Two-stream instability in two-dimensional degenerate systems

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The collisionless Boltzmann equation has been used to study the collective excitations of a separated two-component two-dimensional plasma in the presence of external drift velocities, and a possibility of inducing growth instability in the low-frequency acoustic plasma mode is postulated.

Plasmons in multilayered two-dimensional electron systems (heterojunctions and superlattices) have been studied extensively, both experimentally and theoretically.[1] Das Sarma and Madhuker[2] have used the generalized random-phase approximation to obtain dispersion relations for collective modes in the simplest of these systems—two charge sheets separated by distance d. Krasheninnikov and Chaplik[3] obtained conditions for the amplification of the so-called optic plasmons in a system of two-dimensional electron-hole layers. Also, instability in one-component plasma has been considered by Kempa,[4] and the growth of bulk and surface plasmons in type-II superlattices has been probed by Hawrylak and Quinn.[5] Recently, the possibility of plasmon mediated superconductivity in these two-layered charge systems (e.g., double quantum wells) has been investigated,[6] and the role of both acoustic and optic modes was considered. In this Brief Report the self-consistent linearized method of Landau[7] is used to explore for the first time the possibility of growth instability in the (acoustic) space-charge waves associated with a degenerately doped two-component two-dimensional plasma in the presence of external drift velocities.

Consider two charge sheets of charges q_1 and q_2, masses m_1 and m_2, and, described by distribution functions $f_1(\mathbf{r},\mathbf{v})$ and $f_2(\mathbf{r},\mathbf{v})$, respectively, where $\mathbf{r}$ and $\mathbf{v}$ are two-dimensional position and velocity vectors. The equilibrium distributions functions, $f_i^0(\mathbf{r},\mathbf{v})$ in the presence of external drift velocities v_{D_1} and v_{D_2}, may be approximated by the corresponding displaced functions $f_i^0(\mathbf{r},\mathbf{v}-\mathbf{v}_{D_i})$. The coupled Boltzmann equation describing the system, neglecting exchange interaction between the charge sheets and collisions, is[8]

$$\left[\frac{\partial}{\partial t}+v_i\cdot\nabla_r\right]\delta f_i+\frac{q_i}{m_i}\nabla_r\phi_i\cdot\nabla_v f_i^0=0 \quad (i=1,2)\ , \tag{1}$$

where

$$\delta f_i=f_i-f_i^0 \tag{2}$$

and

$$\phi_i=\sum_{j=1}^{2}V_{ij}\ . \tag{3}$$

The two-dimensional potential V_{ij} is

$$V_{ij}=\begin{cases}-\dfrac{2\pi q_i}{\epsilon k}\delta n_j & (i=j)\\[2ex] -\dfrac{2\pi q_i}{\epsilon k}\delta n_j e^{-kd} & (i\neq j)\ ,\end{cases} \tag{4}$$

with

$$\delta n_j=\frac{m_j^2}{2\pi^2\hbar^2}\int d^2v\,\delta f_j(v-v_{D_j})\ , \tag{5}$$

d being the separation between the two charge sheets.

Properties of small amplitude plasmas are studied by seeking solutions of the form $\exp i(\mathbf{k}\cdot\mathbf{r}-\omega t)\exp(\eta t)$, where $\mathbf{k}$ and ω are the (two-dimensional) wave vector and frequency of the plasma wave, and η is small and positive. The two-dimensional nature of the problem, together with the transformation $(\mathbf{v}-\mathbf{v}_{Dj})\rightarrow V_j$, yields

$$\delta n_j=\frac{q_j m_j^2}{2\pi^2\hbar^2}\times\phi_j\int_0^\infty dV_j\int_0^{2\pi}d\theta_j V_j\times\frac{kV_j\cos\theta_j(\partial f_j^0/\partial E)}{(\omega-\mathbf{k}\cdot\mathbf{v}_{D_j}+i\eta)-kV_j\cos\theta_j}\ . \tag{6}$$

