Electromagnetic-resonance-induced optical response of a thin nonlinear dielectric film

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We show that the optical response of a nonlinear dielectric film may exhibit optical bistability even in the thin-film limit. Using a rigorous numerical treatment, we also show that the resonant excitation of nonlinear guided modes propagating along the film leads to switching intensities 10^4 times lower than those obtained with use of the resonances associated with the Airy function.

I. INTRODUCTION

In a recent paper[1] (henceforth referred to as I) Chen and Mills consider the problem of the optical response of a nonlinear (NL) dielectric film. In I, the emphasis is placed on thin films, i.e., films whose thickness d is small as compared with the wavelength λ_f in the film ($\lambda_f=\lambda/n_3$, where n_3 is the index of refraction of the NL film and λ is the wavelength of the incident plane wave). The formalism developed by the authors of I is an analytical one. One of the conclusions reached by Chen and Mills is that no optical bistability (OB) should exist in the thin-film limit ($d<\lambda_f$).

It is the aim of this paper to reconsider the question of OB in the thin-film limit. We also address the question of whether the NL Airy resonance considered in I is the best-suited electromagnetic (EM) resonance when considering OB in NL thin films.

II. DOES THE OPTICAL RESPONSE OF A NL THIN FILM EXHIBIT OB?

The interest is in the geometry of Fig. 1 of Ref. 1. But for sake of generality, we consider a somewhat different system (Fig. 1) consisting of four different media (relative permittivity ϵ_1, ϵ_2, ϵ_3, ϵ_4, magnetic permeability μ_0). Let e and d be the thicknesses of media 2 and 3, respectively. Medium 3 exhibits optical Kerr effect. A plane wave (frequency ω) is incident (incidence θ) on the interface between media 1 and 2.

We notice that the geometry of Fig. 1 is that of the NL prism coupler (PC). The NL Fabry-Pérot (FP) cavity is obtained by setting $\epsilon_2=\epsilon_1$. The goal is to consider simultaneously these two devices. The reason why we also consider the NL PC will become clear below.

We assume a $e^{-j\omega t}$ time dependence. Throughout this paper, the partial derivatives with respect to z of any function F, $\partial F/\partial z$, are equal to zero. Thus the solutions are either TE polarized (specified by E_z) or TM polarized (specified by H_z). We only consider the TE solution.

Let $\mathbf{E}_q=E_q\mathbf{e}_z$, where $\mathbf{e}_z$ is the unit vector along the z axis ($q=1-4$). In the NL medium, Maxwell's equations read

$$\nabla\times\mathbf{E}_3(x,y)=j\omega\mu_0\mathbf{H}_3(x,y)\ , \quad (1a)$$

$$\nabla\times\mathbf{H}_3(x,y)=-j\omega\epsilon_0\epsilon_3\mathbf{E}_3(x,y)-j\omega\mathbf{P}^{\rm NL}(x,y)\ . \quad (1b)$$

We assume that medium 3 is isotropic, thus[2]

$$\mathbf{P}^{\rm NL}=\mathbf{e}_z P^{\rm NL}\ ,$$

where

$$P^{\rm NL}=\epsilon_0\chi^{(3)}|E_3(x,y)|^2E_3(x,y)\ , \quad (1c)$$

$$\chi^{(3)}=\chi_{zzzz}\ .$$

From Eqs. (1), we derive the equation obeyed by the electric field in the NL medium:

$$\Delta E_3+k_3^2E_3=-\frac{\omega^2}{c^2}\chi^{(3)}|E_3|^2E_3\ . \quad (2)$$

According to the plane-wave excitation, the EM field in media 1–4 is looked for under the following form:

