LO-phonon confinement and polaron effect in a quantum well

F. Comas
Department of Theoretical Physics, University of Havana, San Lazaro y L, Havana, Cuba

C. Trallero-Giner
Department of Theoretical Physics, University of Havana, San Lazaro y L, Havana, Cuba
and Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, 7000 Stuttgart 80, Federal Republic of Germany

R. Riera
Department of Theoretical Physics, University of Havana, San Lazaro y L, Havana, Cuba
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Polaronic corrections are calculated considering electron coupling with confined LO phonons in a quantum well (QW). The electron–LO-phonon Hamiltonian for a confined electron-phonon system in a QW is taken from a previous work. The electron self-energy is calculated by use of the standard perturbation-theory treatment for the weak-coupling polaron assuming parabolic energy band structure and $T=0$ K. Screening effects are included along the lines of the static Thomas-Fermi approximation. Comparison with different experimental works is made and acceptable agreement is achieved. We also present comparisons with other theoretical works on the subject.

I. INTRODUCTION

Low-dimensional semiconductor layered structures [quantum wells (QW), superlattices, etc.] are frequently fabricated from weakly ionic compounds (the $GaAs/Al_xGa_{1-x}As$ case is a typical example) and therefore electron interaction with polar-optical vibrations is in general important for the determination of electron dynamics in such structures, where the electron systems bear a quasi-two-dimensional (Q2D) character. Polaronic corrections in the weak-coupling approximation have been studied by different authors for a Q2D electron system[1–6] employing the usual Fröhlich Hamiltonian for LO phonons in a bulk semiconductor but considering Q2D (or 2D) wave functions for the electrons. In the frame of this kind of approaches the case of the Q2D electron system in a QW (double heterostructure) was studied in Ref. 2 and it was proved that electron self-energy is divergent unless screening is incorporated. If strictly 2D wave functions are utilized for the electron, the self-energy is not divergent but rather a large polaron mass and binding energy are achieved.[3] On the other hand, in general, the Q2D electron system associated with a single heterostructure has convergent self-energies (even in the absence of screening) if the standard 3D Fröhlich Hamiltonian is applied.[3,4] In Ref. 3 good agreement with experimental data reported in Ref. 7 is claimed for the single-heterostructure case from calculations based on the variational wave functions of Fang and Howard which included screening. However, in Refs. 7 and 8 relatively insignificant polaronic corrections were reported in contrast with other experimental works, where larger values for polaron mass and binding energies were measured.[9–11] It should be noticed that deviations from parabolicity in the energy-band structure of these systems have been invoked in order to explain large electron masses measured by cyclotron resonance experiments; but for not very large applied magnetic fields or not very high electron concentrations such deviations could be not so strong.[11] Recent experimental evidence in the InSb case[12] indicates that polaronic corrections are not insignificant. In Ref. 13 new important results concerning polaron effects can be found together with a brief discussion of the existing ambiguities in their determination for low-dimensional semiconductor structures.

In all previous works polar-optical phonons have been treated in the spirit of 3D (bulk) theory regardless of the layered character of the system; this approach seemed to be plausible from the almost complete uniformity of elastic and electric properties throughout the QW or the single heterostructure (as is often invoked in earlier papers on the subject). However, we now propose clear evidence that a rather different model more accurately describes the situation. It has been clearly established that polar-optical phonons in layered structures behave as in a confined system in analogy with the confined electron system.[14,15] Penetration of polar-optical vibrations from a given layer into the adjacent ones is negligible (penetrating less than a monolayer) due to rather different vibrational frequencies in the layers (this situation is quite typical in the $GaAs/Al_xGa_{1-x}As$ superlattice and can be extended to other analogous systems). In the QW case the active layer (for instance the GaAs layer) acts as a kind of "quantum well" for the polar-optical phonons, a phenomenon sharply distinguishable from phonon folding (phonon folding is seen in superlattices for the acoustic phonons) and present both in superlattices and QW's.

From the foregoing discussions it is concluded that electron–LO-phonon interaction must be reconsidered in a QW (and in other layered structures) in order to account for the electron coupling with confined LO phonons. In a recent work Trallero-Giner and Comas have

derived an electron–LO-phonon interaction Hamiltonian for the electron coupling with confined LO phonons in a QW;[16,17] the deduction was based upon a model of LO vibrations entailing complete confinement in the QW active layer and resembling the well-known Born-Huang approximation for the bulk semiconductor case. Some advantages of the model Hamiltonian proposed in Refs. 16 and 17 are (i) it contains the bulk Fröhlich Hamiltonian as a limiting case; (ii) the model for polar-optical vibrations is in close agreement with experimental results (especially with Raman backscattering investigations as discussed by Cardona[18]); (iii) the preliminary obtained results for polaronic corrections seems to be in acceptable agreement with experiments.

