined that $e^*=2e$ and was the first direct experimental evidence for Cooper pairing.

In circumstances where the left-hand side of Eq. (5) may be neglected, the exact value of λ is unimportant and the right-side gives a precise relation, independent of the assumptions in the Ginzburg-Landau theory, since the right-hand side of Eq. (4) depends only on the phase ϕ of the order parameter and not on its magnitude. Similarly, in situations involving uniform translation, Eq. (2) is exact. Of course, these exact relations involve the phenomenological mass parameter m^*, which so far we have determined only approximately.

To derive the exact consequences of Eq. (2), consider now a loop of superconductor, consisting of segments in uniform translation. Choose a gauge which makes $\mathbf{A}$ nonsingular over the closed path Γ (of length L) and over the surface S bounded by Γ. Then we may compute

$$\begin{aligned}\int_S \mathbf{B}\cdot d\mathbf{S}=\int_\Gamma \mathbf{A}\cdot d\mathbf{l}&=(hc/2\pi e^*)\int_\Gamma \nabla\phi\cdot d\mathbf{l}\\ &\quad-(c/e^*)\int_\Gamma [(h/2\pi)\nabla\phi\\ &\quad-(e^*/c)\mathbf{A}]\cdot d\mathbf{l}\end{aligned} \tag{6}$$

so that

$$\int_S \mathbf{B}\cdot d\mathbf{S}=n(hc/e^*)-(m^*c/e^*)\int_\Gamma \mathbf{v}\cdot d\mathbf{l} \ . \tag{7}$$

We argue that since the moving frame is locally inertial, the supercurrent must be zero in the interior as we have shown from Eq. (5) for a superconductor at rest in the laboratory. Thus, in the interior, **v** must be the lattice velocity. Equation (7) gives the generalization of Eq. (5) for superconducting loops in arbitrary (nonrelativistic) steady-state motion.

COMPUTING m^*

Now let us turn to the problem of computing m^*. Since we have related m^* to a gauge-invariant quantity in Eq. (2), we can determine m^* by a straightforward Lorentz boost. We must, however, account for the effect of the boost on both $\nabla\phi$ and on $\mathbf{A}$.

The boosted $\nabla\phi$ is obtained by noting that $P_\mu=[(h/2\pi)\nabla\phi,\ (h/2\pi c)\partial\phi/\partial t]$ is a Lorentz four-vector and that, for a superconductor at rest, the order parameter behaves in space and time according to the Josephson relation $\Psi(\mathbf{x},t)=\Psi(\mathbf{x})e^{2\pi i\mu^* t/h}$. A boost will produce a spatial variation of the phase given by $\phi=2\pi\mu^*\mathbf{v}\cdot\mathbf{x}/hc^2$. Properly, this relation follows if μ^*, the electrochemical potential of a Cooper pair, is referred to the boost-invariant vacuum state. That is, μ^* should be the free energy required to create a pair of electrons at infinity, bring them into the superconductor, and bind them into a Cooper pair (see Fig. 2). At zero temperature

$$\mu^*=2(m_ec^2-W)-\varepsilon\ , \tag{8}$$

where $W>0$ is the work function of the superconducting metal and ε (with $0<\varepsilon\ll W$) is the condensation energy per Cooper pair.[8] This energy $\varepsilon\approx10^{-6}E_F$ and is given by $N(0)\Delta^2$ for a weakly coupled BCS superconductor at zero temperature, where $N(0)$ is the normal density of states at the Fermi surface.

The **A** field which appears in Eq. (2) should be the macroscopic **A** field acting on the Cooper pair. In the superconductor at rest, this is zero; however, the boost gives a contribution to **A** from the total electrostatic potential Φ in which the Cooper pairs move [$A_\mu=(\mathbf{A},\Phi)$ is a Lorentz four-vector]. The term Φ is the spatial mean-field interaction potential of the pair with all of the other electrons and nuclei in the metal. If $\langle\Phi\rangle$ is the correct macroscopic average of this potential, then the **A** induced by the boost is

