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Empirical relation between gate voltage and electrostatic potential in the one-dimensional electron gas of a split-gate device

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We study the magnetic depopulation of one-dimensional subbands in a narrow channel in a GaAs-$Al_xGa_{1-x}As$ heterojunction. The channel is sufficiently short that transport through the channel is ballistic and the quantized resistance due to the subband structure is readily observed. We have made a detailed comparison between the experimental data and a variational calculation with a simple model potential. This potential is flat in the middle and parabolic at the edges and it is found to be clearly better than the commonly used parabolic potential. Both the channel thickness and the one-dimensional carrier concentration in the channel have been extracted as a function of gate voltage. The good agreement found with only two adjustable parameters indicates that this model potential gives a good description of the actual potential in the one-dimensional electron gas in a split-gate device.

There has been much interest in transport in quasi-one-dimensional systems in recent years. A convenient way to create a one-dimensional electron gas (1D EG) is to utilize a layered structure with a two-dimensional electron gas (2D EG) and define a split gate on top of the 2D EG using electron-beam lithography. By applying a negative bias to the gate, the electron gas under the gate is depleted and a narrow channel with a 1D EG can be formed. This technique of electrostatic squeezing is very convenient because the 2D EG is perturbed by external means and one can continuously change the carrier density and the channel width by changing the gate voltage. One disadvantage with this method is that the relation between the gate voltage and the electrostatic potential in the 1D EG is difficult to determine. This potential has often been approximated by a parabola. A self-consistent solution of Schrödinger and Poisson's equations has been performed for an infinitely long channel by Laux, Frank, and Stern.[1] This involves a considerable amount of numerical work and would become very laborious if the finite length of the channel were taken into account. Their calculation indicates that the potential is rather flat in the middle and increases roughly parabolically at the edges of the channel. This potential shows a strong resemblance with a simple model potential which has been put forward by Berggren and co-workers.[2,3] We use this potential, include the effect of a magnetic field, and determine the magnetic fields at which the 1D electron subbands are depopulated. By a suitable choice of the adjustable parameters, we have obtained very good agreement with experiment and from this we have been able to deduce empirical relations for the channel width and the carrier density as a function of gate voltage.

The experimental determination of the magnetic depopulation utilizes the recently discovered phenomenon of quantized resistance,[4,5] which gives a clear indication when the 1D subbands are depopulated. It has been demonstrated that a constriction in the 2D EG of a GaAs-$Al_xGa_{1-x}As$ heterojunction, which is sufficiently narrow that one-dimensional quantization occurs and sufficiently short that electrons pass through constriction ballistically, possesses a quantized resistance, $h/2ie^2$, where i is the number of occupied 1D subbands. The value of the quantized resistance, at temperatures sufficiently low that electron-electron and electron-phonon scattering mechanisms can be ignored in the channel regions, may be derived by considering the current contributed by a single one-dimensional subband:

$$I=\int n(v)ev\,dv\ , \tag{1}$$

where $n(v)$ is the appropriate 1D density of states in velocity space, g_sm^*/h, v is the velocity, and g_s is the spin degeneracy. Using conservation of energy across the channel, $m^*v_i^2+2eV=m^*v_f^2$, where v_i and v_f are the initial and final velocities, and integrating over occupied velocity space leads directly to

$$I=2e^2V/h\ , \tag{2}$$

where $g_s=2$ has been assumed and V is the potential dropped between the ends of the channel. The total resistance of the channel is then given by summing i identical resistances in parallel:

$$R=h/2ie^2\ , \tag{3}$$

where i, the number of occupied subbands, is dependent upon the carrier concentration in the constriction and the width of the narrow channel. This result may also be readily derived from the Landauer[6] multichannel formula, $g=\mathrm{Tr}\,tt^*$, where g, the dimensionless conductance, is determined for the case of perfect transmission in the absence of scattering between channels. It essentially states that the current contributed by each subband is the same; a consequence of the exact compensation that exists in one dimension between the density of states at the Fermi energy and the Fermi velocity.