The angular integration, in the limit $\eta\rightarrow 0$, gives

$$\lim_{\eta\rightarrow 0}\int_0^\pi d\theta_j\frac{kV_j\cos\theta_j}{\omega-\mathbf{k}\cdot\mathbf{v}_{D_j}+i\eta-kV_j\cos\theta_j}=-\pi g(\lambda_j)\ ,$$

where

$$g(\lambda_j)=\begin{cases}1+i\lambda_j/(1-\lambda_j^2)^{1/2}, & \lambda_j^2<1\\ 1-\lambda_j/(\lambda_j^2-1)^{1/2}, & \lambda_j^2>1\end{cases} \tag{7}$$

with $\lambda_j=(\omega-\mathbf{k}\cdot\mathbf{v}_{D_j})/kV_j$. Also, one has, for a degenerate system,

$$-\frac{\partial f_j^0}{\partial E}=\delta(E-E_{F_j})\ , \tag{8}$$

where E_{F_j} is the Fermi energy of charges in the jth sheet, and, corresponds to a Fermi velocity of v_{F_j}. Then, Eqs. (4) and (6)–(8) yield

$$1+\chi_1+\chi_2+\chi_1\chi_2 p=0\ , \tag{9}$$

where

$$p=1-e^{-2kd}\ ,$$

$$\chi_i=\frac{Q_i}{k}g\left[\frac{\omega-\mathbf{k}\cdot\mathbf{v}_{D_i}}{kv_{F_i}}\right]\ , \tag{10}$$

and Q_i is the Thomas-Fermi wave vector

$$Q_i=\frac{2m_i e^2}{\epsilon\hbar^2}\ . \tag{11}$$

The dispersion relation for plasmons in a coupled-quantum well system is obtained by solving Eq. (9) for ω and is independent of the signs of the charges forming the two-component plasma. The plasma waves are undamped in the high-frequency region, $\omega-\mathbf{k}\cdot\mathbf{v}_{D_i}>kv_{F_i}$ $(i=1,2)$, since both χ_1 and χ_2 are real in this case. The frequencies of the two allowed modes, in the long-wavelength limit and in the absence of external drift velocities, are

$$(\omega_\pm)^2=\frac{Q_1 v_{F_1}^2+Q_2 v_{F_2}^2}{4}\times k\left[1\pm\left[1-\frac{4Q_1Q_2 v_{F_1}^2 v_{F_2}^2 p}{(Q_1 v_{F_1}^2+Q_2 v_{F_2}^2)^2}\right]^{1/2}\right]\ . \tag{12}$$

The low-frequency mode ω_- corresponds to the anomalous acoustic plasmon described by Das Sarma and Madhukar, and exists only when the two charge species are physically separated by a critical distance d_c or more, where $d_c=Q_2^{-1}+(v_{F_2}^2/v_{F_1}^2)Q_1^{-1}$. Here $v_{F_2}>v_{F_1}$, and the present value of d_c differs slightly from that of Ref. 2, and is in disagreement with the prediction of $d_c=0$ for $v_{F_1}=v_{F_2}$ by Santoro and Giuliani.[9] It is found that the optic mode ω_+ becomes disallowed for very large separations ($d\to\infty$) and $k>Q_2/2$ (once again, assuming $v_{F_2}>v_{F_1}$).

The value of χ_2 is complex in a low-frequency region, $\omega-\mathbf{k}\cdot v_{D_1}>kv_{F_1}$ and $\omega-\mathbf{k}\cdot v_{D_2}<kv_{F_2}$, so that Eq. (9) admits to complex solutions for ω, $\omega=\omega_k+i\gamma_k$. In general, one has to solve the coupled transcendental equations for ω_k and γ_k. However, in the approximation $\gamma_k\ll\omega_k$,[10] it is sufficient to retain the first term of the Taylor expansion of $g(\lambda_i)$ in terms of γ_k, and one obtains

$$\omega_k=\frac{Q_1(1+Q_2p/k)+k(1+Q_2/k)}{\{(k+Q_2)[k+Q_2+2Q_1(1+Q_2p/k)]\}^{1/2}}\times v_{F_1}k+\mathbf{k}\cdot\mathbf{v}_{D_1}=\omega_k^0+\mathbf{k}\cdot v_{D_1} \tag{13}$$

and

$$\gamma_k=-\frac{m_2}{m_1}\frac{(\lambda_1^2-1)^{3/2}}{(1-\lambda_2^2)^{1/2}}\frac{(\omega_k^0+\mathbf{k}\cdot v_{D_1}-\mathbf{k}\cdot v_{D_2})}{(1+Q_2p/k)^2}e^{-kd}\ , \tag{14}$$

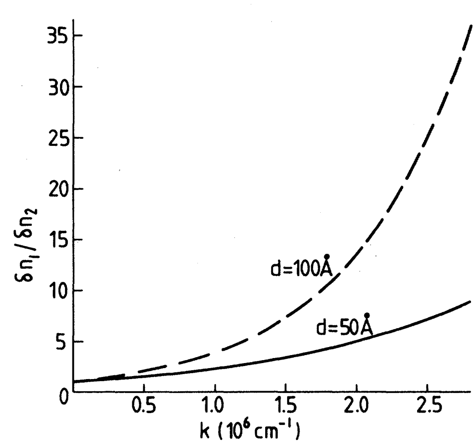

FIG. 1. Wave-vector dependence of the ratio $\delta n_1/\delta n_2$ of the distribution of the acoustic plasma wave amplitude between the two charge sheets.

