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Hall effect in quantum wires

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Calculations of the Hall resistance of quasi-one-dimensional electron systems on a GaAs/$Al_xGa_{1-x}As$ heterointerface are performed. It is shown in a weak-link model of Hall probes that the Hall resistance strongly depends on the way the total current is divided among subbands.

Recent experiments[1-6] on the Hall resistance of quantum wires in the ballistic regime have revealed quite new features of transports. Quantum wires have been fabricated in modulation-doped GaAs/$Al_xGa_{1-x}As$ heterostructures containing a two-dimensional (2D) electron system. The width of wires is much smaller than the mean free path of electrons and comparable to the Fermi wavelength, giving only a few subbands occupied by electrons. In this paper we perform calculations of the Hall resistance of the laterally confined 2D electron gas, and clarify some important aspects of the Hall effect in such systems.

In the low-magnetic-field regime and at low temperatures, the Hall resistance (R_{xy}) exhibits several peculiar behaviors which are absent in two- and three-dimensional systems. Timp *et al.*[1] and Simmons, Tsui, and Weimann[2] observed a plateau at a field where R_{xy} in 2D systems shows the classical linear behavior on the magnetic field. More surprisingly, Roukes *et al.*,[3] followed by Timp *et al.*[4] and Ford *et al.*,[5] discovered a quenching of the Hall effect, i.e., R_{xy} of narrow wires is much lower than the classical line below a certain threshold magnetic field. The threshold field is higher for a narrower wire.

Several theoretical works have been done on R_{xy} of the laterally confined 2D electron gas. In relation to the quantum Hall effect, edge states have been discussed,[7-12] which are localized within about a cyclotron radius of the wire edges but extended along the wire direction. In the presence of a confining potential, their energies rise up as approaching the boundaries, giving a continuous energy spectrum covering the gap regions between bulk Landau levels. It has been noted that in the absence of scatterings R_{xy} is exactly quantized to $h/2Ne^2$ (N: the number of occupied subbands) at the Fermi level between neighboring bulk Landau levels if the Hall voltage is regarded as the chemical potential difference between the edge states in opposite sides. Extrapolation of this formula to the low-field limit leads to a wrong result: R_{xy} does not become zero because N is finite. This indicates that the above definition of the Hall voltage must be refined when the extent of the edge states becomes comparable to the width of the wire W. Beenakker and van Houten[13] argued that the quenching takes place when the extent of the edge state in the lowest subband coincides with W. Peeters[14] calculated R_{xy} by using Landauer's formula for the four-terminal configuration[15] and a weak-link model of Hall probes. Although finding no quenching of R_{xy} at nonzero fields, he obtained a distinct downward deviation of R_{xy} from the quantized values $h/2Ne^2$ in the low-field range and the vanishing of R_{xy} at zero field by considering the presence of an electron transfer between the edge state and the Hall probe in opposite sides.

When more than one subband is occupied by electrons, it is necessary to specify the current distribution among the subbands, which inevitably affects the Hall voltage or R_{xy}. All the theoretical works mentioned above have been done in the simplest distribution such that each subband contributes an equal amount of current. In the present paper, the formula of R_{xy} is derived for arbitrary current distributions within the weak-link model of Hall probes, and R_{xy} of quantum wires is calculated in two typical models of the current distribution.

Consider a noninteracting 2D electron system confined in the space with the width W in the xy plane by infinite barriers. In the presence of a magnetic field H perpendicular to the system, the Hamiltonian is

$$\mathcal{H}=\frac{1}{2m}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]^2+V(x)\,, \tag{1}$$

where m is the effective mass and $\mathbf{A}=(0,Hx)$. The confining potential is given by $V(x)=0$, $-W/2<x<W/2$, and $V(x)=\infty$, otherwise. We neglect the effects of the impurities and phonons because we consider a region in the vicinity of Hall probes, whose size is much smaller than the mean free path. Each eigenstate is specified by the subband index $n=0,1,2,\ldots$ and the wave vector along the y direction k_y.

Suppose the wire is connected at its ends to the current source and sink, which are electron reservoirs with chemical potentials μ_a and μ_b, respectively (Fig. 1). Here we consider the case that $\mu_a-\mu_b$ is positive infinitesimal and the temperature is zero. The total current along the wire is $I=(2e/h)\sum_n\Delta\mu_n$ including spin degeneracy,[15,16] where $\Delta\mu_n$ is the chemical potential difference between electrons in the nth subband coming from the current source ($k_y>0$) and those from the current sink ($k_y<0$). When electrons suffer scatterings on the path from the current source and to the current sink, $\Delta\mu_n$ is not necessarily equal to $\mu_a-\mu_b$. Furthermore, each subband can have occupied and unoccupied states coexisting in the energy range between μ_a and μ_b. The chemical potential μ of each branch ($k_y>0$ or $k_y<0$) in each subband is defined in such a way that the number of occupied states with energy above μ is equal to the number of unoccupied states with