In a transverse magnetic field the above analysis

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remains valid if the magnitude of the magnetic field is sufficiently low that one can neglect the spin splitting. This would give rise to additional plateaus at values of resistance $h/[2(i+\frac{1}{2})e^2]$ but experiments performed in parallel magnetic fields have shown that such spin splitting does not become apparent for fields less than 10 T (Ref. 5). The number of occupied subbands is now dependent not only upon carrier concentration and channel width, but also upon B, the applied magnetic field. The effect of a magnetic field on an electron in a parabolic potential has been considered by many authors. In this case, the problem can be solved analytically[7] and the essential effects of a transverse magnetic field upon the 1D subbands are twofold:[2] first, the magnetic field introduces an effective "magnetic" mass which is larger than the normal 2D effective mass and hence produces a flattening of the subband dispersions and an increase in the density of states. Additionally, the separation of the subband energies increases with B and hence leads to depopulation of the subbands as their energies pass through the Fermi energy.

Although the common use of a parabolic confinement potential is convenient for displaying qualitative features, we have found that it is insufficient to give a quantitative agreement with experiment. Instead, following Berggren and co-workers,[2,3] we model the confining potential in our system using a boxlike potential with parabolic walls.

$$V(x)=\begin{cases} m^*\omega_0^2([x]-t/2)^2/2, & [x]>t/2; \\ 0, & [x]<t/2; \end{cases} \quad (4)$$

where t may be regarded as the width of the channel and ω_0 is an empirical parameter determining the parabolicity of the channel walls. The results by Laux, Frank, and Stern[1] indicate that the numerically determined potential qualitatively behaves like (4) and that ω_0 is almost independent of the applied gate voltage for a given sample. The transverse magnetic field in the z direction is included by addition of an extra confining potential in the Hamiltonian:

$$V_B(x)=m^*\omega_c^2(x-x_0)^2/2\,, \quad (5)$$

where $\omega_c=eB/m^*$ is the cyclotron frequency and $x_0=\hbar k_y/eB$ is the center of the magnetic parabola which, in the absence of any electric confinement, may be equated directly with the center of the Landau state.[8]

Berggren and Newson[3] used the semiclassical Wentzel-Kramers-Brillouin approximation to find the energy levels in the potential given by (4) and (5). One disadvantage with this approach is that the lowest-energy levels become inaccurate. We have, therefore, performed a fully quantum-mechanical calculation with the use of a novel variational approach. We have first solved Schrödinger's equation for a finite square well and used the wave functions so found as a basis set for expanding the wave functions of the potential given by (4) and (5). The width of the square well was chosen to be slightly greater than t, the actual channel width, as this was found empirically to provide the optimum basis set for the expansion and hence also the most accurate eigenvalues. With these basis functions, which are easy to generate, the matrix elements of the kinetic energy and the potentials (4) and (5) are straightforward to evaluate analytically[9] and the energy levels are obtained by numerical diagonalization of a small matrix. For given values of B and t, the energies are determined for a series of values of k_y (or equivalently x_0) and in this way the Fermi level, which is determined by n_e, the density of electrons per unit length, is calculated numerically. By doing this for different magnetic fields we determine the B values at which different subbands are depopulated. This was described in more detail in Ref. 3. By comparison with the experimental results we have thus been able to determine t and n_e (and thus estimate $N_s \approx n_e/t$, the areal electron density in the constriction) empirically as a function of the applied gate voltage. The value of $\hbar\omega_0$ was taken to be constant, 7.5 meV.

