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Magnetic line groups. II. Corepresentations of the magnetic line groups isogonal to the point groups C_n, S_{2n}, and C_{nh}

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The irreducible corepresentations of the 13 families of the magnetic line groups isogonal to the point groups C_n, S_{2n}, and C_{nh} are derived. As an illustration the degeneration of the magnon energy bands is discussed.

I. INTRODUCTION

Line groups[1] are the symmetry groups of physical systems with translational periodicity in one direction. The usual examples are polymers and quasi-one-dimensional systems. Also, these groups can be used to analyze three-dimensional crystals in an analogous way as the point groups are applied, when only one direction is of interest (e.g., in study of some subsystems, especially in cases with high anisotropy[2]). When spin is considered, the time reversal Θ becomes nontrivial additional operation, and the magnetic line groups[3] should be used. Hence, there appears the interest for the corepresentations of the magnetic line groups.

It is well known[4] that each magnetic group is of the form $\mathbf{L}'+g\Theta\mathbf{L}'=\mathbf{L}(\mathbf{L}')$, where $\mathbf{L}'$ is the index-two subgroup which can be represented by linear operators in the Hilbert space, while the second term is the coset which is represented by antilinear operators in the same space. The magnetic group is called grey group, if g can be chosen to be the identity, and black and white group otherwise.

All the nonequivalent irreducible corepresentations of the magnetic group $\mathbf{L}(\mathbf{L}')$ can be found by the method of $*$ induction,[5] based on the knowledge of the irreducible representations of the group $\mathbf{L}'$. It has been shown[3] that the group $\mathbf{L}'$ of the magnetic line group, is always a line group. The irreducible representations of the line groups are known,[6] which enables one to use the $*$-induction method to construct the irreducible corepresentations of the magnetic line groups.

Until now only the corepresentations of the grey magnetic line groups have been found.[7] In this paper we derive the corepresentations of the families of the magnetic line groups whose isogonal point groups are C_n, S_{2n}, and C_{nh}.

In Sec. II it is explained how the method of $*$ induction is adapted to this purpose, and in Sec. III the tables of the irreducible corepresentations are listed. A brief summary of the necessary results of the previous papers is given in the Appendix.

II. $*$ INDUCTION ADAPTED TO LINE GROUPS

$*$ induction is a procedure by which one can construct all the irreducible corepresentations of a black and white magnetic line group $\mathbf{L}(\mathbf{L}')$ out of all the irreducible representations of its subgroup $\mathbf{L}'$ of index 2. Let $d(\mathbf{L}')=\{d(h)|h\in\mathbf{L}'\}$ be an irreducible representation of $\mathbf{L}'$. The $*$-g-conjugated irreducible representation is defined by

$$d_g^*(\mathbf{L}')=\{d_g^*(h)=d^*(g^{-1}hg)|h\in\mathbf{L}'\}\ ,$$

where g is a coset representative. When $d_g^*(\mathbf{L}')$ and $d(\mathbf{L}')$ are equivalent, then there exists a unitary matrix Z such that $d_g^*(h)=Z^{-1}d(h)Z$, for each h from $\mathbf{L}'$, and $ZZ^*=\pm d(g^2)$. The plus or minus sign determines type 1^* or 2^*a of the representation $d(\mathbf{L}')$ (these are types I and II in Wigner's classification). When $d_g^*(\mathbf{L}')$ and $d(\mathbf{L}')$ are not equivalent, then one has type 2^*b (III) representation.

The procedure of $*$ induction is completely analogous to that of induction. Namely, one forms a matrix of double dimension for each element h of $\mathbf{L}'$:

$$\bar{d}(h)=\begin{bmatrix} d(h) & 0 \\ 0 & d_g^*(h)\end{bmatrix}\ ,$$

and for each element $g\Theta h$ of the coset an antimatrix:

$$\bar{d}(g\Theta h)=\begin{bmatrix} 0 & d(g^2) \\ I & 0\end{bmatrix} K_0 \begin{bmatrix} d(h) & 0 \\ 0 & d_g^*(h)\end{bmatrix} = \begin{bmatrix} 0 & d(g^2) \\ I & 0\end{bmatrix} \begin{bmatrix} d^*(h) & 0 \\ 0 & d_g(h)\end{bmatrix} K_0\ .$$

Note that here the homomorphism is preserved. In the theory of corepresentations one omits K_0, but it is implicitly encountered in calculations.