$$E_1=(A_1e^{jk_{1y}y}+B_1e^{-jk_{1y}y})e^{jk_xx}\ , \quad (3a)$$

$$E_2=(A_2e^{jk_{2y}y}+B_2e^{-jk_{2y}y})e^{jk_xx}\ , \quad (3b)$$

$$E_3=A_3(y)e^{jk_xx}\ , \quad (3c)$$

$$E_4=A_4e^{jk_{4y}y}e^{jk_xx}\ , \quad (3d)$$

with

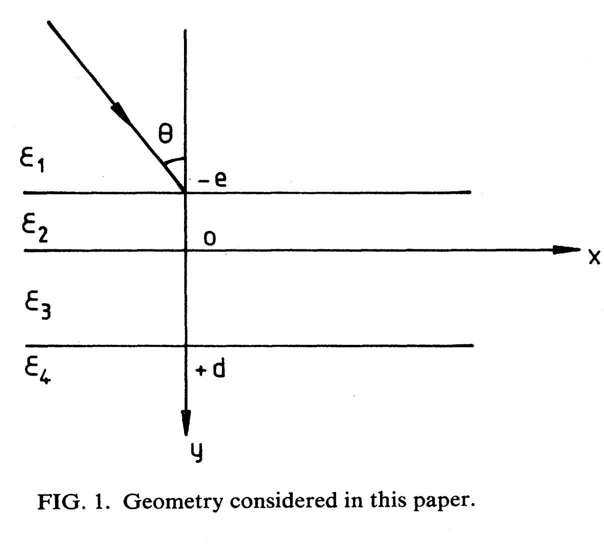

FIG. 1. Geometry considered in this paper.

$$k_{qy}^2 + k_x^2 = k_q^2 = \epsilon_q \frac{\omega^2}{c^2} , \quad q = 1,2,4$$

$$k_x = k_1 \sin\theta .$$

A_1 is the amplitude of the incident plane wave.

Inserting Eq. (3c) into Eq. (2) yields

$$\frac{d^2 A_3}{dy^2} + \left[k_{3y}^2 + \frac{\omega^2}{c^2} \chi^{(3)} |A_3|^2 \right] A_3 = 0 \tag{4a}$$

with $k_{3y}^2 = k_3^2 - k_x^2$. Equation (4a) is solved numerically together with the boundary conditions at $y = -e, 0, d$ (continuity of E_q and dE_q/dy, $q = 1-4$) and Eqs. (3a), (3b), and (3d). One then ends with the following set of equations:

$$E_3(d) = E_4(d) = A_4 e^{jk_{4y}d} e^{jk_x x} , \tag{4b}$$

$$\left.\frac{dE_3}{dy}\right|_{y=d} = \left.\frac{dE_4}{dy}\right|_{y=d} = jk_{4y} E_4(d) , \tag{4c}$$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} A_3(0) \\ \left.\dfrac{dA_3}{dy}\right|_0 \end{bmatrix} , \tag{4d}$$

$$p = \frac{1}{4} \left[\left[1 + \frac{k_{2y}}{k_{1y}} \right] e^{-jk_{2y}e} + \left[1 - \frac{k_{2y}}{k_{1y}} \right] e^{jk_{2y}e} \right] e^{jk_{1y}e} ,$$

$$q = \frac{1}{4jk_{2y}} \left[\left[1 + \frac{k_{2y}}{k_{1y}} \right] e^{-jk_{2y}e} - \left[1 - \frac{k_{2y}}{k_{1y}} \right] e^{jk_{2y}e} \right] e^{jk_{1y}e} ,$$

