Effects of spin-dependent impurity scattering in itinerant ferromagnets

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The effects of spin-dependent impurity scattering in interacting systems have been studied to investigate the role of spin flipping. While charge-density fluctuations continue to be undamped, spin-density fluctuations are found to be damped due to spin flipping. This leads to damping of spin waves and Stoner excitations in the ferromagnetic phase. Also, spin flipping works against ferromagnetism and leads to a modification of the Stoner criterion for the onset of ferromagnetic instability. Another manifestation of the tendency of spin flipping to inhibit ferromagnetism is in the distortion of the minority-spin band, especially near the band edge. When localization corrections are included, spin-orbit impurity scattering is found to lead to identical reduction in the spin-diffusion constant and the spin-relaxation rate which is due to the quantum-interference effect of weak localization. This is in contrast to the enhancement of the charge-diffusion constant due to antilocalization and demonstrates the dissimilarity between diffusion of spin- and charge-density fluctuations. The dynamical localization mechanism that effectively reduces the spin-flipping rate is also found to correspondingly enhance the band shift and thus cause a stiffening of spin waves and a widening of the Stoner excitation gap. In the ultralocalized limit, random spin-orbit coupling is found to lead to competing interactions in a lattice system with a half-filled band. This points towards the possibility of having frustration in a system in which the spin-orbit strength is of the same order as the hopping term.

I. INTRODUCTION

Some of the combined effects of normal disorder due to spin-independent impurity scattering and electronic correlation in an itinerant ferromagnetic system near the magnetic instability have recently been studied[1] with respect to spin- and charge-density fluctuations, spin-wave mode, and Stoner excitations. Typical Fermi-liquid behavior was obtained in lowest order with two separate diffusion constants for spin- and charge-density fluctuations, and the diffusion constants acquire identical weak-localization corrections. In the ferromagnetic phase Stoner excitations are localized, and while the dc conductivity is reduced due to weak-localization effect, the stiffness constant of the spin-wave mode remains unaffected by localization. This suggests that the system can exist in an insulating state and exhibit long-range magnetic order—a "ferromagnetic Anderson insulating state."

In a system which has only spin-independent impurity scattering, the only available mode for spin relaxation is spin diffusion, and hence a dynamical effect such as localization affects the spin-fluctuation mode just as it does the dc conductivity. In the absence of any scattering which flips the spin, dynamical processes have no effect whatsoever on magnetization, and hence the spin-wave stiffness constant is unaffected by localization. In this paper we make some observations on the possible consequences of spin-dependent impurity scattering processes (e.g., spin-orbit coupling, magnetic impurity scattering) on the magnetic behavior of an itinerant system. Spin-flipping impurity scattering processes provide an intrinsic mechanism for spin relaxation which will work in conjunction with the simple diffusion process in a disordered system. We expect this combined relaxation process to show up in the dynamical spin-response function. Spin-flip processes work against ferromagnetism and this should result in a modification of the Stoner criterion. We also expect spin relaxation due to flipping to cause a distortion of the spin bands in the ferromagnetic phase and effectively reduce the magnetization, and hence make it susceptible to dynamical processes such as localization.

Indeed, we find that for spin-orbit impurity scattering (which leads to singular localization corrections owing to its symmetry under time reversal), weak-localization corrections due to the quantum-mechanical interference of scattering amplitudes lead to an enhanced probability of finding a particle in the same spin state and in the same spatial region. This enhancement is reflected in the reduction due to localization in the spin-diffusion constant and the spin-relaxation rate. Localization therefore favors magnetization by effectively reducing the spin-relaxation rate, and this has significant effects on the spin-wave mode, unlike the case with normal impurities only.

The presence of spin-independent scattering processes, it was argued in Ref. 1, does not lead to any change in the qualitative nature of magnetization, and at finite temperatures itinerant magnetic systems with nonmagnetic impurities behave like random magnets. In the electron-gas model with a parabolic band, the system undergoes a ferromagnetic transition with increasing interaction

strength. In a lattice system with a half-filled band, to take another case, the magnetic phase is of the disordered antiferromagnetic type, as was shown using the exact-eigenstate approach.[2] We believe the situation to be quite different in this regard when strong spin-dependent scattering is present. We have shown that in the localized limit, presence of strong spin-orbit scattering generates competing interactions possibly leading to frustration and spin glassiness.

Recent neutron inelastic scattering experiments[3] on the spin dynamics of amorphous $Fe_{90-x}Ni_xZr_{10}$ have revealed a temperature-independent spin-wave line width which goes as q^4. This is contrary to the T^2 dependence which is expected from the contribution to damping of spin waves by magnon-magnon interaction.[4] The temperature independence of the spin-wave broadening was taken to be suggestive of the relevance of intrinsic broadening mechanisms arising, possibly, due to magnetic disorder in the system. Our study of the effects of normal impurity scattering on spin waves[1,5] at zero temperature has indicated a damping term, proportional to q^4, which arises from the diffusive mode indicating the importance of intrinsic broadening mechanisms at low temperatures. In this paper, we consider in detail the effects of spin-dependent impurity scattering processes as well, and study their effects on spin waves and Stoner excitations.

A systematic way to incorporate the effects of both disorder and interaction is to introduce a generalized N-species model of interacting fermions[1] and to study it within the $1/N$ expansion technique. The ferromagnetic transition is accessible in the $N\rightarrow\infty$ limit, and hence the method affords a convenient starting point for the investigation of effects of spin-dependent impurity scattering in the ferromagnetic phase. Spin-dependent scattering could be due to random spin-orbit coupling or due to magnetic impurity scattering. These two impurity scattering processes play identical roles in the $N\rightarrow\infty$ approximation [provided only the s-wave part of the spin-orbit vertex is kept, which is valid in the limit where the momentum exchange is much smaller than the inverse mean free path $(ql\ll 1)$]. When weak-localization corrections are incorporated, by including in the impurity susceptibility maximally crossed diagrams[6] which are of $O(1/N)$, one must distinguish between the two, owing to their different symmetries.[6] Spin-orbit scattering obeys time-reversal symmetry with spin, and weak-localization corrections in this case indicate localization of spin-density fluctuations and antilocalization of charge-density fluctuations. Magnetic impurities break time-reversal symmetry, and weak localization corrections are rendered nonsingular by τ_m^{-1} which acts as a cutoff in the diffusion pole.

Formally, we have extended the generalized N-orbital model of interacting fermions (see Ref. 1 and references therein) by including spin-dependent impurity scattering processes, and have studied it within the $1/N$-expansion technique which systematizes terms in perturbation theory involving both the interaction and the disorder strengths. The spin response of the system has been studied exactly in the $N\rightarrow\infty$ limit, and localization effects have been studied systematically by including all $O(1/N)$ localization corrections. Advantage has been taken of the Ward identities which are obeyed order by order in perturbation theory.

II. LARGE N LIMIT

Spin-dependent scattering leads to two more effective impurity scattering vertices in addition to the spin-independent vertices introduced earlier.[1] These effective scattering vertices are shown in Fig. 1. First, we consider the $N\rightarrow\infty$ limit, in which case terms of order $1/N$ and higher can be neglected. Spin-orbit scattering and magnetic impurity scattering play identical roles in this limit, so we shall collectively use the term spin-flip scattering to refer to both. The self-energy correction due to impurities and interaction is shown diagrammatically in Fig. 2, and represents the following Dyson equation:

$$\Sigma^{\sigma}(\omega)=n^{-\sigma}U+(\gamma_n+\gamma_{sf}/3)\int d^dk\frac{1}{(2\pi)^d}G^{\sigma}(\mathbf{k},\omega)$$
$$+(2\gamma_{sf}/3)\int d^dk\frac{1}{(2\pi)^d}G^{-\sigma}(\mathbf{k},\omega)\ . \qquad (1)$$

Here, $n^{-\sigma}$ is the spin $-\sigma$ electronic density, and γ_n and γ_{sf} are, respectively, the strengths of normal and spin-flip impurity scattering processes. In terms of the elastic lifetimes, we have

$$\tau_n^{-1}=2\pi N(0)\gamma_n$$
$$\tau_{sf}^{-1}=2\pi N(0)\gamma_{sf}\ . \qquad (2)$$

The contribution due to interaction is just the Hartree term proportional to the particle density of opposite spin. There are two additional impurity terms due to spin-dependent scattering in the parallel and antiparallel spin channel, their respective strengths being $\gamma_{sf}/3$ and $2\gamma_{sf}/3$. This complicates the evaluation of self-energy in the ferromagnetic phase which we shall take up later.