In the present paper we aim to proceed with more detailed calculations of polaronic corrections in a QW using the electron–LO-phonon Hamiltonian reported in Refs. 16 and 17. We should limit ourselves to the weak-coupling polaron case for $T=0$ K ignoring all deviations from parabolicity in the energy-band structure. However, screening effects will be introduced along the lines of a Thomas-Fermi-like approach including a temperature-dependent screening factor. This latter approximation should provide temperature-dependent polaronic corrections for low enough temperatures (i.e., $k_B T < \hbar\omega_0$, where ω_0 is the LO-phonon limiting frequency). Of course, a consistent temperature-dependent theory must consider temperature in all the steps and would be valid for a wider temperature range. In the frame of our present approach we want to give further contributions to the general understanding of the role of polaron corrections in a QW and to provide a quantitative estimation of LO-phonon confinement effects upon polaron effective mass and binding energy.

This paper is organized in the following way. In Sec. II we introduce some relations of general character concerning the confined LO-phonon model we use and briefly summarize the electron–LO-phonon Hamiltonian; in Sec. III calculations of polaronic corrections are reported; Sec. IV is devoted to a discussion of our results and comparison is made with other authors and experiment.

II. GENERAL RELATIONS

Let us briefly summarize some details concerning the electron–LO-phonon interaction Hamiltonian for the case of confined LO phonons in a QW.[16,17] In the usual bulk theory the Fröhlich Hamiltonian can be deduced from[19]

$$H_I(\mathbf{r})=\int \frac{e[-\mathrm{div}\mathbf{P}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|}\exp\left[-\frac{|\mathbf{r}-\mathbf{r}'|}{\lambda}\right]d^3r' , \quad (1)$$

where $\mathbf{P}(\mathbf{r})$ is the polarization-field operator associated with the LO vibrations and the exponential incorporates the screening effect along the lines of static Thomas-Fermi approximation through the screening length λ. The classical expression for $\mathbf{P}$ is obtained applying the hydrodynamic Born-Huang theory where $\mathbf{P}$ is proportional to the displacement field $\mathbf{u}(\mathbf{r})$ (see Ref. 18). In Refs. 16 and 17 an analogous approach was developed, but $\mathbf{u}(\mathbf{r})$ was obtained for a QW located between $z=0$ and $z=d$ and requiring complete confinement of the LO vibrations. We shall not give anymore details here about the model. For $\mathbf{P}$ the following expression is found:

$$\mathbf{P}(\mathbf{r})=\sum_{\mathbf{q}_\perp}\sum_{n=-N(q_\perp)}^{N(q_\perp)}\left[\frac{\hbar\omega_0^2}{8\pi\epsilon^* V\omega_q}\right]^{1/2}\frac{\mathbf{q}}{|\mathbf{q}|}(e^{i\mathbf{q}\cdot\mathbf{r}}b_{\mathbf{q}}+\mathrm{H.c.}) , \quad (2)$$

where $(\epsilon^*)^{-1}=(\epsilon_\infty)^{-1}-(\epsilon_0)^{-1}$, $\epsilon_\infty(\epsilon_0)$ is the high-frequency (static) dielectric constant; $V=Sd$ is the volume, S being the typical interface area; $\mathbf{q}=\mathbf{q}_\perp+\mathbf{e}_z q_z$ is the phonon wave vector; and $\omega_q^2=\omega_0^2-\beta^2 q^2$ is the phonon dispersion relation, where β is a parameter with velocity dimensions. b_q is the annihilation operator for a phonon with the wave vector $\mathbf{q}$. The confined character of the LO phonons in the QW is described by the following features. (i) $\mathbf{P}(\mathbf{r})$ is defined just for $0\le z\le d$; (ii) q_z is discrete and given by $q_z=n\pi/d$, where $n=\pm1,\pm2,\ldots$; (iii) for $q_\perp=0$ discrete LO-phonon frequencies are given by $\omega_q^2=\omega_0^2-(n\pi\beta/d)^2$ where ω_0 is the (bulk) LO-phonon limiting frequency and $n=1,2,\ldots$; this is in rather good agreement with Raman backscattering experiments (see Ref. 14). $N(q_\perp)$ is the higher integer less than $(d/\pi)[(\omega_0^2/\beta^2)-q_\perp^2]^{1/2}$ (for a fixed value of $q_\perp$), while summation over $\mathbf{q}_\perp$ covers all vectors $\mathbf{q}_\perp$ inside a circle with radius ω_0/β. The latter restrictions upon summations in expression (2) are consistent with the dispersive phonon model employed in Refs. 16 and 17 but can be obviously avoided in the nondispersive approximation formally contained in the limit $\beta\to0$. In this case $\omega_q\to\omega_0$, $N(q_\perp)\to\infty$, and all summations will cover infinite intervals.