$$r = p(-k_{1y}) ,$$

$$s = q(-k_{1y}) .$$

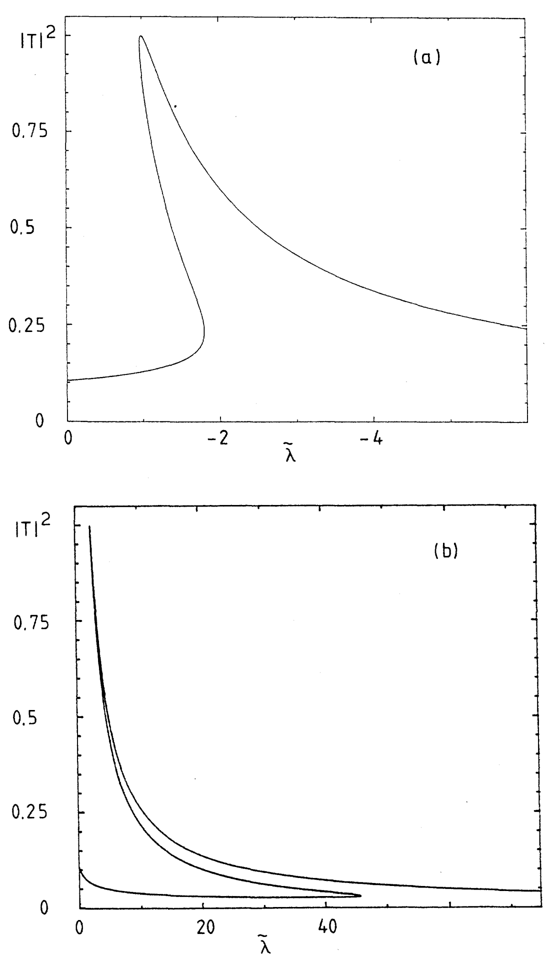

FIG. 2. Transmittivity $|T|^2$ as a function of the dimensionless quantity $\tilde{\lambda}$ for the NL geometry of Fig. 1 working in the NL FP regime: $n_3 = (\epsilon_3)^{1/2} = 10$; $n_q = (\epsilon_q)^{1/2} = 1$, $q = 1,2,4$; $\theta = 0$. (a) $\tilde{\lambda} < 0$, $d = 0.1\lambda_f$; (b) $\tilde{\lambda} > 0$, $d = 0.6\lambda_f$.

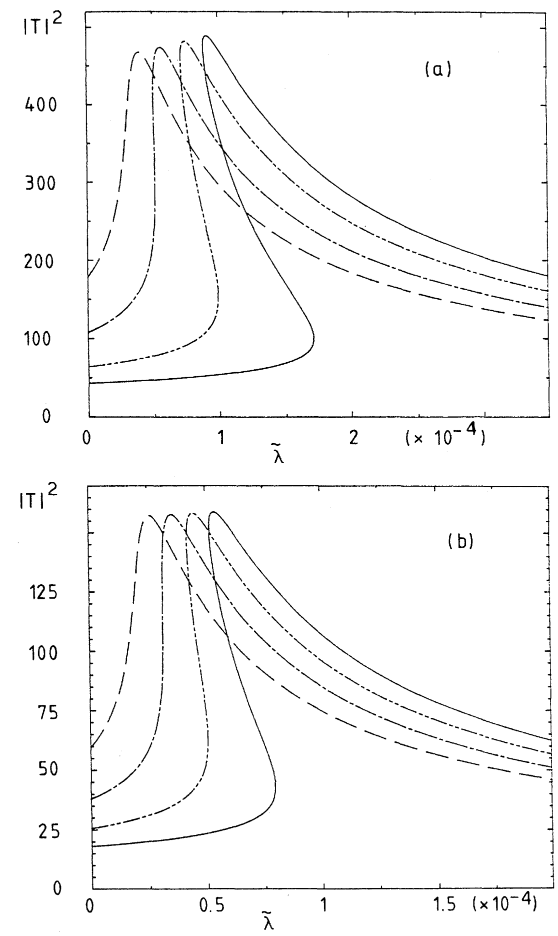

FIG. 3. Transmittivity $|T|^2$ as a function of the dimensionless quantity $\tilde{\lambda}$ for the NL geometry of Fig. 1 working in the NL PC regime: $\lambda = 1$ μm, $n_1 = (\epsilon_1)^{1/2} = 5$, $n_2 = (\epsilon_2)^{1/2} = 1$, $n_3 = (\epsilon_3)^{1/2} = 4$, $n_4 = (\epsilon_4)^{1/2} = 1$, $\tilde{\lambda} > 0$. (a) $d = 0.1\lambda_f$, $e = 0.233$ μm, $\Delta\theta = 3 \times 10^{-3}$ rad [half width at half maximum of the linear $|T(\theta)|^2$ curve] $\theta = 17.3°$, — — —; $\theta = 17.39°$, —·—·—·; $\theta = 17.5°$, ——-——-; $\theta = 17.6°$, ——. (b) $d = 0.4\lambda_f$, $e = 0.11076$ μm, $\Delta\theta = 3 \times 10^{-3}$ rad. $\theta = 37.26°$, — — —; $\theta = 37.34°$, —·—·—·; $\theta = 37.42°$, ——-——-; $\theta = 37.50°$, ——.