For the representations of type 1^* this construction is reducible representation which is reduced to two equivalent irreducible corepresentations:

$$\bar{d}(\mathbf{L}(\mathbf{L}'))=\{d(h),d(gh)=Zd^*(h)|h\in\mathbf{L}'\}\ .$$

For the representations of types 2^*a and 2^*b $*$ induction gives the irreducible corepresentations. The only difference is that in case 2^*a one can utilize the equivalent Wigner's form

$$\begin{bmatrix} d(h) & 0 \\ 0 & d(h)\end{bmatrix}$$

for each element h of $\mathbf{L}'$, and

$$\begin{bmatrix} 0 & Z \\ -Z & 0 \end{bmatrix} \begin{bmatrix} d^*(h) & 0 \\ 0 & d^*(h) \end{bmatrix}$$

for the elements of the coset. However, in the tables the original $*$-induction form is presented, sparing the calculations of Z in 2^*a case.

To determine the type of $d(\mathbf{L}')$ one uses Dimmock's character test: $d(\mathbf{L}')$ is of type 1^*, 2^*a, and 2^*b iff the quantity $X=(1/|\mathbf{L}'|)\sum_{h\in \mathbf{L}}\mathrm{Tr}[d(gh)^2]$ is equal to 1, -1, and 0, respectively (here $|\mathbf{L}'|$ stands for the order of $\mathbf{L}'$).

Obviously, the problem of construction of the irreducible corepresentations of $\mathbf{L}(\mathbf{L}')$ is reduced to calculation of X for each irreducible representation of $\mathbf{L}'$, and determination of Z in the case when $X=1$. In the case of the line groups, X can be calculated easily since the infinite

TABLE I. Irreducible corepresentations of the line groups $\mathbf{L}n_p'$. Here $\mathbf{L}'=\mathbf{L}(n/2)_{p'}=\{(\mathbf{C}_{n/2}^s|t+Fr(2sp/n))|s=0,1,\ldots,n/2-1;\ t=0,\pm1,\ldots\}$ with $P'=(n/2)Fr(2p/n)$, $g=(\mathbf{C}_n|p/n)$.

Corepresentations	Type	g	$(\mathbf{C}_{n/2}^s\|Fr(2sp/n)+t)$
	$p<n/2,\ \mathbf{L}'=\mathbf{L}(n/2)_p$		
The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_m, {}_0A_{-m})$, $({}_kA_m, {}_{-k}A_{-m})$, $({}_{\pi/a}A_m, {}_{\pi/a}A_{m'})$, where $m'=-m-p$ for $-p-m\in(-n/4,n/4)$ and $m'=n/2-m-p$ for $n/2-m-p\in(-n/4,n/4)$.			
${}_0\overline{A}_0$	1^*	1	1
${}_0\overline{\overline{A}}_{n/4}$	2^*a	$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$	$(-1)^sI$
${}_0\overline{\overline{A}}_m \quad m\in(0,n/4)$	2^*b	$\begin{vmatrix} 0 & e^{-im\alpha} \\ 1 & 0 \end{vmatrix}$	$M(m,s)$
${}_{\pi/a}\overline{A}_{-p/a}$	1^*	1	$(-1)^{t+\mathrm{Int}(2sp/n)}$
${}_0\overline{\overline{A}}_{n/4-p/2}$	2^*a	$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$	$(-1)^{s+t+\mathrm{Int}(2sp/n)}I$
${}_{\pi/a}\overline{\overline{A}}_m \quad m\in(-p/a,n/4-p/2)$	2^*b	$W^*(2p\pi/na,m)$	$(-1)^tK[\pi/a,Fr(2sp/n)]M(m,s)$
${}_k\overline{\overline{A}}_m \quad {}^{k\in(0,\pi/a)}_{m\in(-n/4,n/4)}$	2^*b	$W^*(2pk/n,m)$	$K[k,t+Fr(2sp/n)]M(m,s)$
	$p\geq n/2,\ \mathbf{L}'=\mathbf{L}(n/2)_{p-n/2}$		
The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_m, {}_0A_{-m})$, $({}_kA_m, {}_{-k}A_{-m})$, $({}_{\pi/a}A_m, {}_{\pi/a}A_{m'})$, where $m'=n/2-m-p$ for $n/2-p-m\in(-n/4,\ n/4)$ and $m'=n-m-p$ for $n-m-p\in(-n/4,n/4)$.			
${}_0\overline{A}_0$	1^*	1	1
${}_0\overline{\overline{A}}_{n/4}$	2^*a	$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$	$(-1)^sI$
${}_0\overline{\overline{A}}_m \quad m\in(0,n/4)$	2^*b	$\begin{vmatrix} 0 & e^{-im\alpha} \\ 1 & 0 \end{vmatrix}$	$M(m,s)$
${}_{\pi/a}\overline{A}_{n/2-p/2}$	1^*	1	$(-1)^{t+\mathrm{Int}(2sp/n)}$
${}_0\overline{\overline{A}}_{n/4-p/2}$	2^*a	$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$	$(-1)^{s+t+\mathrm{Int}(2sp/n)}I$
${}_{\pi/a}\overline{\overline{A}}_m \quad m\in(n/4-p/2,n/2-p/2)$	2^*b	$W^*(2p\pi/na,m)$	$(-1)^tK[\pi/a,Fr(2sp/n)]M(m,s)$
${}_k\overline{\overline{A}}_m \quad {}^{k\in(0,\pi/a)}_{m\in(-n/4,n/4)}$	2^*b	$W^*(2pk/n,m)$	$K[k,t+Fr(2sp/n)]M(m,s)$