The structure of the sample is similar to that used in previous experiments.[5] In the z direction it contained the following sequence of layers, grown by molecular-beam epitaxy on a semi-insulating GaAs substrate: several μm of undoped GaAs, a spacer layer of 200-Å undoped $Al_{0.3}Ga_{0.7}As$, 400 Å of n-type $Al_{0.3}Ga_{0.7}As$ with $n=4\times10^{17}$ cm^{-3}, and a capping layer of 100-Å GaAs. We used electron-beam lithography to define a short split gate on top of this structure. The channel length was less than 0.3 μm with a width smaller than 0.5 μm, while the heterostructure material had a mobility in excess of 10^6 cm^{2}V^{-1}s^{-1} at liquid-helium temperature and a carrier concentration of approximately 4×10^{11} cm^{-2}, yielding an elastic mean free path of approximately 10 μm. Thus the electron motion through the narrow channel of our device may be regarded as truly ballistic with negligible scattering. This is in contrast to our earlier experiments which utilized structures that were considerably longer than both the elastic and inelastic mean free paths.[2,10]

The depopulation experiments were performed at temperatures ~100 mK in the mixing chamber of a dilution refrigerator. All measurements were made using phase-sensitive techniques between optically defined voltages probes separated by 100 μm and thus included an additional resistance from the 2D EG in series with the constriction. In these experiments we chose to fix B and

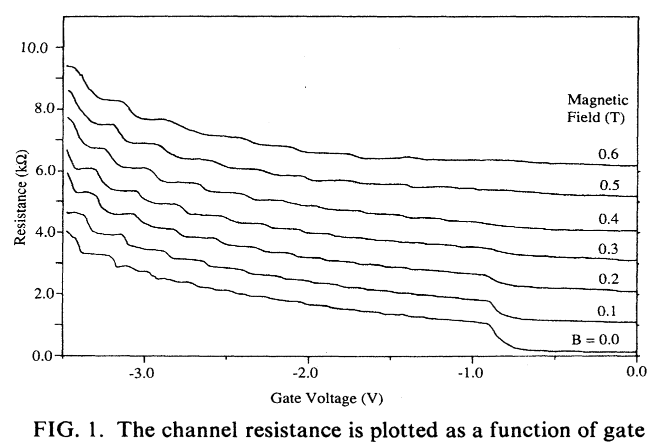

FIG. 1. The channel resistance is plotted as a function of gate voltage for different magnetic fields as indicated. The curves are offset vertically for clarity.

sweep V_g, the gate voltage, and hence were easily able to distinguish between the contribution to the resistance from the 1D channel and that due to the surrounding 2D EG.

Figure 1 shows the depopulation of the subbands as a function of V_g for several values of the magnetic field. As the magnetic field is increased, the gate voltage required to move from one quantized value of resistance to another decreases, showing that the subbands depopulate in wider channels at higher fields. We have chosen to regard the point of inflection between plateaus as defining the gate voltage required for depopulation of a particular subband. Figure 2 shows the experimental depopulation curves thus obtained, as a function of the electric and magnetic confinement together with the calculated values. We see that with only two adjustable parameters, the channel width t and the electron density n_e, we have managed to obtain an excellent fit of the depopulation of up to five subbands (for a given V_g). This good fit is a strong indication that the model potential (4) indeed gives a good description of the potential in the channel region and that the present method can be used to deduce the important device parameters t and n_e, which would be difficult to determine accurately by other means. In Fig. 3 we display these empirically determined values as a function of gate voltage. As can be seen, the channel width t varies linearly with voltage until we approach pinchoff when the channel narrows more rapidly. Fitting data at small values of applied gate voltage was difficult because of the inherent uncertainty of the point of depopulation when the step in resistance is small. However, assuming the channel variation to be essentially linear with voltage and extrapolating to the point when the channel is first defined ($V_g = -0.8$ V) gives us an estimated maximum channel width of 3200 Å. This value may seem small compared to the lithographically defined channel width, but we would expect the lateral depletion, due to the gates, to be similar in magnitude to the depletion depth beneath the gates. Taking this fact into account gives us good agreement with our lithographically defined width of 0.5 μm. The linear variation of the carrier concentration per unit length furthermore suggests a pinchoff voltage of -4.0 V in agreement with our experimental data.