A. Paramagnetic phase

First, we consider the paramagnetic case in which particle densities and the Green's functions are spin independent, and so evaluation of the self energy can be carried through with γ replaced by $(\gamma_n+\gamma_{sf})$. The Green's func-

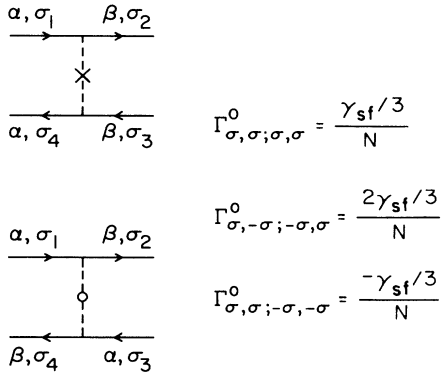

FIG. 1. Effective spin-dependent impurity scattering processes.

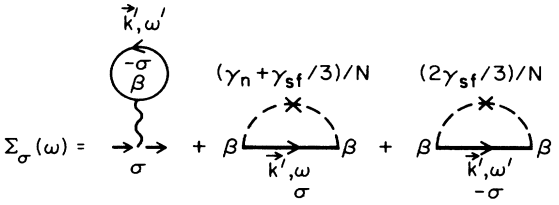

FIG. 2. Self-energy corrections in the $N\rightarrow\infty$ limit.

tion is then given by

$$G(\mathbf{k},\omega)=\frac{2}{q^2(\omega)-k^2}\ ,$$
$$G(\mathbf{r}-\mathbf{r}';\omega)=-\frac{1}{2\pi}\frac{\exp[iq(\omega)|\mathbf{r}-\mathbf{r}'|]}{|\mathbf{r}-\mathbf{r}'|}\ , \tag{3}$$

where

$$q(\omega)=\frac{i}{2l}+\mathrm{sgn}(\omega)[2(\omega+\omega_F)]^{1/2}\ , \tag{4}$$

with the condition that $\mathrm{Im}[2(\omega+\omega_F)]^{1/2}\geq 0$. The mean free path and the elastic lifetime are now given by

$$l=\frac{\pi}{\gamma_n+\gamma_{sf}}\ ,$$
$$\tau^{-1}=\frac{k_F}{\pi}(\gamma_n+\gamma_{sf})\ . \tag{5}$$

We now evaluate the impurity-averaged transverse magnetic susceptibility in order to study the spin response of the system. Evaluation of the transverse susceptibility is more convenient than that of the longitudinal susceptibility which involves the spin-flip scattering vertices as well. In the paramagnetic phase there is no difference between these two susceptibilities apart from a factor of 2. In the $N\rightarrow\infty$ limit the relevant two-particle propagator consists of ladder diagrams involving ladders of interaction vertices and the normal and spin-flip impurity scattering vertices. The diagrammatic series for the impurity vertex $\Gamma_{\mathrm{imp}}^{\uparrow\uparrow,\downarrow\downarrow}(\mathbf{q},\omega)$ is shown in Fig. 3, and it corresponds to the following self-consistent equation:

$$\Gamma=1+\gamma_n J\Gamma+\left[-\frac{\gamma_{sf}}{3}\right]J\Gamma\ . \tag{6}$$

The impurity-averaged transverse magnetic susceptibility is thus given by

$$\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega)=i\int\frac{d\omega'}{2\pi}\frac{J^{\uparrow\downarrow}(\mathbf{q},\omega;\omega')}{1-(\gamma_n-\gamma_{sf}/3)J^{\uparrow\downarrow}(\mathbf{q},\omega;\omega')}\ . \tag{7}$$

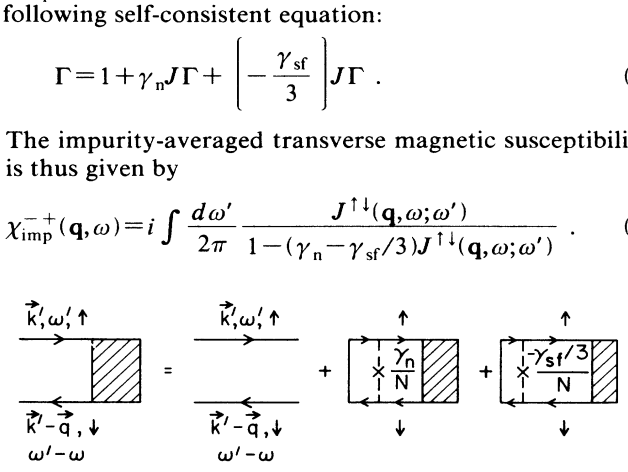

FIG. 3. Diagrammatic representation of the impurity vertex relevant to the transverse magnetic susceptibility in the $N\rightarrow\infty$ limit.

Here,

$$J^{\uparrow\downarrow}(\mathbf{q},\omega;\omega')=\int_{k'}G^{\uparrow}(\mathbf{k}',\omega')G^{\downarrow}(\mathbf{k}'-\mathbf{q},\omega'-\omega)\ .$$

Evaluation of the frequency integral is done as before[1] and in the diffusive limits of small frequency ($\omega\tau\ll 1$) and long wavelength ($1/q\gg l$), the susceptibility to order (ω,q^2) can be expressed as

$$\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega)=\chi_{\mathrm{imp}}^{-+}(\mathbf{q},0)\left[1+\frac{i\eta\omega}{\frac{4}{3}\tau_{sf}^{-1}+Dq^2-i\omega}\right]\ , \tag{8}$$

where $\eta\equiv N(0)/\chi_{\mathrm{imp}}^{-+}(\mathbf{q},0)$ and $\tau_{sf}^{-1}\equiv k_F\gamma_{sf}/\pi$ is the relaxation rate due to spin-flip scattering and $N(0)=k_F/2\pi^2$, $D=l^2/3\tau$ as before. It is characteristic of systems with spin-dependent scattering that the static susceptibility is smaller than the one-particle density of states so that $\eta>1$. This is simply because spin flipping tends to reduce the net magnetization for a given applied magnetic field. This reduction in the static susceptibility has bearing on the Stoner criterion for onset of ferromagnetism and will be discussed in more detail later.

In the limit when $\eta\omega\ll\tau_{sf}^{-1}$, we can write Eq. (8) in a form which clearly shows the damped nature of spin-density fluctuations:

$$\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega)=\chi_{\mathrm{imp}}^{-+}(\mathbf{q},0)\left[\frac{\frac{4}{3}\tau_{sf}^{-1}+Dq^2}{\frac{4}{3}\tau_{sf}^{-1}+Dq^2-i\eta\omega}\right]\ . \tag{9}$$

In the $N\rightarrow\infty$, the dynamical susceptibility for the interacting system can now be written in terms of $\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega)$ in the form of a (RPA) random-phase approximation type expression,[1]

$$\chi^{-+}(\mathbf{q},\omega)=\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega)/[1-U\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega)]\ .$$

We obtain

$$\chi^{-+}(\mathbf{q},\omega)=\frac{\chi_{\mathrm{imp}}^{-+}(0,0)}{1-U\chi_{\mathrm{imp}}^{-+}(0,0)}\left[\frac{\frac{4}{3}\tau_{\sigma}^{-1}+D_{\sigma}q^2}{\frac{4}{3}\tau_{\sigma}^{-1}+D_{\sigma}q^2-i\eta\omega}\right]\ . \tag{10}$$

Thus, the dynamical spin-response function is seen to exhibit typical diffusive behavior in the small frequency and long-wavelength approximation. Up to this level of approximation, interaction is seen to lead to a Fermi-liquid–type renormalization of the various physical quantities—the static susceptibility, the spin-orbit relaxation rate, and the diffusion constant for spin-density fluctuations

$$\tau_{\sigma}^{-1}=[1-U\chi_{\mathrm{imp}}(0,0)]\tau_{sf}^{-1}\ ,$$
$$D_{\sigma}=[1-U\chi_{\mathrm{imp}}(0,0)]D\ . \tag{11}$$

From Eq. (10) we observe that due to spin flipping associated with the spin-dependent scattering term, spin fluctuations in the system are damped with a relaxation rate $\sim\tau_{\sigma}^{-1}$, and this leads to an "infinite-wavelength broadening" of the spin resonance. This result has been obtained previously by treating the spin-flip term to lowest order.[7] We find essentially the same result in the $N\rightarrow\infty$ limit in which spin-flip is treated to all orders, but with one

difference: The energy scale is found to be renormalized by the spin-flipping term which tends to suppress static susceptibility. Spin flipping leads to an effectively slower spin relaxation due to diffusion ($D/\eta < D$).