We use the following wave functions for the electrons:

$$\Psi(\mathbf{r}_\perp,z)=\frac{1}{\sqrt{S}}\exp(i\mathbf{k}_\perp\cdot\mathbf{r}_\perp)\rho_l(z) , \quad (3)$$

where, for complete electron confinement we choose

$$\rho_l(z)=\begin{cases}\left[\frac{2}{d}\right]^{1/2}\sin k_z z & \text{for } 0\le z\le d\\ 0 & \text{otherwise} ,\end{cases} \quad (4)$$

where $k_z=\pi l/d$, and the corresponding electron energies are

$$\varepsilon_{\mathbf{k}}^{(0)}=\varepsilon_{\mathbf{k}_\perp}^{(0)}+E_0 l^2, \quad E_0=\frac{\pi^2\hbar^2}{2md^2} , \quad (5)$$

with $\varepsilon_{\mathbf{k}_\perp}^{(0)}=\hbar^2 k_\perp^2/2m$ and m the effective electron mass.

As proved in Refs. 16 and 17 the electron–LO-phonon Hamiltonian will be given by

$$H_I=\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}[C_{\mathbf{q},\lambda}F_\lambda(\mathbf{q},\mathbf{k},\mathbf{k}')b_{\mathbf{q}}a_{\mathbf{k}}^\dagger a_{\mathbf{k}'}+\mathrm{H.c.}] , \quad (6)$$

where $a_{\mathbf{k}}^\dagger$ ($a_{\mathbf{k}}$) are creation (annihilation) operators for electrons in the states $|\mathbf{k}_\perp,l\rangle$ with the notation $\mathbf{k}=\mathbf{k}_\perp+\mathbf{e}_z k_z$.

The parameters in (6) are defined as

$$F_\lambda(\mathbf{q},\mathbf{k},\mathbf{k}')=\Delta(\mathbf{k}_\perp-\mathbf{k}'_\perp-\mathbf{q}_\perp)G_\lambda(\mathbf{q},k_z,k_z') , \tag{7}$$

$$G_\lambda(\mathbf{q},k_z,k_z')=\int_{-\infty}^{\infty}\rho_l^*(z)\rho_l(z)\exp(iq_z z)\Phi_{\mathbf{q},\lambda}(z)dz , \tag{8}$$

$$C_{\mathbf{q},\lambda}=-ie\left[\frac{2\pi\hbar\omega_0^2}{\epsilon^*\omega_q V}\right]^{1/2}\frac{1}{[q^2+(1/\lambda)^2]^{1/2}} , \tag{9}$$

and

$$\Delta(\mathbf{k})=\begin{cases}1 & \text{if } \mathbf{k}=\mathbf{0}\\ 0 & \text{if } \mathbf{k}\neq\mathbf{0}\end{cases} .$$

$\Phi_{\mathbf{q},\lambda}(z)$ is a certain modulating function defined in Ref. 17. Then, we have

$$G_\lambda(\mathbf{q},k_z,k_z')=I_{ll'n}+J_{ll'n}(Q) , \tag{10}$$

where

$$I_{ll'n}=\tfrac{1}{2}[\Delta(n-l-l')+\Delta(n+l-l')-\Delta(n-l-l')-\Delta(n+l+l')] , \tag{11}$$

$$J_{ll'n}(Q)=\frac{2\pi^2 Qdll'[1+(-1)^{l+l'+n}][(-1)^n\exp(-Qd)-1]}{[Q^2d^2+(l+l')^2\pi^2][Q^2d^2+(l-l')^2n^2]}, \quad \text{with } Q^2=q_\perp^2+1/\lambda^2 . \tag{12}$$

It should be noticed that $J_{ll'n}(Q)$ is a rapidly decreasing function of Q different from zero only when $l+l'+n$ is an even integer. $I_{ll'n}$ invokes a kind of momentum conservation in the z direction as far as $I_{ll'n}$ differs from zero only for $q_z=\pm(k_z\pm k_z')$. Of course, this latter interpretation is only formal if we realize that momentum is not a well-defined quantity in the confinement direction.