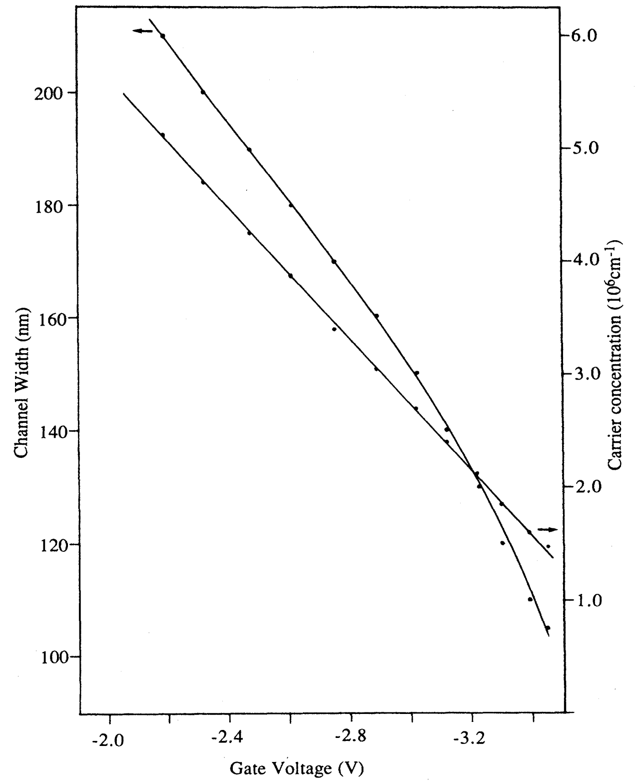

FIG. 3. The derived channel width (left scale) and carrier density per unit length (right scale) are plotted as a function of gate voltage.

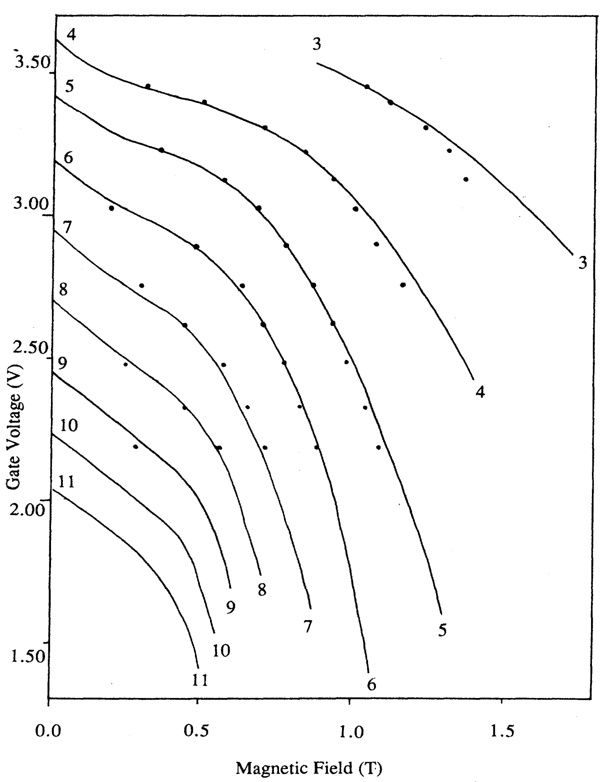

FIG. 2. The gate voltage required for subband depopulation is plotted as a function of the applied magnetic field. The solid curves illustrate the experimental results while the plotted points are the best fit values derived from the theoretical model. The subband index is indicated for each of the subbands plotted.

In conclusion, we have determined the magnetic field at which depopulation of 1D subbands occurs in a split-gate device. This has been done both experimentally, using the quantized resistance, and theoretically, using a variational calculation with a simple model potential for the 1D confinement. From the good agreement we have been able to deduce how the channel thickness and the carrier concentration vary with applied gate voltage. Although this method has been applied to a specific sample, the results obtained can be used as a reference for samples with different material parameters. For example, a thinner spacer layer and a higher donor concentration can be expected to increase the channel width and the carrier concentration at a given gate voltage. Our work indicates that a good description of the 1D confinement potential in a split-gate device is a flat base with parabolic side walls.

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