$$\mathbf{A}=(\mathbf{v}/c)\langle\Phi\rangle\ . \tag{9}$$

In the BCS theory, $\langle\Phi\rangle$ is given[7] by

$$\langle\Phi\rangle\equiv\int d^3k\,|\Psi_{\rm BCS}(\mathbf{k})|^2\langle\mathbf{k}|\Phi|\mathbf{k}\rangle\ , \tag{10}$$

where the average is formed from the expectation values of single electron states weighted by the BCS ground-state occupation density in **k** space (nonzero only for $|E_F-E_{\mathbf{k}}|\approx\Delta$). Since $\Delta/E_F\approx10^{-3}$ to 10^{-4}, it is sufficient to average over states at the Fermi surface, and $\langle\Phi\rangle\approx\langle\mathbf{k}_F|\Phi|\mathbf{k}_F\rangle_{\rm ave}$, as pointed out by CGL.[7]

Combining the Lorentz boosts of Eq. (8) and (9) with Eq. (2) we obtain

$$m^*c^2=\mu^*-e^*\langle\Phi\rangle=2(m_ec^2+\langle T\rangle)-\varepsilon\ , \tag{11}$$

where in the last expression all energies have been written as positive quantities (see Fig. 2). We have used the relation

$$|e\langle\Phi\rangle|-W\equiv\langle T\rangle\ , \tag{12}$$

to introduce $\langle T\rangle$, the single-particle kinetic energy of an electron, averaged over the Fermi surface. Note that m^*c^2 is the chemical potential (including the rest mass). Equation (12) makes it clear that, in estimating m^* from single electron states, one cannot use the familiar psuedo-potential cancellation between the kinetic and potential energies near the ion cores. The chemical potential is an intrinsic property of the bulb metal[10] and is independent of electric charges and electric dipole layers on the surface, a statement not true of either μ^* or $e^*\langle\Phi\rangle$.

The expectation value of Φ includes the interaction potential of the Cooper-pair electrons with all of the other electrons and nuclei in the metal, but it does not include the binding energy of the Cooper pair itself. Thus m^* does not directly contain any interaction energies other than the small condensation energy ε per pair. It is a good approximation, however, to neglect this small quantity. One then recovers the result of CGL.[7]

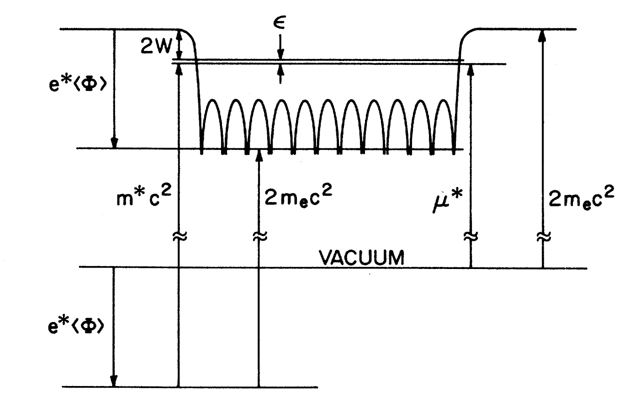

FIG. 2. Energy diagram for Cooper pairs which includes the work function $W<0$, the electrochemical potential μ^*, and the electrostatic potential $e^*\langle\Phi\rangle$.

LORENTZ BOOSTED SUPERCURRENT

Having unambiguously defined m^*, let us consider a Lorentz boost of Eq. (4). For this purpose it is convenient to define Lorentz four-vectors for the macroscopic quantities, the current density $j_\mu=(\mathbf{j},\ ce^*\Psi^*\Psi)$, the canonical momentum $P_\mu=[(h/2\pi)\nabla\phi,\ (h/2\pi c)\partial\phi/\partial t]$, and the vector potential $A_\mu=(\mathbf{A},\ \langle\Phi\rangle)$. As before, we are only interested in the nonrelativistic Galilean limit. We obtain

$$\mathbf{j}-\mathbf{v}e^*\Psi^*\Psi=(e^*/m^*)\Psi^*\Psi[(h/2\pi)\nabla\phi-(e^*/c)\mathbf{A}-m^*\mathbf{v}]\ , \tag{13}$$