The presence in Eq. (10) of a relaxation term, which cuts off the diffusion pole at $q,\omega \to 0$ has important consequences[8] on the temperature behavior of the specific heat of a nearly magnetic system. For temperatures much smaller than the relaxation rate, the specific heat goes as T^3 and crosses over to a $T^{3/2}$-like behavior only when $T \sim \tau_\sigma^{-1}$. As will be discussed in Sec. III, weak-localization corrections reduce the effective spin-relaxation rate which suggests that the temperature range for the T^3 behavior of specific heat is actually much smaller.

A careful evaluation of the static susceptibility leads to

$$\chi^{-+}_{\rm imp}(\mathbf{q},0)=N(0)\left[1-\frac{2}{3k_F l_{\rm sf}}\tan^{-1}\left(\frac{3k_F l_{\rm sf}}{2}\right)-\alpha q^2/12k_F^2\right] . \tag{12}$$

Here $l_{\rm sf}=k_F\tau_{\rm sf}$ is the mean free path due to spin-flip scattering. The factor α is obtained numerically from the q dependence of the static susceptibility. The variation of α with increasing strength of spin-flip impurity scattering is shown in Fig. 4 for different values of $(k_F l_{\rm n})^{-1}$. We notice that spin-flipping impurity scattering is about an order of magnitude more effective in reducing α than normal impurity scattering (see inset in Fig. 4). The reduction in α indicates that spin-density fluctuations become more and more localized spatially. This behavior of α also has bearing on the spin-wave energy, as will be discussed later.

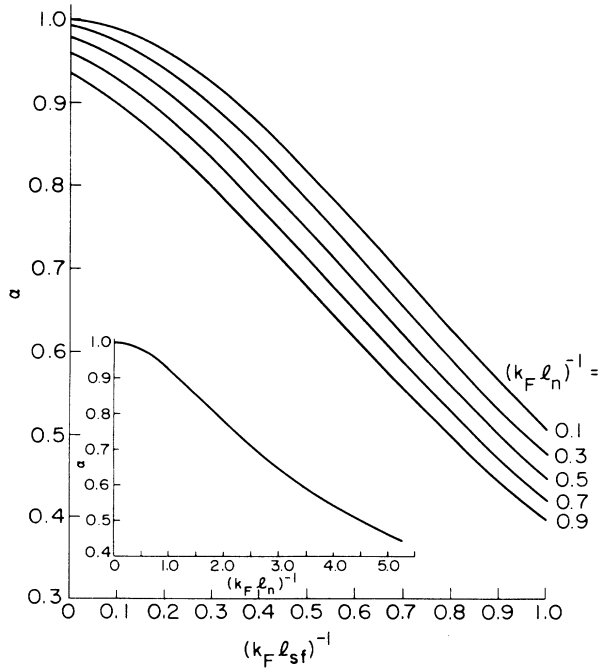

FIG. 4. Variation of α with $(k_F l_{\rm sf})^{-1}$.

The second term in Eq. (12) for the static susceptibility vanishes in the limit of $k_F l_{\rm sf} \to \infty$ and approaches 1 in the limit of strong spin-flip scattering ($k_F l_{\rm sf} \to 0$). Thus, spin flip processes lead to a reduction in the static susceptibility from its normal value which equals the density of states. An inference to be drawn from this reduction is that of suppression of spin-density fluctuations due to spin-flip impurity scattering. This reduction in the static susceptibility also results in a modification of the Stoner criterion. The criterion for the onset of the ferromagnetic instability is now given by

$$1-U_c N(0)\left[1-\frac{2}{3k_F l_{\rm sf}}\tan^{-1}\left(\frac{3k_F l_{\rm sf}}{2}\right)\right]=0 . \tag{13}$$

In the absence of spin-flip scattering ($k_F l_{\rm sf} \to \infty$) we recover the Stoner criterion, independent of strength of normal disorder. However, the critical interaction strength, U_c, required to make the system magnetic, is seen to increase with increasing spin-orbit or magnetic-impurity scattering strength. This is simply because spin flipping associated with these scattering processes works against ferromagnetism.

B. Ferromagnetic phase

The Green's function in the ferromagnetic phase, with magnetization $m = n^{\uparrow} - n^{\downarrow}$, is given by

$$G_\sigma(\mathbf{q},\omega)=\frac{2}{q_\sigma^2(\omega)-k^2} , \tag{14}$$

where the complex wave vector

$$q_\sigma(\omega)=[2(\omega-\Sigma_\sigma(\omega)]^{1/2}$$

is obtained from Eq. (1) for the spin-dependent self-energy, which yields after the momentum integration

$$\omega-\frac{q_\sigma^2}{2}=n^{-\sigma}U+\left[\gamma_{\rm n}+\frac{\gamma_{\rm sf}}{3}\right]\left[-\frac{i}{2\pi}q_\sigma\right]+\frac{2\gamma_{\rm sf}}{3}\left[-\frac{i}{2\pi}q_{-\sigma}\right] . \tag{15}$$

Solving Eq. (15) for the two spins self-consistently, we find that to first order in $\Delta=-(n^{\uparrow}-n^{\downarrow})U$, we can express $q_\sigma(\omega)$ in the following form:

$$q_\sigma(\omega)=q(\omega)-\sigma\Delta\delta q(\omega)/2 , \tag{16}$$

where $q(\omega)$ is given in Eq. (4), and $\delta q(\omega)\equiv q_{\uparrow}(\omega)-q_{\downarrow}(\omega)$ is given by

$$\delta q(\omega)=\frac{1}{{\rm sgn}(\omega)\sqrt{2(\omega+\omega_F)}+i(2/3l_{\rm sf})} . \tag{17}$$

This indicates that in the ferromagnetic phase the spin-flip term causes the mean free path and the elastic lifetime to be *spin dependent*. Basically, this spin dependence arises because while the spin-dependent densities of

states are symmetric about the average value, the scattering amplitudes for the spin-flip and non-spin-flip processes due to spin-orbit are not equal (see Fig. 2).

If no spin-flipping impurity scattering is present, Δ also measures the relative band shift between the majority and minority spin bands; there are no minority-spin states up to energy Δ measured from the bottom of the majority-spin band. However, if spin-flip processes are present, one finds that the band edges of the two spin bands must coincide. Keeping in mind that the density of states is related to the real part of $q(\omega)$ [see Eq. (3)], this can be seen from Eq. (15), wherein if $\mathrm{Re}q_\sigma$ vanishes, it is then impossible to satisfy the equation unless $\mathrm{Re}q_{-\sigma}$ also vanishes simultaneously. However, if spin-flip scattering is small, then the density of states in the minority spin band does become significant but only outside this region. For energies far from the band edge, Δ, apart from a renormalization, still measures the band shift.