III. POLARONIC CORRECTIONS

According to standard perturbational treatment the electron self-energy due to the electron–LO-phonon interaction in the weak-coupling approximation is given by Ref. 19:

$$\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}}^{(0)}=\sum_{\mathbf{k}',\mathbf{q}}\frac{|\langle 1_{\mathbf{k}'},0_{\mathbf{k}},1_{\mathbf{q}}|H_I|0_{\mathbf{k}'},1_{\mathbf{k}},0_{\mathbf{q}}\rangle|^2}{\varepsilon_{\mathbf{k}}^{(0)}-\varepsilon_{\mathbf{k}-\mathbf{q}}^{(0)}-\hbar\omega_{\mathbf{q}}} , \tag{13}$$

where $\varepsilon_{\mathbf{k}}^{(0)}$ are unperturbed electron energies and the matrix elements are written explicitly for the $T=0$ K case, when in the initial state we do not have phonons. Really, we only need the condition $N_q=[\exp(\hbar\omega_0/k_BT)-1]^{-1}\ll 1$, which is usually fulfilled for $k_BT<\hbar\omega_0$, and the temperature is not strictly required to be equal to zero. Expression (13) contains all matrix elements between initial and intermediate states involving electron transitions induced by electron–LO-phonon interaction. Hence, $|0_{\mathbf{k}'},1_{\mathbf{k}},0_{\mathbf{q}}\rangle$ denotes the initial state with one electron in the state $\mathbf{k}$, zero electrons in the $\mathbf{k}'$ and zero phonons. The interpretation of intermediate states $|1_{\mathbf{q}},0_{\mathbf{k}},1_{\mathbf{k}'}\rangle$ is analogous.

In (13) we use Hamiltonian (6) assuming that electrons are completely confined in the quantum well, i.e., form factor (7) is expressed through (10), (11), and (12), and energies $\varepsilon_{\mathbf{k}}^{(0)}$ shall be given by (5). If we realize that $G_\lambda(\mathbf{q},k_z,k_z')$ is an even function of q_z (i.e., of n) it is not difficult to obtain

$$\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}}^{(0)}=\frac{16m}{\hbar^2}\sum_{\mathbf{q}_\perp,l'}\frac{K_{ll'}(Q)}{2k_\perp q_\perp\cos\theta-q_\perp^2-\frac{\pi^2}{d^2}(l'^2-l^2)-\frac{2\omega_0}{\hbar}} \tag{14}$$

with

$$K_{ll'}(Q)=\sum_{n=1}^{\infty}|G_\lambda(\mathbf{q},k_z,k_z')|^2+\tfrac{1}{8}|G_\lambda(\mathbf{q}_\perp,k_z,k_z')|^2 . \tag{15}$$

The second term at the right-hand side of (15) means we take $G_\lambda(\mathbf{q},k_z,k_z')$ with $q_z=0$. In expression (14) θ is the angle between vectors $\mathbf{k}_\perp$ and $\mathbf{q}_\perp$. The denominator in (14) must be expanded up to quadratic terms in $k_\perp$. Straightforward summation over $\mathbf{q}_\perp$ (everywhere we are assuming a nondispersive approximation for the phonons and the summations cover infinite intervals, as was clarified in Sec. II) leads us to the following results:

$$\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}}^{(0)}\cong-\alpha_l\hbar\omega_0-\frac{\hbar^2k_\perp^2}{2m}\gamma_l , \tag{16}$$

where

$$\alpha_l=16\pi\alpha\left[\frac{\hbar\omega_0}{E_0}\right]^{1/2}\sum_{l'}\int_0^{\infty}\frac{F_{ll'}(x)x\,dx}{x^2+\pi^2\left[l'^2-l^2+\frac{\hbar\omega_0}{E_0}\right]} , \tag{17}$$

$$\gamma_l=32\pi^3\alpha\left[\frac{\hbar\omega_0}{E_0}\right]^{3/2}\times\sum_{l'}\int_0^{\infty}\frac{F_{ll'}(x)x^3dx}{\left[x^2+\pi^2\left[l'^2-l^2+\frac{\hbar\omega_0}{E_0}\right]\right]^3} . \tag{18}$$