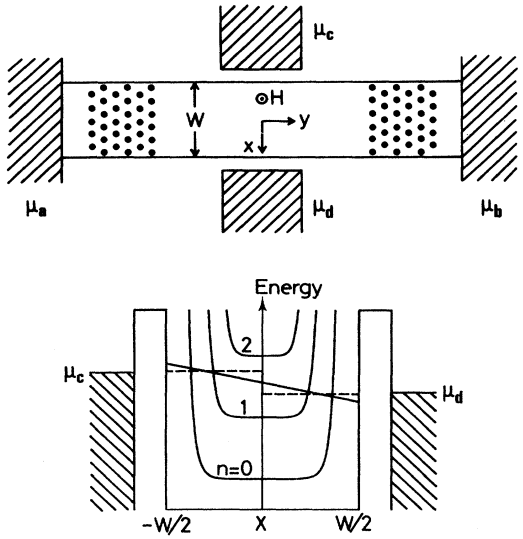

FIG. 1. Quantum wire connected to four reservoirs: current source (chemical potential μ_a), current sink (μ_b), and Hall contacts (μ_c and μ_d). Dotted areas are scattering regions. Lower figure shows energy eigenvalues of a wire with $W=0.1$ μm in magnetic field $H=4$ T as a function of X, the expectation value of x. Two models of the current distribution (chemical potentials of subbands) are shown: step model (dashed line) and slope model (solid line). High barriers are placed between the wire and the Hall contacts.

energy below μ.

In four-terminal measurements of the Hall resistance, the quantum wire is connected via Hall probes to electron reservoirs (Hall contacts). Chemical potentials of the Hall contacts are determined to give vanishing net currents between the wire and the contacts. In the small conductors considered here, the Hall probes are parts of the sample, possibly affecting electronic states in the wire. However, we restrict ourselves to the weak-link model given by Peeters[14] for simplicity. In this model two equivalent high barriers are symmetrically placed between the wire and the Hall contacts (Fig. 1), although this symmetry is not essential to the conclusions obtained below. The presence of the Hall contacts does not disturb electronic states in the wire, but introduces electron transfers through the barriers with extremely low probability. By considering infinite barriers, the transition probability of an electron between the wire and one of the Hall contacts is proportional to the square of the derivative of its wave function at the wire edge.

In the case that only a single subband ($n=0$) is occupied, we have two states at the Fermi level, one with positive k_y and the other with negative k_y. By denoting the transition probability between the state with $k_y>0$ and one of the Hall contacts in $x<-W/2$ by P_1 and that for the state with $k_y<0$ by $P_2(P_1>P_2)$, we have the following condition for the vanishing of the net current: $(\Delta\mu_0-\Delta\mu_H)P_1=(\Delta\mu_0+\Delta\mu_H)P_2$, where $\Delta\mu_H$ is the chemical potential difference between the two Hall contacts. Combining this equation with the current formula given before, we obtain

$$R_{xy}=\frac{h}{2e^2}\frac{P_1-P_2}{P_1+P_2}. \tag{2}$$

In the absence of a magnetic field, R_{xy} vanishes because the amplitude of the wave function becomes independent of k_y, giving $P_1=P_2$. In the high-field limit, on the other hand, the states at the Fermi level are well localized on each edge and $P_1\gg P_2$, leading to a quantized value of R_{xy}. Generalization to multisubband cases gives

$$R_{xy}=\frac{h}{2e^2}\frac{\sum_n(P_{n1}-P_{n2})\Delta\mu_n}{\sum_n(P_{n1}+P_{n2})\sum_n\Delta\mu_n}, \tag{3}$$

where P_{n1} and P_{n2} are the transition probability for the nth subband. Here we assume that the total number of electrons in each subband is the same as in the absence of current. Equation (3) can be obtained from Landauer's conductance formula for the four-terminal configuration.[15]