The two spin-coupled equations in Eq. (15) can be decoupled resulting in fourth-order equations for q_σ. Solution for this quartic equation with complex coefficients is obtained numerically, and results for the real part of q_σ, which is proportional to the density of states, is shown in Fig. 5 for some representative values of magnetization and impurity scattering strength. Spin flipping leads to a distortion of the minority-spin band in the low-energy region and $\varepsilon_k^{\uparrow}-\varepsilon_k^{\downarrow}$ becomes a strong function of k for $k \ll k_F$. This should lead to interesting experimental consequences, in that if one depopulates the very low-energy states of the majority-spin band and looks for spin-flip transitions, one should observe no energy gap.

The transverse static susceptibility [Eq. (7)] involves $q_\uparrow(\omega')+q_\downarrow(\omega')$, which, to first order in Δ, is independent of magnetization from Eq. (16). Therefore, to first order in Δ, $\chi_{\mathrm{imp}}^{-+}(\mathbf{q},0,\Delta)$ in the ferromagnetic phase is given by the same expression as in the paramagnetic phase [Eq. (12)] but where $k_F=\sqrt{2\omega_F}$ is the Fermi momentum in the ferromagnetic phase. As shown in the Appendix the static susceptibility is just equal to $1/U$.

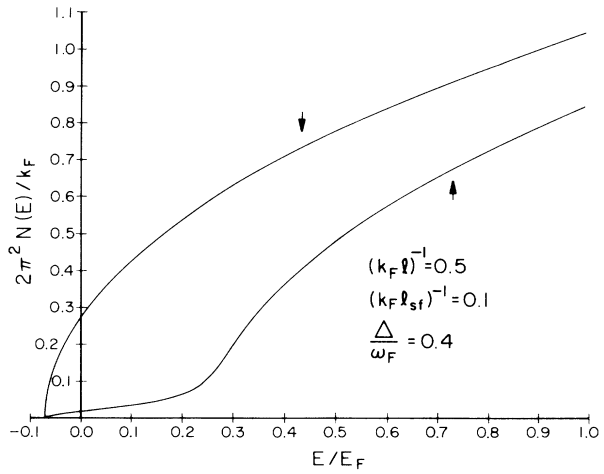

FIG. 5. Density of states in the ferromagnetic phase showing distortion in the minority-spin band.

The frequency dependent part of $\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega)$ involves $q_\uparrow(\omega')+q_\downarrow(\omega'-\omega)$, where $0<\omega'<\omega$. Using Eqs. (16) and (17) we obtain, to first order in ω and Δ,

$$q_\uparrow(\omega')+q_\downarrow(\omega'-\omega)=\frac{i}{l}+\frac{\omega-\bar{\Delta}}{k_F}\ , \tag{18}$$

where

$$\bar{\Delta}\equiv\Delta/[1+(2/3k_F l_{\mathrm{sf}})^2]\ . \tag{19}$$

We observe that the spin-flip term renormalizes the relative band shift, Δ. This effective reduction in Δ comes essentially from the reduction in the difference of the up-spin and down-spin densities of states arising from the distortion of the bands due to spin flipping. Using Eq. (7) and evaluating the frequency dependent part of $\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega)$, we obtain, after adding the static part,

$$\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega,\Delta)=\chi_{\mathrm{imp}}^{-+}(0,0,\Delta)-\bar{N}(0)\frac{\alpha q^2}{12k_F^2}+\bar{N}(0)\frac{\omega}{(\bar{\Delta}-\omega)-\frac{4}{3}i\tau_{\mathrm{sf}}^{-1}-iDq^2/(1+i\bar{\Delta}\tau)}\ , \tag{20}$$

where $\bar{N}(0)\equiv k_F/2\pi^2$; $k_F=\sqrt{2\omega_F}$ now being the Fermi momentum in the ferromagnetic phase.

The equation for the spin-wave mode can now be obtained from the condition

$$1-U\chi_{\mathrm{imp}}^{-+}(\mathbf{q},\omega,\Delta)=0\ , \tag{21}$$

which yields

$$\omega=\left[\bar{\Delta}-i\left[\tfrac{4}{3}\tau_{\mathrm{sf}}^{-1}+\frac{Dq^2}{1+(\bar{\Delta}\tau)^2}\right]\right]\alpha q^2/12k_F^2\ . \tag{22}$$

The stiffness constant of the spin-wave mode is proportional to $\bar{\Delta}\alpha$, which, in view of the variation of α [see Fig. 4], decreases rapidly with increasing spin-flip scattering strength, γ_{sf}. The presence of an imaginary part indicates that spin waves in this system are damped, the damping being proportional to q^2. The damping term (or the spin-wave inverse lifetime) is proportional to $\alpha(k_F l_{\mathrm{sf}})^{-1}$, the behavior of which, with increasing spin-flip scattering strength, is shown in Fig. 6. The spin-wave damping appears to saturate with increasing spin-flip scattering strength.

We comment here on the static susceptibility in the ferromagnetic phase which, in view of Eq. (21) for the spin-wave mode, is of importance in determining the nature of spin-wave spectrum in the disordered system. Evaluation of the static susceptibility can be carried out rigorously for the case of normal impurities only. As in the paramagnetic case, the static susceptibility turns out to be completely independent of normal impurity scattering

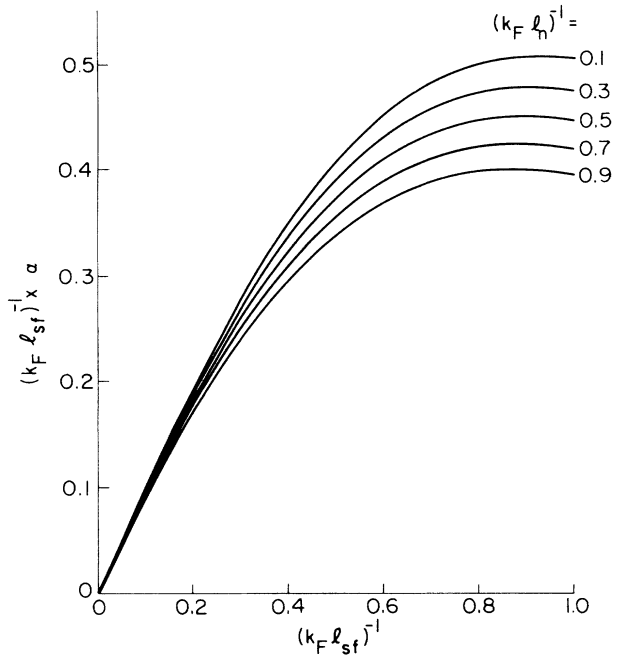

FIG. 6. Variation of spin-wave damping term with $(k_F l_{sf})^{-1}$.

strength and is equal to $1/U$ in the ferromagnetic phase so that there is no energy gap in the spin-wave spectrum. When spin-flip scattering is present, the static susceptibility in the paramagnetic phase does depend upon the spin-flip scattering strength. However, in the ferromagnetic phase, evaluation of static susceptibility carried out to order Δ leads to the result $1/U$, independent of disorder strength. The evaluation has been sketched out in the Appendix. This result should actually hold for all orders in Δ because our system is isotropic and so spin waves should be Goldstone modes and there should not be a gap in their spectrum. The situation does change if one introduces anisotropy which may be due to an external magnetic field or some internal anisotropic interactions arising from crystal-field effects.

We now briefly discuss the nature of Stoner excitations, the spectrum of which is obtained from the pole in the frequency dependent part of the transverse susceptibility. From Eq. (20) we obtain, in the limit $\bar{\Delta}\tau \ll 1$ for simplicity, the dispersion relation for Stoner excitations as

$$\omega = \bar{\Delta} - i\tfrac{4}{3}\tau_{sf}^{-1} - iDq^2 \ . \tag{23}$$

The fact that it is the renormalized band shift, $\bar{\Delta}$, which enters the above expression, is consistent with the distortion of the bands due to spin flipping as discussed earlier. Spin flipping thus leads to damping of Stoner excitations which is proportional to the spin-flip relaxation rate.