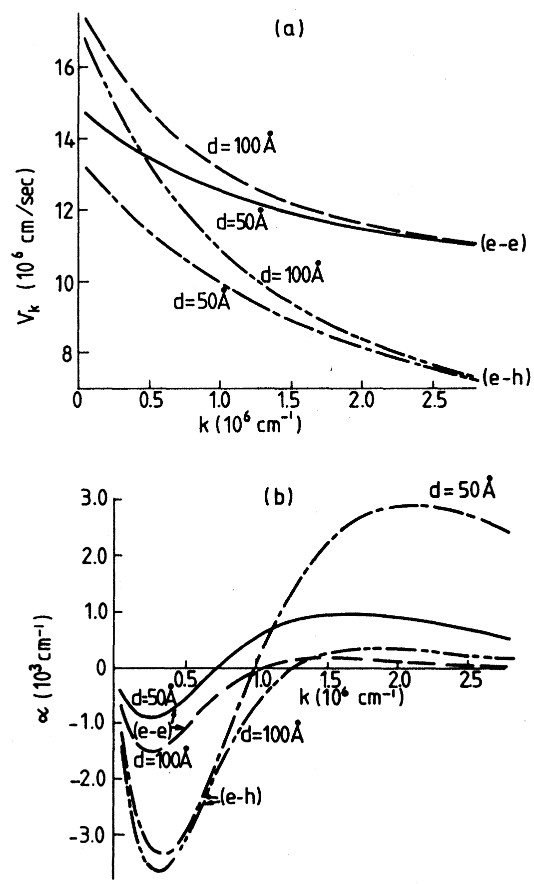

FIG. 2. (a) Phase velocity v_k of the low-frequency plasma mode for the two-stream electron-electron (e-e) plasma and the hole-electron (h-e) plasma. The parameters used in the calculation are listed in Table I. (b) Gain $\alpha=2\gamma_k/v_k$ associated with the acoustic plasma wave in e-e plasma and e-h plasma for the parameters listed in Table I.

TABLE I. Various parameters used for the calculation of phase velocity and gain factor for the low-frequency plasma mode.

	n_1 (10^{12} cm^{-2})	n_2 (10^{12} cm^{-2})	v_{D_1} (10^7 cm s^{-1})	v_{D_2} (10^7 cm s^{-1})
electron-electron plasma	0.05	1	0.13	1.3
hole-electron plasma	0.5	1	0.0	1
electron mass $m_e=0.068m_0$, hole mass $m_h=0.5m_0$.				

where ω_k^0 is the frequency of the plasma mode in the absence of any external drift velocities.

For the case of intermediate coupling, $kd \ll 1$ and $p \approx 2kd$, Eq. (13), in the long-wavelength limit and in the absence of external drift velocities, yields

$$v_k^0=\frac{\omega_k^0}{k}=\frac{1+(m_1/m_2)(1+2Q_2d)}{[1+2(m_1/m_2)(1+2Q_2d)]^{1/2}}v_{F_1} . \quad (15)$$

It emerges from Eq. (15) that v_k^0 is always greater than v_{F_1}, so that there is no lower limit on d for the existence of low-frequency acoustic plasmon mode, contrary to the findings of Das Sarma and Madhukar. Calculation[10] of the ratio $\delta n_1/\delta n_2$ reveals that the distribution of the wave amplitude, between the two charge sheets, is independent of the effective masses, charge densities, and the drift velocities of the charge carriers involved. Also, this low-frequency acoustic mode, which is equally distributed between the two sheets at low k values, is increasingly supported by the charge sheet with lower Fermi velocity as the wave vector k increases. The effect is more pronounced for larger intersheet separations. The variation of $\delta n_1/\delta n_2$ for $d=50$ and 100 Å is shown in Fig. 1.