According to the expected "S shape" of the optical response, A_1 is a single valued function of A_4. Thus the computations have been performed given A_4 and looking for A_1, i.e., $A_1(A_4)$.

In order to allow an easy comparison with I, the parameter $\tilde{\lambda}=(\chi^{(3)}/\epsilon_3)|A_1|^2$ is used. Figures 2(a) and 2(b) are plots of $|T|^2=|A_4e^{jk_{4y}d}/A_1|^2$ as a function of $\tilde{\lambda}$. This has been done in the same conditions as in Figs. 2(b) and 4(a) of I. It is seen that the absence of OB reported in I is due to the fact that the range of values in which $\tilde{\lambda}$ has been swept is not large enough. Let $\tilde{\lambda}\uparrow$ and $\tilde{\lambda}\downarrow$ be, respectively, the switch up and switch down values of $\tilde{\lambda}$. With the parameters of Fig. 2(a) of I, $\tilde{\lambda}\uparrow\simeq 4000$ with a corresponding jump of $|T|^2$, $\Delta|T(\tilde{\lambda}\uparrow)|^2\simeq 0$; $\tilde{\lambda}\downarrow\simeq 35$ with $\Delta|T(\tilde{\lambda}\downarrow)|^2\simeq 1$.

Notice that in Figs. 2(a) and 2(b) the numerical values $\tilde{\lambda}\uparrow$ and $\tilde{\lambda}\downarrow$ are prohibitively high, resulting in huge index variations. To a lesser extent, this is also true in Ref. 1.

This remark prompted us to wonder whether the NL Airy resonance, which is the EM resonance involved in the type of OB reported in I, is the best suited resonance when considering OB in thin NL films.

Keeping in mind that such NL films also propagate NL normal modes,[3] we now investigate the characteristics of NL guided-mode (GM) -induced OB. These GM are excited using the NL PC (Fig. 1) illuminated by a plane wave under oblique incidence θ. The computations are performed using Eqs. (4).

The curves $|T(\tilde{\lambda})|^2$ are reported in Figs. 3(a) and 3(b). Comparison of these figures with Figs. 2(a) and 2(b) shows that the NL GM resonance leads to values of $\tilde{\lambda}\uparrow$ and $\tilde{\lambda}\downarrow$ nearly 10^4 times smaller than those arising from the NL Airy resonance. Notice that the peak value of $|T(\tilde{\lambda})|^2$ can be larger than 1 since the energy balance criterion is not concerned with the evanescent wave in medium 4.

III. CONCLUSION

A rigorous theoretical study of the geometry of Fig. 1 has been achieved by performing a numerical integration of the equation obeyed by the electric field in the NL slab, Eq. (4a), together with Eqs. (4b)–(4d). This formalism applies even in the case of a lossy NL medium. It allows studying simultaneously the NL FP cavity and the NL PC. In fact both devices are similar.

The EM resonance involved when considering the NL FP cavity is the NL Airy resonance; the angle of incidence being smaller than θ_c [θ_c is the critical angle of total reflection at the interface ϵ_3, ϵ_4 (Fig. 1)].

The EM resonance involved when considering the NL PC is the NL GM resonance; the angle of incidence being larger than θ_c.

The lack of OB reported in I in the case of thin films comes from the fact that the range of explored values of $\tilde{\lambda}$ is not large enough. Although the results presented here show that a NL FP cavity exhibits OB even in the thin-film limit, the situation is unrealistic because of the associated huge switching values $\lambda\uparrow$ and $\tilde{\lambda}\downarrow$ mainly due to the low finesse of a dielectric film without any reflecting coating.

This prompted us to investigate the NL GM induced optical response of a NL thin film. We have shown that the NL GM resonance is a very interesting candidate to study OB in such films. Indeed, in realistic experimental situations where the half width at half maximum of the GM resonance curve is of the order of 3 mrad, this EM resonance leads to switching values of $\tilde{\lambda}$ 10^4 times smaller than those obtained with the NL Airy resonance.

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