III. LOCALIZATION EFFECTS ON FERROMAGNETIC FLUCTUATIONS

We now study the localization corrections to the transverse magnetic susceptibility in the ferromagnetic phase in order to determine the effects of localization on such features as the Stoner excitations and spin waves. We take the case of spin-orbit impurity scattering only in the following as it is the more interesting of the two. Magnetic impurity scattering breaks time-reversal symmetry and localization corrections in this case are nonsingular.[3] Spin-orbit scattering preserves time-reversal symmetry with spin, and as is well known, leads to singular corrections which cause antilocalization of charge-density fluctuations.[9] Our interest here is mainly to examine the nature of weak-localization corrections for spin-fluctuations. For the case of normal impurity scattering only, it was found[1] that the band shift is unaffected by impurities and Stoner excitations are localized. The stiffness constant of spin waves was found to be unaffected by localization corrections as a consequence of an exact cancellation. Presence of spin-orbit scattering causes spin relaxation and leads to distortion of bands and a consequent reduction of the band shift, $\bar{\Delta}$. We find that localization corrections effectively reduce the spin flipping and lead to a reduction in the effective spin relaxation rate and an enhancement of the effective band shift. Results for the paramagnetic phase can be obtained simply by setting the magnetization to zero. This should tell us about localization effects on the dynamical spin response and, therefore, how spin diffusion is affected by spin-orbit impurity scattering.

Weak-localization corrections to the transverse magnetic susceptibility are of order $1/N$ within the $1/N$-expansion technique and involve maximally crossed diagrams in the particle-hole propagator.[10] These maximally crossed diagrams become ladder diagrams in the corresponding particle-particle or Cooper propagator. The particle-particle propagators relevant to the transverse magnetic susceptibility are shown in Fig. 7. The vertex parts obey the equations:

$$\begin{aligned}\Gamma_{\uparrow\uparrow;\downarrow\downarrow} &= (\gamma_n+\gamma_{so}/3)+(\gamma_n+\gamma_{so}/3)J_{\uparrow\downarrow}\Gamma_{\uparrow\uparrow;\downarrow\downarrow} \\ &\quad +(-2\gamma_{so}/3)J_{\downarrow\uparrow}\Gamma_{\downarrow\uparrow;\uparrow\downarrow} \ , \\ \Gamma_{\downarrow\uparrow;\uparrow\downarrow} &= (-2\gamma_{so}/3)+(\gamma_n+\gamma_{so}/3)J_{\downarrow\uparrow}\Gamma_{\downarrow\uparrow;\uparrow\uparrow} \\ &\quad +(-2\gamma_{so}/3)J_{\uparrow\downarrow}\Gamma_{\uparrow\uparrow;\downarrow\downarrow} \ .\end{aligned} \tag{24}$$

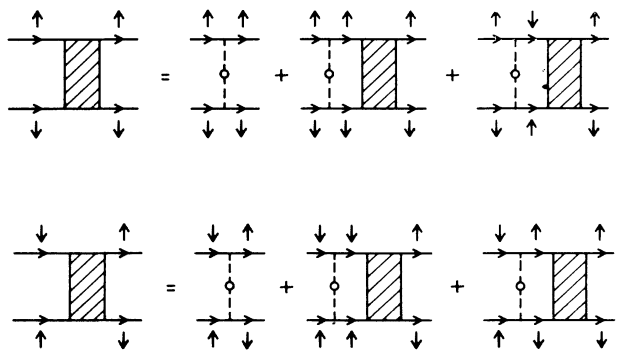

FIG. 7. Diagrammatic representation of Cooper propagators relevant to the transverse magnetic susceptibility.

Notice that the bare amplitudes for the spin-orbit impurity scattering vertices in the Cooper propagator have opposite signs. This is because in the Cooper propagator the total incoming momentum is vanishing and so the vertex involves $(\mathbf{k}\times\mathbf{k}')\cdot(-\mathbf{k}\times-\mathbf{k}')$ and not $(\mathbf{k}\times\mathbf{k}')\cdot(\mathbf{k}'\times\mathbf{k})$. This is precisely where the time-reversal symmetry with spin of spin-orbit scattering comes in. Solving for $\Gamma_{\uparrow\uparrow;\downarrow\downarrow}$ we obtain,

$$\Gamma_{\uparrow\uparrow;\downarrow\downarrow}=\frac{2\gamma_{so}/3+(\gamma_n-\gamma_{so}/3)(1-\gamma J_{\downarrow\uparrow})}{[1-(\gamma_n-\gamma_{so}/3)J_{\uparrow\downarrow}](1-\gamma J_{\downarrow\uparrow})-(2\gamma_{so}/3)(J_{\uparrow\downarrow}-J_{\downarrow\uparrow})} . \tag{25}$$

Other localization diagrams of order $1/N$ which contribute to the transverse magnetic susceptibility and charge-density correlation function are shown in Fig. 8. But before considering the diagrams themselves, we first study the general structure of the transverse susceptibility upon including the localization correction, $L^{\uparrow\downarrow}(q,\omega)$, in the antiparallel spin channel. The $O(1/N)$ localization correction with impurity-ladder diagrams of $O(1)$ also included in on either side is

$$\chi^{-+}_{\text{imp}}(\mathbf{q},\omega,\bar{\Delta})=i\int\frac{d\omega'}{2\pi}\frac{1}{1-(\gamma_n-\gamma_{so}/3)J^{\uparrow\downarrow}}L^{\uparrow\downarrow}\frac{1}{1-(\gamma_n-\gamma_{so}/3)J^{\uparrow\downarrow}} , \tag{26}$$

where $L^{\uparrow\downarrow}$ denotes the complete $O(1/N)$ localization correction in the antiparallel spin channel. Actually, a resummation of the localization diagrams (which are the most singular ones) can be performed and, after adding the $O(1)$ part given in Eq. (7), one arrives at the following expression:

$$\chi^{-+}_{\text{imp}}(\mathbf{q},\omega,\bar{\Delta})=i\int\frac{d\omega'}{2\pi}\frac{J^{\uparrow\downarrow}}{1-(\gamma_n-\gamma_{so}/3)J^{\uparrow\downarrow}-L^{\uparrow\downarrow}/J^{\uparrow\downarrow}} . \tag{27}$$

The correctness of the above expression to $O(1/N)$ can be seen explicitly by expanding the integrand in powers of $1/N$. It is physically more appropriate to bring the localization correction downstairs so that one can see directly the effects on the diffusion constant, relaxation rate, etc.

We first take up the paramagnetic phase in which case $\Delta=0$. The frequency-dependent part of the susceptibility comes from the ω' integration over the interval $0<\omega'<\omega$; it is also in this frequency range that localization corrections are most singular. In the diffusive limit, we have, for $0<\omega'<\omega$

$$1-\gamma J(\mathbf{q},\omega;\omega')=\tau(Dq^2-i\omega) , \tag{28}$$

where $\gamma=\gamma_n+\gamma_{so}$ and $\tau^{-1}=k_F\gamma/\pi$ as given in Eq. (5). Substituting

$$J^{-2}(\mathbf{q},\omega;\omega')\approx\gamma^2[1+2\tau(Dq^2-i\omega)]$$

in Eq. (27), we obtain for the frequency dependent part of the transverse susceptibility

$$i\frac{\omega}{2\pi}\left[\frac{1}{\frac{4}{3}\gamma_{so}+\gamma\tau(Dq^2-i\omega)-\gamma^2[1+2\tau(Dq^2-i\omega)]L^{\uparrow\downarrow}(\mathbf{q},\omega)}\right] . \tag{29}$$

Now we study the various vertex parts in the paramagnetic phase and isolate the ones with most singular contribution. Using $J^{\uparrow\downarrow}=J^{\downarrow\uparrow}$ and Eq. (28), we obtain from Eq. (25) the vertex part in the antiparallel spin channel

$$\Gamma_{\uparrow\uparrow;\downarrow\downarrow}(\mathbf{Q},\omega)=\frac{\gamma}{2}\left[\frac{1}{\tau(DQ^2-i\omega)}+\frac{1}{\tau(DQ^2-i\omega)+\frac{4}{3}\left[\frac{\gamma_{so}}{\gamma_n-\gamma_{so}/3)}\right]}\right] . \tag{30}$$