where we have assumed $\mathbf{v}\ll c$. Note that this equation is identical with Eq. (3) where we have subtracted equal quantities from both sides. In the laboratory frame $\mathbf{j}_{\text{total}}=\mathbf{j}-\mathbf{v}e^*\Psi^*\Psi$, the first term being the supercurrent and the second the normal current. Taking the curl twice and utilizing Maxwell's equation curl $\mathbf{B}=4\pi/c\,\mathbf{j}_{\text{total}}$, we obtain curl $(\text{curl} j_{\text{total}})=-(1/\lambda^2)\mathbf{j}_{\text{total}}$, so that the left side of Eq. (13) is exponentially attenuated many λ away from any surface as was true for the supercurrent alone in Eq. (4). Neglecting the left side of Eq. (13) we obtain Eq. (2), and integrating Eq. (13) around a closed path Γ which remains everywhere within the superconductor yields

$$\int_\Gamma (4\pi\lambda^2/c)\mathbf{j}_{\text{total}}\cdot d\mathbf{l} = n(hc/e^*)-(c/e^*)\int_\Gamma [m^*\mathbf{v}+e^*/c\,\mathbf{A}]\cdot d\mathbf{l}\ , \tag{14}$$

where again the singled-valued condition on the phase was used. For Γ in the interior and many λ away from all surfaces,

$$nh=\int_\Gamma [m^*\mathbf{v}+e^*/c\,\mathbf{A}]\cdot d\mathbf{l}\ , \tag{15}$$

exactly the Bohr-Sommerfeld quantum condition for the canonical momentum. Notice that this exact relation depends solely on the phase of the order parameter and that the Cooper pairs are at rest with respect to the lattice in the interior of the superconductor. This result must hold since locally the moving frame is inertial and cannot be distinguished from the superconductor at rest.

Returning to the pulley arrangement in Fig. 1(a) and picking a path Γ everywhere parallel to $\mathbf{v}$, we obtain $\int_\Gamma m^*\mathbf{v}\cdot d\mathbf{l}=m^*vL$, where L is the path length around Γ. Equation (15) becomes

$$\int_S \mathbf{B}\cdot d\mathbf{S}=n(hc/e^*)-(m^*c/e^*)vL\ . \tag{16}$$

For each integer n there exists a velocity v_n such that

$$m^*v_n=nh/L\ , \tag{17}$$

an explicit form of the de Broglie relation for Cooper pairs.

THE OBSERVABLE MASS m'

Equation (7), which leads to Eqs. (16) and (17), is a fundamental relation for the magnetic flux enclosed by a loop immersed in a moving superconductor. However, a further step is necessary to connect this relation to observable quantities. In general, one observes flux quantization in a superconducting loop by measuring the magnetic flux coupling through a second circuit. This sensor is external to the superconducting material of the sample loop. For a sufficiently thick superconductor at rest, there is no difference in the magnetic flux through nearby loops inside and outside of the material. But in our analysis of a moving superconductor, we need to include the contributions of Eq. (9) from the microscopic atomic electrostatic potential. This vector potential appears only inside the metal, whether it is superconducting or normal, and does not couple to an external magnetometer. Thus, this contribution to the intrinsic mass m^* should not be observable.

To better understand this point, consider a simple example, as shown in Fig. 3, for the atomic electrostatic potential in a spherically symmetric Wigner-Seitz (WS) cell. The electric field produced by each WS cell is nonzero only within each cell. In the lattice reference frame we assume that there is no magnetic field thus the electric field $\mathbf{E}'=-\nabla\Phi'$. Then when viewed from a laboratory frame, there is a magnetic field everywhere given by $\mathbf{B}=-(\mathbf{v}/c)\times\mathbf{E}'=(\mathbf{v}/c)\times\nabla\Phi'=\nabla\times[(\mathbf{v}/c)\Phi']$. Thus to within a gauge transformation $\mathbf{A}=(\mathbf{v}/c)\Phi'$. In our symmetric example the magnetic field also is confined within each WS cell, thus magnetic flux can only couple to surfaces S_Γ bounded by paths Γ within the lattice. The contribution to the total vector potential from the atomic electrostatic potentials is zero outside of the lattice so that surfaces bounded outside of the superconductor, for example at the magnetometer pickup loop, couple no net magnetic flux.