In the latter expressions α is the usual 3D Fröhlich constant

$$\alpha=\frac{e^2}{\epsilon^*}\left[\frac{m}{2\hbar^3\omega_0}\right]^{1/2} , \tag{19}$$

and $F_{ll'}(x)$ a function defined by

$$F_{ll'}(x)=\sum_{n=1} F_{ll'n}(x)+\tfrac{1}{8}F_{ll'}(x)\ , \tag{20}$$

$$F_{ll'n}(x)=\frac{x^2+\pi^2n^2}{[x^2+\pi^2n^2+(d^2/\lambda^2)]^2} \times\left\{\tfrac{1}{2}[\Delta(n)\Delta(l-l')+\Delta(n-|l-l'|)-\Delta(n-l-l')] +\frac{2\pi^2 ll'[x^2+(d^2/\lambda^2)]^{1/2}[1+(-1)^{l+l'+n}]((-1)^n\exp\{-[x^2+(d^2/\lambda^2)]^{1/2}\}-1)}{[x^2+\pi^2(l+l')^2+(d^2/\lambda^2)][x^2+\pi^2(l-l')^2+(d^2/\lambda^2)]}\right\}^2 . \tag{21}$$

It is obvious that (16) can be rewritten in the form

$$\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}}^{(0)}\cong-\alpha_l\hbar\omega_0+\frac{\hbar^2k_\perp^2}{2m_l^*}\ , \tag{22}$$

where

$$m_l^*=\frac{m}{1-\gamma_l} \tag{23}$$

is the polaron effective mass for the "l" subband and

$$\varepsilon_l=\alpha_l\hbar\omega_0 \tag{24}$$

is the polaron binding energy.

IV. DISCUSSION

In the calculations of Sec. III it is assumed that the electron can be localized in a given subband l but the intermediate states in the perturbation-theory formula (13) involve all possible subbands and phonon states. Polaronic mass and binding energy are calculated from the parabolic isotropic band structure depicted in Ref. 5. Screening is introduced from the very beginning in the Hamiltonian (1) through the screening length λ. We should stress that in the unscreened limit $\lambda\to\infty$ our approach predicts finite-electron self-energies in contrast with earlier calculations neglecting LO-phonon confinement (see, for instance, Ref. 2).

From general considerations it is possible to conclude that our results cannot be valid for too small or too large values of $\hbar\omega_0/E_0$. In the first case very small QW widths

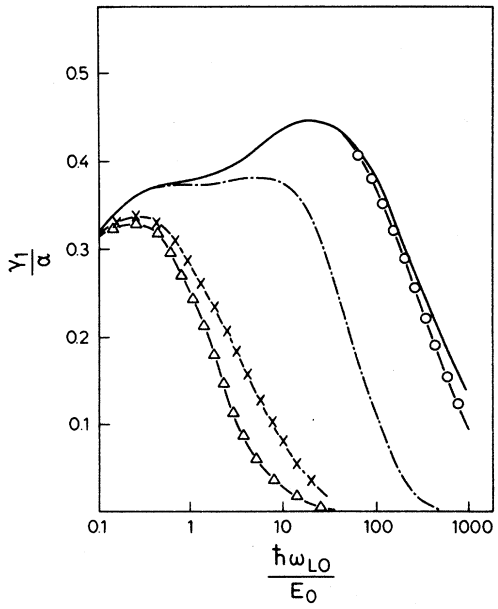

FIG. 1. Parameter γ_l (for $l=1$, in units of α) as a function of $\hbar\omega_0/E_0$. Different curves correspond to different values of the screening length λ: —— for $\lambda\to\infty$; -○-○- for $\lambda=1.36\times10^{-5}$ cm; —·—·— for $\lambda=2.49\times10^{-6}$ cm; -×-×- for $\lambda=10^{-6}$ cm; -△-△- for $\lambda=10^{-6}$ cm correspond to the EQL case.