It is observed from Eq. (14) that external drift velocities can induce two-stream growth instability in the acoustic plasma wave for

$$\mathbf{k}\cdot\mathbf{v}_{D_2} > \omega_k^0+\mathbf{k}\cdot\mathbf{v}_{D_1} . \quad (16)$$

Figure 2(a) presents the variation of the phase velocity $v_k(=\omega_k/k)$ of the wave with k for the cases of electron-electron plasma (e-e) and the hole-electron plasma (h-e) for intersheet separations of $d=50$ and 100 Å. Values of other parameters used in the calculations are listed in Table I. Figure 2(b) shows the k variation of the power gain coefficient $\alpha=2\gamma_k/v_k$ for the same four cases as in Fig. 2(a). It is seen that the acoustic character ($\omega_k \propto k$) of the mode decreases with increasing d. Also, one observes, from Fig. 2(b), that the drift velocities of $v_{D_2}=1.3\times10^7$ cm s^{-1} for the e-e system, and $v_{D_1}=1.0\times10^7$ cm s^{-1} for the h-e system are just sufficient to induce growth instability for k greater than some wave vector, say, k_c. This instability, or, indeed, the damping for $k<k_c$, is greater for the h-e two-stream plasma. The effect of separation between the two charge carriers is to suppress the growth instability, the effect, once again, being larger for the e-h plasma.

Figure 3 presents a plot of the maximum gain α_{max} versus electron drift velocity v_{D_2} for an e-e two-stream plasma. The corresponding value of the wave vector, k_{max}, is also shown. The intercharge sheet separation is $d=50$ Å and values of other parameters are as in Table I. For the present case, the critical drift velocity of $v_{D_2}=1.14\times10^7$ cm s^{-1}, needed to induce growth instability, corresponds to a relative drift velocity of 1.025×10^7 cm s^{-1} between the two charge carriers. The corresponding threshold value of the relative drift velocity for the case of e-h plasma, once again using parameters of Table I, is 0.92×10^7 cm s^{-1}, and α_{max} increases much faster with v_{D_2} in this case. A low value of n_1—the carrier density of charge sheet with lower Fermi energy—also results in a low value of the threshold drift velocity. The onset of plasma instability is not critically dependent on charge density n_2. However, a low value of n_2 results in a higher growth rate.

The requirement of drift velocities in the region of 1×10^7 cm s^{-1} imply that the electrons will be hot, and a more detailed analysis would have to take that into account. It is unlikely however, that hot-electron effects will be strong enough to affect the two-stream coupling significantly.

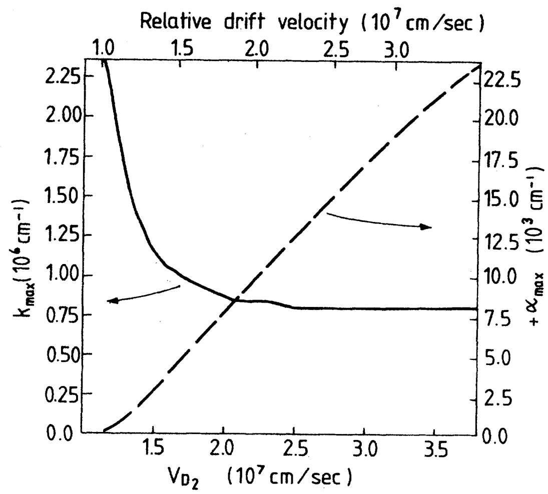

FIG. 3. Variation of α_{max}, the maximum gain, and k_{max}, the wave vector at which the gain is maximum, with drift velocity v_{D_2} for an electron-electron plasma with an intersheet separation of 50 Å, $n_1=5\times10^{10}$ cm^{-2}, $n_2=10^{12}$ cm^{-2}, and $v_{D_1}=0.1v_{D_2}$.

In conclusion, an electron drift velocity of $\sim 1.5\times 10^7$ cm s^{-1} should allow observation of growth instability in a hole-electron plasma with carrier densities $n_1 = 5\times 10^{11}$ cm^{-2}, $n_2 = 10^{12}$ cm^{-2}, respectively, or an electron-electron plasma of carrier densities $n_1 = 5\times 10^{10}$ cm^{-2}, $n_2 = 10^{12}$ cm^{-2}, in a GaAs double quantum well separated by ~ 50 Å, in either case. It is anticipated that any instability will be suppressed for a separation greater than about 100 Å.

ACKNOWLEDGMENT

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