Comparing with the case for normal impurities only, we notice that half of the singularity in the diffusion pole as $Q,\omega\to0$ is killed by the spin-orbit term and that the singularity occurs with the same sign. This, as we shall see, leads to localization of spin-density fluctuations just as for the case of normal impurities only. The vertex parts in the parallel and spin-flip channel are similarly obtained by solving diagrammatic equations. In the presence of spin-orbit impurity scattering, the vertex part in the parallel channel becomes nonsingular:

$$\Gamma_{\sigma\sigma;\sigma\sigma}(\mathbf{Q},\omega)=\gamma\left[\frac{1}{\tau(DQ^2-i\omega)+\frac{4}{3}\left[\frac{\gamma_{so}}{\gamma_n-\gamma_{so}/3}\right]}\right] . \tag{31}$$

The localization effect on the spin-diffusion constant is in contrast with the *antilocalization effect* of spin-orbit impurity scattering on conductivity and the charge-diffusion constant.[3] The localization correction to these two quantities involves the sum $\sum_{\sigma'}\Gamma_{\sigma\sigma';\sigma'\sigma}$ in which the singularity due to the diffusion pole occurs with a negative sign,

$$\sum_{\sigma'} \Gamma_{\sigma\sigma';\sigma'\sigma}(\mathbf{Q},\omega) = -\frac{\gamma}{2}\left[\frac{1}{\tau(DQ^2-j\omega)} - \frac{3}{\tau(DQ^2-i\omega)+\frac{4}{3}\left[\frac{\gamma_{so}}{\gamma_n-\gamma_{so}/3}\right]}\right] . \tag{32}$$

The coefficient $-\frac{1}{2}$ of the singular term is characteristic of this system which belongs to the symplectic ensemble[11] and leads to a quantum enhancement of the conductivity and the charge-diffusion constant. Thus, while spin fluctuations in the system are localized by the $O(1/N)$ localization corrections, the charge fluctuations get delocalized. This antilocalization is due to the symmetry of the spin-orbit term under time reversal. Because of the phase factor of π associated with spin-$\frac{1}{2}$ particles under time reversal, the scattering amplitudes interfere destructively, which leads to a quantum reduction in the backscattering.[9] A fundamental consequence of spin-orbit impurity scattering is, therefore, that it leads to markedly different diffusion constants for spin- and charge-density fluctuations.

Now that we know about the nature of the vertex parts, we consider carefully all the localization diagrams of $O(1/N)$ shown in Fig. 8 which contribute to the transverse susceptibility and the density-correlation function. Of the diagrams for $L^{\uparrow\downarrow}$ [shown in Figs. 8(a) and (b)] we need to consider only those which involve a singular vertex part. $\Gamma_{\sigma\sigma;\sigma\sigma}$ is nonsingular and so we can ignore the diagrams in Fig. 8(b). Now, in general, the contributions from the diagrams can be arranged in powers of ω and q^2 and we keep terms up to $O(\omega,q^2)$. If no spin-orbit scattering is present, the $O(1)$ and $O(\omega)$ terms from diagrams in Fig. 8(a) cancel exactly. These cancellations are connected with Ward identities related to total particle conservation and energy conservation in elastic scattering respectively.[12] Notice, however, that the second and third diagrams in (a) involve an impurity line which contributes $(\gamma_n-\gamma_{so}/3)$ and not $\gamma_n+\gamma_{so}=\gamma$, and so we see immediately that there will be net contributions of $O(1)$ and $O(\omega)$ proportional to $(\frac{4}{3})\gamma_{so}$ in addition to the $O(q^2)$ contribution. The singular $O(\omega)$ contribution does, however, get cancelled in Eq. (29) as it should because spin-orbit scattering is elastic and will not lead to any renormalization of the energy scale. The non-cancellation of $O(1)$ terms leads to a singular correction to the spin relaxation rate due to localization. Evaluation of all the relevant diagrams leads to the following expression for $L^{\uparrow\downarrow}(\mathbf{q},\omega)$:

$$L^{\uparrow\downarrow}(\mathbf{q},\omega) = N(0)4\pi\tau^3\gamma^{-1}C_d(\omega)\left[\tfrac{4}{3}\gamma_{so}+\tau(\gamma_n-\tfrac{5}{3}\gamma_{so})Dq^2+2\tfrac{4}{3}\gamma_{so}i\omega\tau\right] , \tag{33}$$

where

$$C_d(\omega) = \frac{\gamma}{2}\frac{1}{\tau}\int \frac{d^dQ}{(2\pi)^d}\frac{1}{DQ^2-i\omega} \tag{34}$$

expresses the singular nature of the localization corrections and leads to the interesting physical dependences on scale or temperature which act as infrared cutoffs in the system. This has been extensively discussed in Ref. 13 and other references therein.

Substituting for $L^{\uparrow\downarrow}$ in Eq. (29), we see that the singular $O(\omega)$ term vanishes due to cancellation, and the scattering-strength coefficients with the Dq^2 terms add up to γ so that the spin-relaxation rate and the diffusion constant acquire *identical* localization corrections. This is explained in the next paragraph. The frequency dependent part of the transverse susceptibility can finally be written as

$$N(0)\frac{i\omega}{\frac{4}{3}(\tau_{so}^1)^{-1}+D^1q^2-i\omega} , \tag{35}$$

where the effective spin-diffusion constant and spin-relaxation rate involve identical localization corrections and are given by

$$D^1(\omega) = D\left[1-\frac{1}{2\pi N(0)}\int_Q \frac{1}{DQ^2-i\omega}\right] ,$$
$$(\tau_{so}^1)^{-1} = \tau_{so}^{-1}\left[1-\frac{1}{2\pi N(0)}\int_Q \frac{1}{DQ^2-i\omega}\right] . \tag{36}$$

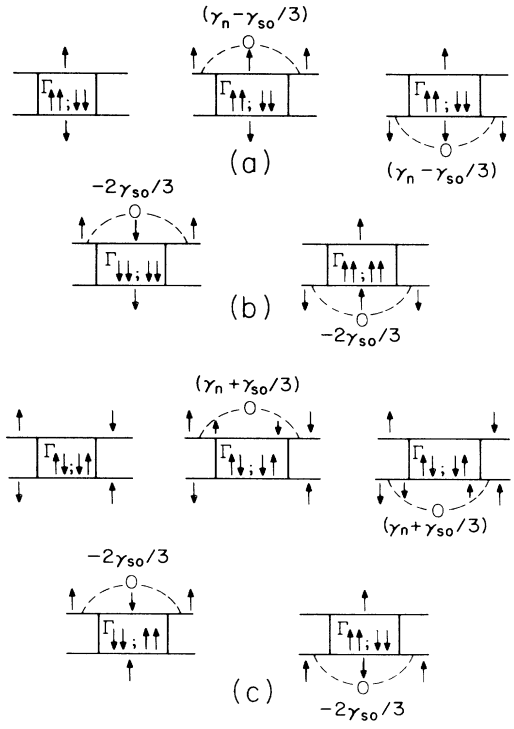

FIG. 8. Localization diagrams of $O(1/N)$ which contribute to the transverse magnetic susceptibility and the density correlation function.

Weak-localization corrections can be interpreted as arising due to interference of scattering amplitudes.[9] The reduction in effective spin-diffusion constant is due to constructive interference which leads to an enhanced probability of finding the spin at a later time t in the vicinity of where it was placed at time 0. This very interference also results in an enhancement for finding the particle in the *same* spin state and leads to an identical reduction in the effective relaxation rate due to spin flipping. As discussed later, in the ferromagnetic phase this effective reduction in the spin flipping causes a corresponding increase in the band shift.