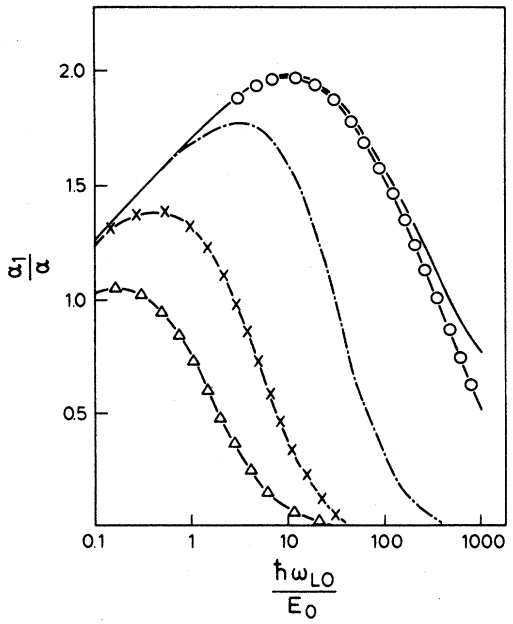

FIG. 2. Parameter α_l (for $l=1$, in units of α) as a function of $\hbar\omega_0/E_0$. Different curves correspond to different values of the screening length λ and are labeled in analogy with Fig. 1.

are involved and we cannot expect our continuum (hydrodynamic) model for LO vibrations to give correct values of α_l or γ_l in this limit; on the other hand, the effective-mass approximation is not applicable for too small values of a QW width. In the second case we must realize that the energy difference between subbands l and l' is $E_0(l^2-l'^2)$; if E_0 is too small the subbands will be so close together that it will be impossible to think in terms of a polaron localized in a given subband l, but actually the polaron will be "shared" by several subbands. Therefore, the obtained results are reasonably reliable for intermediate values of $\hbar\omega_0/E_0$.

From Figs. 1 and 2 it can be seen that γ_l and α_l (for $l=1$) are larger than in the purely 3D case [where $\alpha_1=\alpha$ and $\gamma_1=\alpha/6$ (Ref. 19)]. Our approach predicts enhanced polaronic corrections in agreement with experimental data from Refs. 9, 10, 12, and 13. For a purely 2D system or the case of the Q2D electron system of a single heterostructure as studied in Refs. 3, 20, and 21 rather small polaronic corrections were obtained. The enhanced polaronic corrections found by us are both a peculiarity of the QW structure and of the confined character of LO phonons. In Figs. 1 and 2 the solid curve corresponds to the unscreened quantity while the other curves correspond to different values of the screening length. In particular the curve with triangles ($\lambda=10^{-6}$ cm) corresponds to the extreme quantum limit (EQL) when the electron is localized in the $l=1$ subband and intersubband transitions are neglected (i.e., summation over l' in Ref. 14 only contains the term $l'=1$).

In Fig. 3 we show a comparison of our results for the EQL with those reported in Ref. 2, where the author ap-

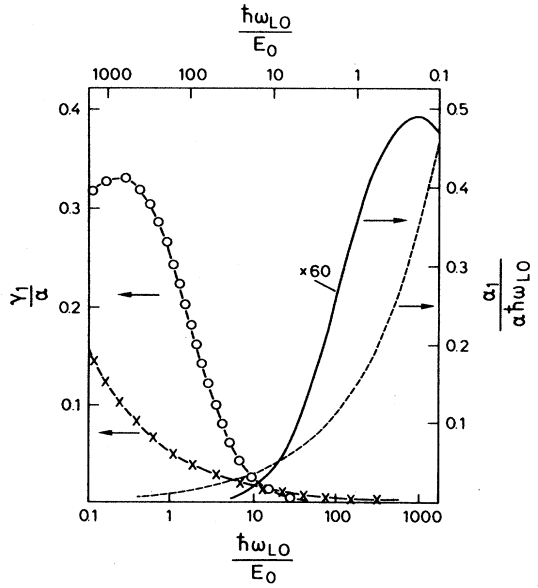

FIG. 3. Curves for γ_1 and α_1 (in units of α and $\alpha\hbar\omega_0$, respectively) as a function of $\hbar\omega_0/E_0$. Curves —— and —·—·—· correspond to α_1 and γ, respectively, in the EQL ($\lambda=10^{-6}$ cm) according to our calculations. Curves — — — and –×–×– correspond to α and γ_1, as calculated in Ref. 2. Curve —— is obtained after multiplying by 60.