The next problem is to determine $\Delta\mu_n$, i.e., how the total current is shared by subbands. This distribution may be affected by scatterings of an electron by impurities, phonons, and the other electrons during its travel along the wire and also by the potential due to the polarization along x proportional to $\Delta\mu_n$. We consider two models for the current distribution in this paper. In the first model, the total current is divided into equal parts, i.e., $\Delta\mu_n$ is independent of n. Since in this model the chemical potential is a step function of X, the expectation value of x, at high magnetic fields (Fig. 1), we call it step model. Note that X and k_y have opposite signs at high fields. Although this distribution is used in many theoretical works as mentioned before, it is realized only when electrons suffer no scatterings. In the step model, the above formula of R_{xy} reduces to that obtained by Peeters:[14]

$$R_{xy}(\text{step})=\frac{h}{2e^2N}\frac{\sum_n P_{n1}-\sum_n P_{n2}}{\sum_n P_{n1}+\sum_n P_{n2}}, \tag{4}$$

which is always smaller than the quantized values due to the presence of P_{n2}. In the second model, which we call slope model, $\Delta\mu_n$ is proportional to X_n, X of the state with $k_y<0$ at the Fermi level in the nth subband (Fig. 1). This distribution is just the same as that produced when a uniform electric field is applied adiabatically in the x direction. In the step model the chemical potential is singular at $X=0$, and becomes a complicated function at low fields because for large n, states with $k_y>0$ have $X>0$ in order to be orthogonal to states with smaller n and the same k_y, which have $X<0$. Such a distribution is unrealistic in the present system. If we consider the field due to the polarization, the slope model is closer to the real systems.

Figure 2 presents the calculated Hall resistance of a quantum wire with $W=0.1$ μm as a function of magnetic field. The sheet density of electrons N_s is 4.0×10^{11} cm^{-2} corresponding to one of the samples of Roukes *et al.*,[3] and $N=5$ at $H=0$. There appear large differences in R_{xy} between the two models for the current distribution. Particularly, in the high-field regime such that $P_{n1}\gg P_{n2}$, R_{xy}

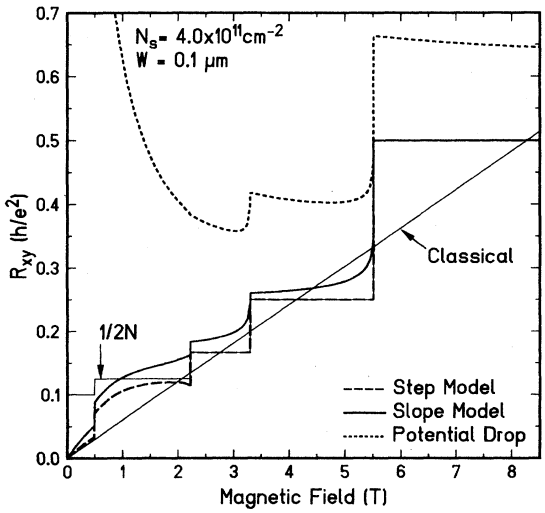

FIG. 2. Hall resistance R_{xy} as a function of magnetic field H for a quantum wire with the width $W=0.1$ μm and the sheet density $N_s=4.0\times10^{11}$ cm^{-2}. The results are shown for two current distributions: step model (thick dashed curve) and slope model (thick solid curve). Thick dotted curve shows R_{xy} calculated with use of the Hall voltage defined by potential drop. Two thin solid lines represent the quantized Hall resistance $1/2N(h/e^2)$ with N the number of occupied subbands and the classical Hall resistance H/N_sec, respectively.

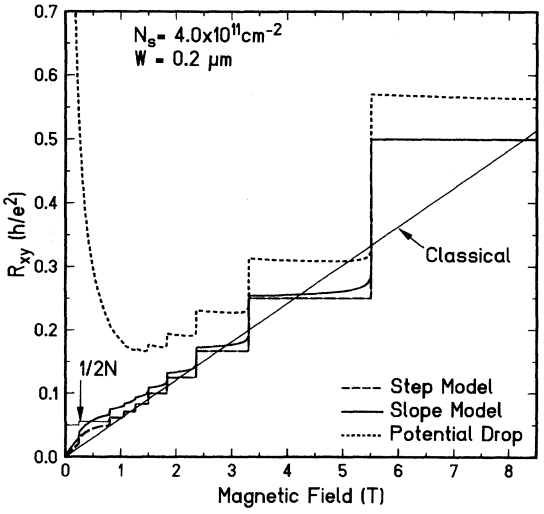

FIG. 3. Hall resistance R_{xy} as a function of magnetic field for a quantum wire with the width $W=0.2$ μm. For more explanations, see the caption of Fig. 2.