We now discuss the nature of the localization correction expressed in Eq. (36) which tends to decrease the spin-diffusion constant and the spin-relaxation rate for low frequencies. The momentum integration leads to $\log\omega$ and $\sqrt{\omega}$ dependences in quasi–two dimensions and three dimensions, respectively.[1] At a nonzero temperature, it is the inelastic-relaxation rate which acts as a cutoff in the diffusion pole and governs the dependence provided that $\omega \ll \tau_{\rm in}^{-1}$. The temperature dependence of the inelastic-relaxation rate then translates into logarithmic (in quasi–two dimensions) or a power-law (in three dimensions) temperature dependence for the localization correction to the spin-diffusion constant and the spin-relaxation rate.

Localization diagrams which yield *singular* contributions to the density correlation function are shown in Fig. 8(c). We notice that contributions from the impurity lines are such that they add up to $\gamma_n+\gamma_{so}/3+2\gamma_{so}/3=\gamma$ when account is taken of the different signs of the singular vertex parts involved. Hence, the situation is virtually identical to the case of normal impurity scattering only, and one sees immediately that the $q,\omega\rightarrow 0$ contribution to the density-correlation function vanishes due to exact cancellation. This is only to be expected because the density-correlation function for zero frequency and momentum is related to the well-behaved thermodynamic density of states, $\partial N/\partial\mu$, and hence should not exhibit any singular behavior.

In order to study localization effects on spin waves and Stoner excitations, we now turn to localization corrections in the ferromagnetic phase, the relevant vertex part for which is given in Eq. (25). In the ferromagnetic phase, the term $J^{\uparrow\downarrow}-J^{\downarrow\uparrow}$ is proportional to $\bar{\Delta}$, and this term cancels the $O(\bar{\Delta})$ contribution from the first term in the denominator, so that to first order in $\bar{\Delta}$, the denominator yields a diffusion pole for $Q,\omega\rightarrow 0$. This is unlike the case of normal impurities only (where the diffusion pole occurs at $\omega\rightarrow\Delta$) and has, as discussed below, important consequences on spin waves and Stoner excitations. For $\gamma_{so}\neq 0$, the most singular contribution to the vertex part is obtained as

$$\Gamma_{\uparrow\uparrow;\downarrow\downarrow}(\mathbf{Q},\omega;\bar{\Delta})=\frac{\gamma}{2}\frac{1}{(DQ^2-i\omega)}\left[1-\frac{\gamma_0-\gamma_{so}/3}{2\gamma_{so}/3}i\bar{\Delta}\tau\right], \tag{37}$$

where we have used

$$1-\gamma J^{\downarrow\uparrow}(\mathbf{Q},\omega;\bar{\Delta})=\tau[DQ^2-i(\omega+\bar{\Delta})], \tag{38}$$

$$1-\gamma J^{\uparrow\downarrow}(\mathbf{Q},\omega;\bar{\Delta})=\tau[DQ^2-i(\omega-\bar{\Delta})], \tag{39}$$

and have kept terms to first order in Q^2, ω, and $\bar{\Delta}$ in evaluating the denominator in Eq. (25). From Eq. (37) we notice that unlike the case of normal impurities only, low-frequency spin fluctuations in this case continue to be affected by localization correction in the ferromagnetic phase which will have an important consequence on the spectrum of spin waves. On the other hand, Stoner excitations involve frequencies of the order of $\bar{\Delta}$ and thus, the corrections are rendered nonsingular by $\bar{\Delta}$ which acts as a lower cutoff in the Q integration.

Unlike the singular ω-dependent terms which cancel exactly, a cancellation of the $\bar{\Delta}$ terms is avoided due to the nature of the diffusion pole in Eq. (37). This is briefly discussed below. Evaluation of the diagrams in Fig. 8 using Green's functions appropriate for the ferromagnetic phase leads to terms like $i(\omega-\bar{\Delta})$ in place of the $i\omega$ terms, and one obtains for the frequency-dependent part of the transverse susceptibility

$$i\frac{\omega}{2\pi}\left[\frac{2}{\frac{4}{3}\gamma_{so}+\gamma\tau[Dq^2-i(\omega-\bar{\Delta})]-\gamma^2\{1+2\tau[Dq^2-i(\omega-\bar{\Delta})]\}L^{\uparrow\downarrow}(\mathbf{q},\omega,\bar{\Delta})}\right]. \tag{40}$$

But the diffusion pole in the vertex part, $\Gamma^{\uparrow\uparrow;\downarrow\downarrow}$, which goes into the localization correction, $L^{\uparrow\downarrow}$, continues to be of the $1/(DQ^2-i\omega)$ type. Therefore, the contribution from terms like $\int DQ^2/(DQ^2-i\omega)$ goes as $i\omega$ and *not* as $i(\omega-\bar{\Delta})$. That the singular $O(\omega)$ terms cancel, therefore, necessarily indicates that the singular $O(\bar{\Delta})$ terms do not.

By evaluating the singular localization corrections in the ferromagnetic phase using diagrams shown in Fig. 8 for $L^{\uparrow\downarrow}$ and then isolating terms of order $\bar{\Delta}$, we obtain

$$L^{\uparrow\downarrow}(\mathbf{q},\omega,\bar{\Delta})-L^{\uparrow\downarrow}(\mathbf{q},\omega,0)=N(0)4\pi\tau^3\gamma^{-1}C_d(\omega)\times[-i\bar{\Delta}\tau(\gamma_n+\tfrac{11}{3}\gamma_{so})], \tag{41}$$

where, $L^{\uparrow\downarrow}(\mathbf{q},\omega,0)$ is given in Eq. (33). Substituting in Eq. (40), we find that the scattering strength coefficients with the $\bar{\Delta}$ term add up to γ, so that the frequency-dependent part of the transverse susceptibility in the ferromagnetic phase can be written as

$$N(0)\frac{i\omega}{\frac{4}{3}(\tau_{so}^{1})^{-1}+D^{1}q^{2}-i(\omega-\bar{\Delta}^{1})} , \tag{42}$$

where the effective band shift, $\bar{\Delta}^{1}$ is seen to be enhanced by localization corrections which are of exactly the same form as for the effective spin-diffusion constant and spin-relaxation rate but with a difference in sign:

$$\bar{\Delta}^{1}=\bar{\Delta}\left[1+\frac{1}{2\pi N(0)}\int_{Q}\frac{1}{DQ^{2}-i\omega}\right] . \tag{43}$$

As far as spin waves are concerned, these localization corrections to the band shift and the spin-relaxation rate translate into corresponding enhancement of the stiffness constant and reduction in spin-wave damping, respectively. Dropping the q^4 term for simplicity, we can write, in analogy with Eq. (22), the equation for the spin-wave mode as

$$\omega=[\bar{\Delta}^{1}-i\tfrac{4}{3}(\tau_{so}^{1})^{-1}]\alpha q^{2}/12k_F^{2} . \tag{44}$$

To summarize, then, for spin fluctuations, weak-localization corrections which are viewed as arising from the quantum-mechanical interference of scattering amplitudes lead to an enhanced probability of finding a particle in the same spatial region and the same spin state. This results in an effective reduction in spin diffusion and spin-relaxation rate due to spin flipping. Since spin flipping distorts the spin bands, and hence reduces the band shift in the ferromagnetic phase, an effective reduction in the spin-flipping rate due to the dynamical localization phenomenon results in a corresponding increase in the band shift, and hence in the spin-wave stiffness constant.