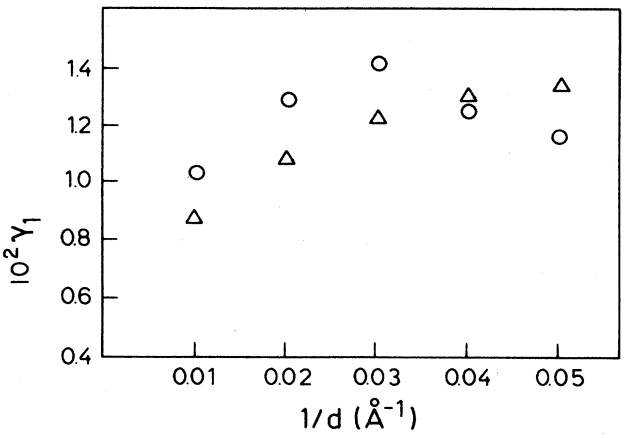

FIG. 4. Parameter γ_1 as a function of $1/d$ for the GaAs case. The points ○ correspond to our calculations for $T=50$ K and $n_s=10^{+11}$ cm^{-2}. Points △ correspond to calculations from Ref. 5.

plied the usual Fröhlich Hamiltonian (for nonconfined LO phonons) to a Q2D electron systems and used the mass-conservation approximation (MCA). As can be seen we obtained (for the same screening length) larger polaron corrections.

In Fig. 4 we show our results for γ_1 as a function of $1/d$ (circle) and those of Ref. 5 (triangles). We assumed $m=0.067m_0$, $\hbar\omega_0=0.036$ eV, and $\alpha=0.07$ for the GaAs case, $n_s=10^{+11}$ cm^{-2}, and $T=50$ K [λ was evaluated by the classical formula $\lambda=(\epsilon_0 k_B T/4\pi e^2 n)^{1/2}$ and in the computations we have set $n=n_s/d$]. Again larger results are predicted from our calculations up to a certain value of $1/d$. For smaller values of d (larger values of $1/d$) we cannot expect our calculations to be reliable enough as stated above.

In Fig. 5 we show the polaron mass m^* (in units of bare electron mass m_0) as a function of temperature. The curve was calculated from our expressions using the GaAs parameters and $d=100$ Å. The temperature was

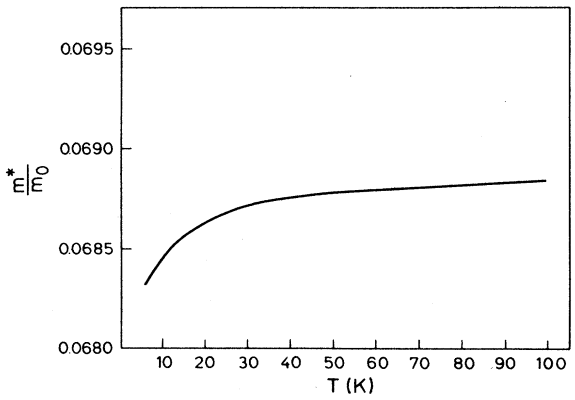

FIG. 5. Polaron effective mass m^* (in units of bare electron mass m_0) as a function of T for the GaAs case.

introduced through λ using the classical formula mentioned above. Of course, this approximation can only be valid for $k_B T < \hbar\omega_0$.

Let us also compare with experimental results in the GaAs case from Ref. 10. The authors determined $n_s = 4.0 \times 10^{11}$ cm^{-2} and $d = 250$ Å. For $T = 220$ K they measure $m^* = (0.071 \pm 0.015) m_0$ and we obtain $m^* = 0.0685 m_0$. For $T = 5$ K they measure $m^* = (0.069 \pm 0.001) m_0$ and we obtain $m^* = 0.0690 m_0$. This gives us a measure of pure polaronic contribution to the effective mass. As far as we ignore nonparabolicity effects (and also the effect of the magnetic field), etc., rigorous comparison with the results of Ref. 10 cannot be done. However, even within our relatively coarse estimations good agreement is found.

Another interesting comparison is the case of a InP/$Ga_x In_{1-x}$As superlattice from Ref. 22. For calculations we used $m = 0.041 m_0$, $\alpha = 0.06$, and $\hbar\omega_0 = 0.0345$ eV. We can compare with experimental determinations of m^* for $T = 125$ K using two experimental techniques: Cyclotron resonance (CR) and magnetophonon resonance (MPR) (see Table I). As can be seen from Table I we obtain better agreement with the $d = 150$-Å case (especially comparing with the MPR measurement). Of course, for too small values of d our theory works worse (as explained above) and also we should expect stronger deviations from parabolicity.