TABLE II. Irreducible corepresentations of the line groups $\mathbf{L}_c n_p$. Here $\mathbf{L}'=\mathbf{L}''n_{p/2}=\{(C_n^s|t+Fr(sp/2n))|s=0,1,\ldots,n-1;\ t=0,\pm 1,\ldots\}$, $g=(E|\frac{1}{2})$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_m, {}_0A_{-m})$, $({}_kA_m, {}_{-k}A_{-m})$, $({}_{\pi/a}A_m, {}_{\pi/a}A_{m'})$, where $m'=-m-p/2$ for $-m-p/2\in(-n/2,n/2]$ and $m'=n-m-p/2$ for $n-m-p/2\in(-n/2,n/2]$.

Corepresentations	Type	g	$(\mathbf{C}_n^s\|Fr(sp/2n)+t)$
${}_0\overline{A}_0$	1^*	1	1
${}_0\overline{A}_{n/2}$	1^*	1	$(-1)^s$
${}_0\overline{\overline{A}}_m$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$M(m,s)$
${}_{\pi/a}\overline{\overline{A}}_{-p/4}$	2^*a	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{t+\mathrm{Int}(sp/2n)}I$
${}_{\pi/a}\overline{\overline{A}}_{n/2-p/4}$	2^*a	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{s+t+\mathrm{Int}(sp/2n)}I$
${}_{\pi/a}\overline{\overline{A}}_m$ $m\in(-p/4,n/2-p/4)$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^tK(\pi/a,Fr(sp/2n))M(m,s)$
${}_k\overline{\overline{A}}_m$ $k\in(0,\pi/a)$ $m\in(-n/2,n/2)$	2^*b	$\begin{vmatrix}0 & e^{-ika}\\ 1 & 0\end{vmatrix}$	$K(k,t+Fr(sp/2))M(m,s)$

sums occurring in the expression for X are sums of the trigonometric functions. Namely, whenever one-dimensional representation of $\mathbf{L}'$ is considered, X is found using the equality (δ is the Kronecker's symbol):

$$\sum_{s=0}^{n-1}\exp(i2\pi ms/n)=n\sum_{z\in\mathbf{Z}}\delta_{m,zn}\ ,$$

while for the two-dimensional representations besides this the following sum appears:

$$\sum_{s=0}^{n-1}\cos(2\pi ms/n+\phi)=n\cos\phi\sum_{z\in\mathbf{Z}}\delta_{m,zn}\ .$$

Instead of the method for determination of Z originally

TABLE III. Irreducible corepresentations of the line groups $\mathbf{L}_s n_p$. Here $\mathbf{L}'=\mathbf{L}''n_{(n+p)/2}=\{(C_n^s|t+Fr(sp/2n))|s=0,1,\ldots,n-1;\ t=0,\pm 1,\ldots\}$, $g=(E|\frac{1}{2})$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_m, {}_0A_{-m})$, $({}_kA_m, {}_{-k}A_{-m})$, $({}_{\pi/a}A_m, {}_{\pi/a}A_{m'})$, where $m'=-m-(p+n)/2$ for $-m-(p+n)/2\in(-n/2,n/2]$ and $m'=(n-p)/2-m$ for $(n-p)/2-m\in(-n/2,n/2]$.