IV. COMPETING INTERACTIONS

As pointed out earlier, a model with repulsive interaction and spin-independent impurity scattering can be effectively mapped to a spin-$\frac{1}{2}$ Heisenberg model with random (but all with same sign) exchange constants. And so, the critical behavior of itinerant ferromagnets with nonmagnetic impurities is expected to be identical to that of random ferromagnets. In contrast, we believe the situation to be quite different in this regard when spin-dependent scattering processes are present. As we show below, in the localized limit the presence of strong spin-orbit scattering effectively generates competing interactions leading, possibly, to frustration and thereby to spin-glass behavior.[14]

In this section we consider a lattice system with a half-filled band in the site-localized limit in which each site is occupied precisely by one electron. We evaluate the second-order correction to the ground-state energy by treating as perturbation the hopping term and the random spin-orbit coupling term which is also off-diagonal in site indices

$$\Delta\mathcal{E}_l=\sum_{m\neq l}\frac{|\langle l|H'|m\rangle|^{2}}{\mathcal{E}_l-\mathcal{E}_m} . \tag{45}$$

In a tight-binding representation of the Hamiltonian the perturbation is

$$H'=-V\sum_{i,j,\sigma}{}'(a_{i\sigma}^{\dagger}a_{j\sigma}+a_{j\sigma}^{\dagger}a_{i\sigma}) + \sum_{i,j,\sigma,\sigma'}{}'(f_{\sigma\sigma'}^{ij}a_{i\sigma}^{\dagger}a_{j\sigma'}+f_{\sigma\sigma'}^{ji}a_{j\sigma}^{\dagger}a_{i\sigma'}) , \tag{46}$$

where a prime to the sum indicates that the sum is done only over nearest-neighbor pairs of sites. The second term represents the spin-orbit interaction and $f_{\sigma\sigma'}$ may be written in the following form[15]

$$f_{\sigma\sigma'}=i[\boldsymbol{\sigma}\cdot\mathbf{f}]_{\sigma\sigma'} , \tag{47}$$

where $\mathbf{f}$ is real and antisymmetric in the site indices.

Consider now a single pair of sites labeled 1 and 2, with on-site energies ϵ_1 and ϵ_2. If the spins of electrons occupying these sites are aligned parallel to each other, the second-order correction due to the first term in H', which is diagonal in spin, vanishes. This is because the virtual state is a doubly occupied state with parallel spins which is excluded from Pauli's exclusion principle. For antiparallel alignment, the second-order correction due to the first term is

$$\Delta\mathcal{E}_1^{\uparrow\downarrow}=\left[\frac{-2U}{U^{2}-(\epsilon_1-\epsilon_2)^{2}}\right]V^{2} . \tag{48}$$

If the singly occupied system forms the ground state, as has been assumed, then it is easy to see that this correction energy is always negative. The implication is that if only normal disorder is present, it is the *antiferromagnetic* arrangement of spins which is favored in the ground state.

Consider now the correction due to the spin-orbit term in H'. For parallel and antiparallel occupancy, the corrections are

$$\Delta\mathcal{E}_2^{\uparrow\uparrow}=\left[\frac{-2U}{U^{2}-(\epsilon_1-\epsilon_2)^{2}}\right](f_x^{2}+f_y^{2}) \tag{49}$$

$$\Delta\mathcal{E}_2^{\uparrow\downarrow}=\left[\frac{-2U}{U^{2}-(\epsilon_1-\epsilon_2)^{2}}\right]f_z^{2} . \tag{50}$$

It is not clear now whether it is the parallel or the antiparallel arrangement which is favored in the ground state. The answer depends on the relative magnitude of $f_z^2+V^2$ and $f_x^2+f_y^2$. A parallel arrangement is favored if $f_z^2+V^2<f_x^2+f_y^2$ and vice versa.

If the spin-orbit term is of the order of the hopping term, then due to the randomness in $\mathbf{f}$, it is likely that a parallel spin arrangement is favored energetically for some pairs, whereas the antiparallel spin arrangement is favored for others. Thus, in this system, both ferromagnetic and antiferromagnetic interactions between spins are effectively present and this leads to *frustration* and possibly to a spin-glass phase.

V. CONCLUSION

In this paper we have considered the effects of magnetic and spin-orbit impurity scattering in interacting systems and have shown how the spin dependence of impurity-scattering processes leads to basic differences from the case of normal impurity scattering where no

spin flipping is present. Thus, while charge-density fluctuations continue to be undamped, spin-density fluctuations become damped due to spin flipping. This leads to damping of spin waves and Stoner excitations in the ferromagnetic phase. Also, spin flipping works against ferromagnetism and, indeed, we find that the Stoner criterion for the onset of ferromagnetic instability is modified, and the critical interaction strength increases with increasing spin-flip scattering strength. Another manifestation of the tendency of spin flipping to inhibit ferromagnetism is in the distortion of the minority spin band, especially near the band edge. Far from the band edge, the effective band shift is reduced by spin flipping.

When localization corrections are included, we find that spin-orbit impurity scattering leads to identical reduction in the spin-diffusion constant and the spin-relaxation rate which is due to the quantum interference effect of weak localization. This is in contrast with the enhancement of charge diffusion constant due to antilocalization, and indicates that the similarity between diffusion of spin- and charge-density fluctuations, as found in the case of normal impurities only, is broken. The reduction in the effective spin-relaxation rate and also its temperature dependence will have important implications on the low-temperature specific heat of a nearly-magnetic system which is being investigated. We also find that the dynamical localization mechanism which effectively reduces the spin-flipping rate also correspondingly enhances the band shift, and thus causes a stiffening of spin waves and increases the Stoner excitation gap.

Finally, we have shown that in the ultralocalized limit random spin-orbit coupling leads to competing interactions in a lattice system with a half-filled band. This points towards the possibility of having frustration in a system in which the spin-orbit strength is of the same order as the hopping term.

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APPENDIX

The evaluation of static susceptibility in the ferromagnetic phase for a system with normal impurity scattering as well as spin-dependent impurity scattering is briefly sketched out here for the sake of completeness. From Eq. (7) the static susceptibility is given by

$$\chi^{-+}_{\rm imp}(0)=i\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{J^{\uparrow\downarrow}(\omega)}{1-(\gamma_n-\gamma_{sf}/3)J^{\uparrow\downarrow}(\omega)}\ , \tag{51}$$

where we have[1]

$$J^{\uparrow\downarrow}(\omega)=\frac{i}{\pi[q_\uparrow(\omega)+q_\downarrow(\omega)]}\ . \tag{52}$$

From Eqs. (16) we find that to first order in Δ, $q_\downarrow(\omega)+q_\downarrow(\omega)$ is explicitly independent of Δ and apparently independent of U as well. However, the dependence is hidden in the Fermi energy ω_F which, in the ferromagnetic phase, implicitly depends upon U.

Since there is no explicit dependence on Δ one can proceed as for the paramagnetic case and obtain [see Eq. (12)]

$$\chi^{-+}_{\rm imp}(0)=\frac{k_F}{2\pi^2}\left[1-\frac{2}{3k_Fl_{sf}}\tan^{-1}\left[\frac{3k_Fl_{sf}}{2}\right]\right]\ , \tag{53}$$

where $k_F=\sqrt{2\omega_F}$.

In order to determine the manner in which k_F depends, in the ferromagnetic phase, implicitly on U, we need to evaluate the particle densities for both spins and relate the difference to Δ. We proceed with the density of states obtained from the real part of $q_\sigma(\omega)$

$$N_\sigma(\omega)=\frac{1}{2\pi^2}\sqrt{2(\omega+\omega_F)}\left[1+\frac{\sigma\Delta/2}{2(\omega+\omega_F)+(2/3l_{sf})^2}\right]\ . \tag{54}$$

From which we obtain the particle densities

$$n_\sigma=\int_{-\omega_F}^{0}d\omega N_\sigma(\omega)\ . \tag{55}$$

Since $n^\uparrow-n^\downarrow=\Delta/U$, we get

$$\frac{\Delta}{U}=\frac{1}{2\pi^2}\Delta\int_{-\omega_F}^{0}d\omega\frac{\sqrt{2(\omega+\omega_F)}}{2(\omega+\omega_F)+(2/3l_{sf})^2}$$
$$=\Delta\frac{k_F}{2\pi^2}\left[1-\frac{2}{3k_Fl_{sf}}\tan^{-1}\left[\frac{3k_Fl_{sf}}{2}\right]\right]\ . \tag{56}$$

Comparing with Eq. (53) we immediately see that in the ferromagnetic phase ($\Delta\neq0$), the static susceptibility is equal to $1/U$.

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