From the comparisons outlined in the foregoing discussion we can conclude that polaron corrections are sufficiently important and should be considered together with deviations from parabolicity in order to explain the experimental data. Also the influence of surface optical phonons must be analyzed in a QW which has been already done for a single heterostructure.[23] It is also quite clear that the interaction of electrons with confined LO phonons is responsible for important features of the polaron properties in a QW, which have not been considered in earlier works.

TABLE I. InP/$Ga_x In_{1-x}$As superlattices.

n_s (10^{11} cm^{-2})	d (Å)	$\frac{m^*}{m_0}$ (CR)	$\frac{m^*}{m_0}$ (MPR)	$\frac{m^*}{m_0}$ (ours)
1.8	150	0.0407	0.0422	0.041 93
2.2	100	0.0458	0.0465	0.041 92
1.2	80	0.0543	0.0560	0.041 89

More detailed calculations on the basis of the Hamiltonian used in this work can be done, especially taking into account temperature effects in all the steps and including nonparabolic energy band structure. This should produce theoretical results having better correspondence with experiment.

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[1] S. Das Sarma and A. Madhukar, Phys. Rev. B **22**, 2823 (1980).
[2] B. K. Ridley, J. Phys. C **15**, 5899 (1982).
[3] S. Das Sarma, Phys. Rev. B **27**, 2590 (1983).
[4] W. Xiaoguang, F. M. Peeters, and J. T. Devreese, Phys. Status Solidi **133**, 229 (1986); Phys. Rev. B **34**, 2621 (1986).
[5] B. A. Mason and S. Das Sarma, Phys. Rev. B **31**, 5223 (1985).
[6] Wu Xiaoguang, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **34**, 8800 (1986).
[7] G. Lindemann, W. Seidenbusch, R. Lassnig, J. Edlinger, and E. Görnick, Physica B+C **117&118B**, 649 (1983).
[8] J. Scholz, F. Koch, J. Ziegler, and H. Maier, Solid State Commun. **46**, 665 (1983).
[9] D. C. Tsui, Th. Englert, A. Y. Cho, and A. C. Gossard, Phys. Rev. Lett. **44**, 341 (1980).
[10] Th. Englert, D. C. Tsui, J. P. Portal, J. Beerens, and A. Gossard, Solid State Commun. **44**, 1301 (1982).
[11] F. Thiele, U. Merkt, J. P. Kotthaus, G. Lommer, F. Malcher, U. Rössler, and G. Weimann, Solid State Commun. **62**, 841 (1987).
[12] M. Horst, U. Merkt, and J. P. Kotthaus, Phys. Rev. Lett. **50**, 754 (1983).
[13] U. Merkt, M. Horst, T. Evelbauer, and J. P. Kotthaus, Phys. Rev. B **34**, 7234 (1986).
[14] C. Colvard, R. Fischer, T. A. Gant, M. V. Klein, H. Morkoç, and A. Gossard, Superlatt. Microstruct. **1**, 81 (1985).
[15] M. V. Klein, IEEE J. Quantum Electron, **QE-22**, 1760 (1986).
[16] C. Trallero-Giner and F. Comas, Phys. Status Solidi B **144**, K-19 (1987).
[17] C. Trallero-Giner and F. Comas, Phys. Rev. B **37**, 4583 (1988).
[18] M. Cardona, in *Lectures on Surface Science,* edited by M. Cardona and G. R. Castro (Springer-Verlag, Berlin, 1987), Vol. 86, p. 143.
[19] H. Haken, *Quantum Field Theory of Solids* (North-Holland, Amsterdam, 1976).
[20] M. A. Hopkins, R. J. Nicholas, M. A. Brummell, J. J. Harris, and C. T. Foxon, Phys. Rev. B **36**, 4789 (1987).
[21] F. M. Peeters, X. Wu, and J. T. Devreese, Solid State Commun. **65**, 1505 (1988).
[22] M. A. Brummell, R. J. Nicholas, J. C. Portal, M. Razeghi, and M. A. Poisonn, Physica B+C **117&118B**, 753 (1983).
[23] C. Y. Chen, T. Z. Ding, and D. L. Lin, Phys. Rev. B **35**, 4398 (1987).