(step) is well quantized, whereas R_{xy} (slope) is considerably higher except in the high-field limit ($N=1$). This difference is explained as follows. Under a constant current ($\Sigma_n\Delta\mu_n=$const), $\Delta\mu_n$ for the outermost edge states ($n=0$) is larger in the slope model than that in the step model (Fig. 1). For example, in the case that two subbands ($n=0,1$) are occupied, $\Delta\mu_0$(slope)$=2X_0/(X_0+X_1)\Delta\mu_0$(step). The chemical potentials of the outermost edge states mainly determine the Hall voltage because of their higher transition probability to the Hall contacts. Therefore, R_{xy}(slope) is larger than R_{xy}(step). In the case of $N=2$, the deviation of R_{xy}(slope) from the quantized values is approximately $(X_0-X_1)/W$, which is proportional to l/W, where l is the cyclotron radius defined by $(c\hbar/eH)^{1/2}$. At the low field where P_{n2} becomes comparable with P_{n1}, both R_{xy}(step) and R_{xy}(slope) start to curve downward and finally vanish at zero field, although a quenching of R_{xy} at nonzero fields does not occur in the present model. As a result of small N at low fields, a hump appears above the classical line, although a distinct plateau as observed in the experiments[1-3,5,6] is not found.

Figures 3 and 4 show the calculated results for wider wires with $W=0.2$ μm and 0.5 μm. Both R_{xy}(step) and R_{xy}(slope) approach the quantized values as W becomes larger. R_{xy}(step) is almost quantized in the high-field region satisfying $W\gg l_{N-1}$, and its downward deviation is proportional to $\Sigma_nP_{n2}/\Sigma_nP_{n1}\sim\exp[-\alpha(W/l_{N-1})^2]$, where $l_n=l\sqrt{2n+1}$ is the cyclotron radius for the nth subband and α is a factor of the order of unity. On the other hand, the deviation of R_{xy}(slope) is proportional to l/W and remains large for wide wires. A hump appears above the classical line around the low field satisfying $W=l_{N-1}$. This condition reduces for large N to Beenakker's classical result for the critical field[13] $H_{\rm crit}\propto 1/W$. The field position of the hump coincides roughly with that observed by Roukes *et al.*,[3] although the height of the hump is much larger than the observed one.

Ono and Ohtsuki[17] and Gudmundsson *et al.*[18] calculated R_{xy} by identifying the Hall voltage with a potential drop which is an applied electric field across the wire multiplied by W. Figures 2, 3, and 4 also show the Hall resistance obtained in this way [R_{xy} (potential)]. The current distribution is identical to that in the slope model. R_{xy} (potential) is always larger than R_{xy}(slope) because the potential drop and $\Delta\mu_n$ are proportional to W and

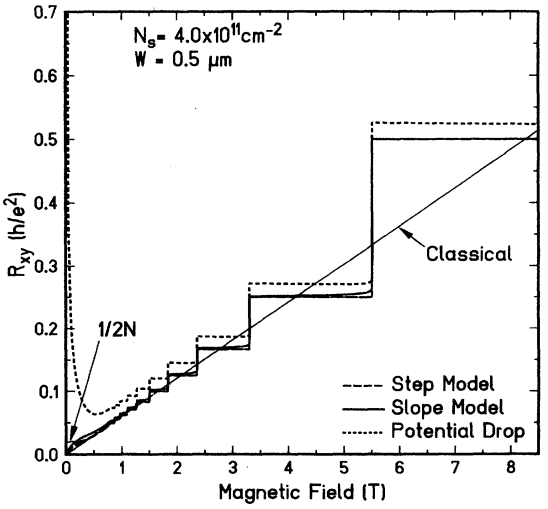

FIG. 4. Hall resistance R_{xy} as a function of magnetic field for a quantum wire with the width $W=0.5$ μm. For more explanations, see the caption of Fig. 2.

$2X_n (< W)$, respectively. In particular, as $H \rightarrow 0$, R_{xy}(potential) becomes infinite because the Hall voltage is constant although the current is infinitesimal, proportional to X_n. This indicates that the potential drop is quite different from the measured Hall voltage in the case of quantum wires.

In actual Hall bars, the width of Hall probes is the same as that of a wire, just like at a crossing. At high fields electrons at the Fermi level in every subband move along the edge and turn to the left at the crossing. Since $\Delta\mu_H$ becomes a certain average of $\Delta\mu_n$ with respect to n, we expect that R_{xy}(slope) approaches the quantized values. In the case of narrow Hall probes, however, the situation is expected to be close to that in the present weak-link model. The magnitude of the deviations of R_{xy} in actual systems is much affected by such structures of Hall probes, but the direction of the deviations is likely to be upward because the slope model is more realistic than the step model. Calculations of current distributions and R_{xy} of actual systems are now under way.

In conclusion, we have calculated the Hall resistance of quantum wires in two models of the current distribution among subbands; one is that of a ballistic current (step model) and the other is that in a transverse electric field (slope model). In the weak-link model of Hall probes, R_{xy}(step) is well quantized at plateaus with only small downward deviations, whereas R_{xy}(slope) deviates considerably upward. In the low-field regime, R_{xy} deviates downward from the quantized values, shows a hump, and finally vanishes at zero field without quenching at nonzero fields.

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