To properly account for the effects of the microscope contributions to $\mathbf{A}$, which contribute to the shift in the order-parameter phase but are not detectable outside of the material, we write

$$\mathbf{A}=\mathbf{A}_{\text{obs}}+(\mathbf{v}/c)\langle\Phi\rangle\ , \tag{18}$$

where $\mathbf{A}_{\text{obs}}$ contains the contributions from all macroscopic currents. Note that for uniform translation curl $\mathbf{A}=$ curl $\mathbf{A}_{\text{obs}}$ so that the discussion following Eq. (13) is unaffected. It is also convenient to write

$$m^*=m'-e^*\langle\Phi\rangle/c^2\ , \tag{19}$$

and from Eq. (11) we can identify m' with μ^*/c^2, the electrochemical potential for a Cooper pair which includes twice the electron rest energy. The canonical momentum remains an invariant since

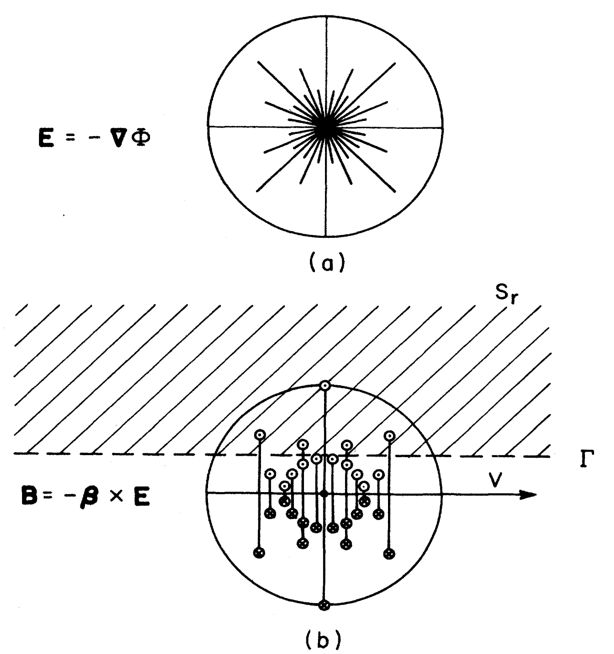

FIG. 3. (a) Electrostatic potential of Wigner-Seitz cell in lattice rest frame; (b) transforms to magnetic vector potential in laboratory frame.

$$m^*\mathbf{v}+(e^*/c)\mathbf{A}=m'\mathbf{v}+(e^*/c)\mathbf{A}_{obs} \tag{20}$$

as it must to maintain the quantization condition. All of the previous equations can be written in terms of quantities observable from outside of the superconductor by replacing $\mathbf{A}$ and m^* with $\mathbf{A}_{obs}$ and m', respectively.

To illustrate these results, let us consider the predictions for niobium in detail. D. Liberman[11] has obtained $\langle T\rangle \approx 91.6$ eV from a self-consistent calculation on a symmetric WS cell with the volume of the unit cell in bulk niobium. The cell is embedded in a sea of electrons with Fermi energy equal to that of niobium. From an independent estimate based on the Herman-Skillman tables for a single neutral niobium atom, CGL (Ref. 7) obtained $\langle T\rangle \approx 92$ eV. The work function for niobium is ~ 4.0 eV.[12] From Eq. (11) we obtain for the intrinsic Cooper-pair mass $m^*c^2 \approx 2m_ec^2+$ 184 eV or $(m^*/2m_e)_{Nb} \approx 1.000\,180$, an increase over twice the electron rest mass of 180 ppm dominated by the kinetic energy for two electrons averaged over the Fermi surface. Also, we find the observable Cooper-pair mass in an experiment to be $m'c^2 \approx 2m_ec^2$ -8 eV or $(m'/2m_e)_{Nb}$ $\approx 0.999\,992$, a decrease over twice the electron rest mass of 8 ppm which is dominated by the work function for two electrons averaged over the Fermi surface.

One might argue that the intrinsic mass m^* is in principle measurable using a direct probe of the magnetic field through the interior bulk material of a uniformly rotating superconductor. Such a probe could be constructed using neutron interferometer. The probe would sense the interior field by the precession of the neutron magnetic moment along a path through the material. However, for a rotation velocity of 100 Hz the differential field $\mathbf{B}-\mathbf{B}_{obs} \approx 10^{-8}$G is too small to detect with current technology using cold neutrons. In addition, one would expect the field contribution from the interior electrostatic potential as measured by a neutron probe to differ from that derived from Eqs. (9) and (10), because the neutrons will sample the interior nearly uniformly. Thus for neutrons Eq. (10) would be replaced by a nearly uniform spatial average of the electrostatic potential. For Nb the uniform spatial average of the electrostatic potential is about 15 V, six times smaller than its expectation value for the Cooper pairs.

Note that an additional but smaller surface electrostatic potential exists ($\Phi_{surface} \approx -1$ eV) because electrons extend out further at the surface, giving the surface WS cells a net electric dipole moment, and because adsorbed surface gas layers can contribute an additional electric dipole layer. This term, which can change depending on the sample surface preparation, is included in the work function and does contribute to m' but not to m^*. Such variations of less than 1 ppm are not of concern in the present treatment, but may determine the ultimate resolution for measuring m'.

UNIFORMLY ROTATING RING

Beginning with London, a number of authors have analyzed the case of uniform rotation.[2-7] We argue that Eqs. (13)–(17) are also applicable to the case of uniform rotation at angular velocity $\boldsymbol{\omega}$ by making the substitution $\mathbf{v}=\boldsymbol{\omega}\times\mathbf{r}$ and assuming $|\boldsymbol{\omega}\times\mathbf{r}| \ll c$. In the interior, many λ away from any surface, the left-hand side of Eq. (13) is negligibly small and the curl of the right side yields

$$\mathbf{B}=-(m^*c/e^*)\text{curl}\mathbf{v}$$

or

$$\mathbf{B}_{obs}=-(m'c/e^*)\text{curl}\mathbf{v}\ , \tag{21}$$

where $\mathbf{B}_{obs}=\text{curl}\,A_{obs}$. The magnetic field strength inside of the material is $\mathbf{B}$, whereas the field strength on the inside of a spherical void in the material is $\mathbf{B}_{obs}$. From Eq. (21), those portions of a superconductor under uniform translation have $\mathbf{B}=\mathbf{B}_{obs}=\mathbf{0}$, whereas for uniform rotation curl $(\boldsymbol{\omega}\times\mathbf{r})=2\boldsymbol{\omega}$ and

$$\mathbf{B}=-(2m^*c/e^*)\boldsymbol{\omega}$$

or

$$\mathbf{B}_{obs}=-(2m'c/e^*)\boldsymbol{\omega}\ , \tag{22}$$

the London relation of Eq. (1). To illustrate uniform rotation, Fig. 4 shows the vector potential and magnetic field strength plotted as a function of radius for a uniformly rotating solid cylinder [Fig. 4(a)] and for a uniformly rotating hollow cylinder [Fig. 4(b)]. In the interior of the material there exists a larger value for the expectation values of the vector potential $\mathbf{A}$ and magnetic field $\mathbf{B}$, and any measurements performed with magnetometers either outside of the cylinder or in the central core of a hollow cylinder are sensitive only to $\mathbf{A}_{obs}$ or $\mathbf{B}_{obs}$.

Turning again to the case of uniform rotation, it has been argued that, since the mass appearing in the London

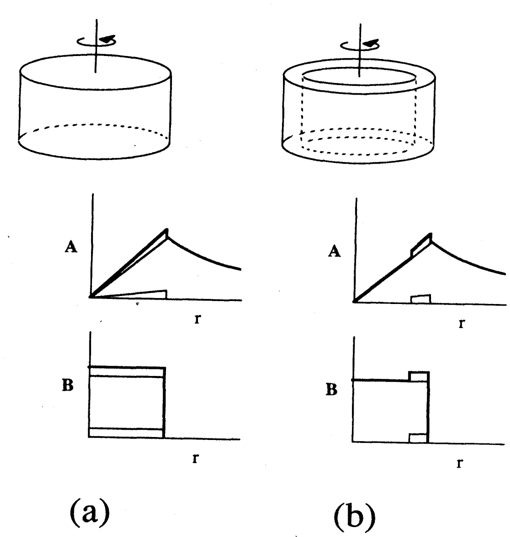

FIG. 4. The magnetic vector potential $\mathbf{A}$ (entirely azimuthal) and the magnetic field $\mathbf{B}$ (entirely along the spin axis) plotted as a function of radius (a) for a uniformly rotating solid cylinder and (b) for a uniformly rotating hollow cylinder. The contributions from the electrostatic potential are greatly exaggerated so as to be seen.

expression can be derived from the Larmor theorem of the kinematic equivalence between a uniform rotation and the application of a uniform magnetic field, it is not related to the m^* appearing in the Ginzburg-Landau equations.[8] We point out that the results of Eqs. (13) and (14) are more general than those derived from an assumption of uniform rotation. The term containing m^* or m' will appear on the right-side of Eq. (15) whenever the circulation $\int_\Gamma \mathbf{v}\cdot d\mathbf{l}$ around a closed path is not zero. Note also that in Fig. 1(a) the contributions from the sections undergoing uniform translation are equal per unit path length to those undergoing uniform rotation, even though for the former curl $\mathbf{v}=\mathbf{0}$ locally.

It is important to point out that precise experiments based on Eq. (14) are performed on thin-film superconducting rings. Then the left-hand side of Eq. (14) is not negligible; however, Eq. (17) still holds since both $\mathbf{j}_{\text{tot}}$ and $\mathbf{A}_{\text{obs}}$ become zero together for each n. Thus, although the magnetic flux becomes a smaller fraction of a fluxoid with decreasing film thickness, $v_n - v_{n-1}$ remains exact.

The most precise measurement utilizing Eq. (17) has been made on a uniformly rotating ring with its spin and symmetry axes coincident.[13] The analysis has been based on uniform rotation, but can be easily obtained from Eq. (23) by setting $L=2\pi R$, where R is the radius of the ring, and $v=\omega R$. Then $m'\omega_n = nh/2\pi R^2$ and setting the area $S=\pi R^2$ and $\Delta\omega=\omega_n-\omega_{n-1}$ we obtain

$$h/m' = 2\,\Delta\omega\, S \ . \tag{23}$$

It is m' in Eq. (23) which appears in the Larmor theorem for uniformly rotated superconductors (not $2m_e$) but more fundamentally m' and m^* are derived from the quantum-mechanical canonical momentum $\mathbf{P}=m^*\mathbf{v}+(e^*/c)\mathbf{A}=m'\mathbf{v}+(e^*/c)\mathbf{A}_{\text{obs}}$ of the Cooper pairs.

CONCLUSIONS

In the nonrelativistic limit for single-electron velocities, $m'=m^*=2m_e$. To include the first-order relativistic corrections, we find that we must consider the single-electrons states which make up the pairing function (those very near the Fermi surface). The relativistic shifts to the intrinsic mass m^* come from the velocity-mass shift of the single electrons and not from the center-of-mass velocities of the pairs, which are highly nonrelativistic. We find m^*c^2 is the chemical potential (including the rest masses) for two single electrons on the Fermi surface, defined as the difference between the electrochemical potential and expectation value of the electrostatic energy, minus ε (the condensation energy per Cooper pair), a negligible quantity. The mass m^* is an intrinsic property of the bulk metal and could in principle be deduced from measurements with a neutron interferometer. For most superconductors m^* is 100–200 ppm greater that in $2m_e$. Experiments with magnetometers do not directly measure the intrinsic mass m^*, but rather the observable mass m', which is smaller than twice the free electron mass by twice the work function. For most superconductors m' is about 10 ppm less than $2m_e$. As with the work function, m' is affected by changes in the electric dipole layer on the surface of the superconductor caused by differing surface structure and adsorbates. However, these variations are expected to be of order 1 ppm.

The treatment presented here is in agreement with previous treatments[4–6] which are based on the macroscopic approach and we conclude that the treatment of Cabrera, Gutfreund, and Little[7] correctly computed m^*, but did not include the entire contribution to the magnetic vector potential from the interior electrostatic fields when analyzing experiments with magnetometers.

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