Corepresentations	Type	g	$(\mathbf{C}_n^s\|Fr(sp/2n)+t)$
${}_0\overline{A}_0$	1^*	1	1
${}_0\overline{A}_{n/2}$	1^*	1	$(-1)^s$
${}_0\overline{\overline{A}}_m$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$M(m,s)$
${}_{\pi/a}\overline{\overline{A}}_{-(n+p)/4}$	2^*a	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{t+\mathrm{Int}[s(p+n)/2n]}I$
${}_{\pi/a}\overline{\overline{A}}_{(n-p)/4}$	2^*a	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{s+t+\mathrm{Int}[s(p+n)/2n]}I$
${}_{\pi/a}\overline{\overline{A}}_m$ $m\in(-(n+p)/4,(n-p)/4)$	2^*a	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^tK[\pi/a,Fr(s(p+n)/2n)+t]M(m,s)$
${}_k\overline{\overline{A}}_m$ $k\in(0,\pi/a)$ $m\in(-n/2,n/2)$	2^*b	$\begin{vmatrix}0 & e^{-ika}\\ 1 & 0\end{vmatrix}$	$K[k,t+Fr(s(p+n)/2n)]M(m,s)$

suggested,[5] the direct calculation of Z is made. The method was effective due to the fact that the matrices of the irreducible representations of the line groups are either diagonal or off diagonal.

III. RESULTS AND DISCUSSION

The line groups isogonal to $\mathbf{C}_n$ ($n=0,1,\dots$) are $\mathbf{L}n_p$ ($p=0,1,\dots,n-1$); the line groups isogonal to $\mathbf{S}_{2n}$ are $\mathbf{L}(\bar{2}\bar{n})$ ($n=0,2,4,\dots$), $\mathbf{L}\bar{n}$ ($n=1,3,5,\dots$); finally the line groups isogonal to $\mathbf{C}_{nh}$ are $\mathbf{L}n/m$ and $\mathbf{L}n_{n/2}/m$ ($n=0,2,\dots$) and $\mathbf{L}(\bar{2}\bar{n})$ ($n=1,3,\dots$). Black and white magnetic line groups derived from $\mathbf{L}n_p$ are $\mathbf{L}_c n_p$, $\mathbf{L}_s n_p$ and only for n even $\mathbf{L}n_p'$. In the class of $\mathbf{L}(\bar{2}\bar{n})$ (respectively, $\mathbf{L}\bar{n}$) are the black and white groups $\mathbf{L}(\bar{2}\bar{n})'$ and $\mathbf{L}_c(\bar{2}\bar{n})$ ($\mathbf{L}\bar{n}'$ and $\mathbf{L}_c\bar{n}$, respectively). In the class of $\mathbf{L}n/m$ [$\mathbf{L}(\bar{2}\bar{n})$, respectively] are $\mathbf{L}n/m'$, $\mathbf{L}_c n/m$ ($\mathbf{L}(\bar{2}\bar{n})'$, $\mathbf{L}_c(\bar{2}\bar{n})$), and $\mathbf{L}n'/m'$, $\mathbf{L}n'/m$, $\mathbf{L}_s n/m$. Finally, the black and white groups corresponding to $\mathbf{L}(2k)_k/m$ are $\mathbf{L}(2k)_k/m'$, $\mathbf{L}(2k)_k'/m'$, $\mathbf{L}(2k)_k'/m$.

For each of these magnetic line groups, Tables I–XIII containing all the irreducible corepresentations are presented. Basically, the same symbol of the irreducible representation of $\mathbf{L}'$ is retained to denote the $*$-induced corepresentation of $\mathbf{L}(\mathbf{L}')$. In case 1^* this symbol is barred, while in cases 2^*a and 2^*b it is double barred.

TABLE IV. Irreducible corepresentations of the line groups $\mathbf{L}(\bar{2}\bar{n})'$, $\mathbf{L}\bar{n}'$. Here $\mathbf{L}'=\mathbf{L}n=\{(\mathbf{C}_n^s|t)|s=0,1,\dots,n-1;\ t=0,\pm1,\dots\}$, $g=(\sigma_h C_{2n}|0)$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_m,{}_0A_{-m})$, $({}_kA_m,{}_{-k}A_{-m})$, $({}_{\pi/a}A_m,{}_{\pi/a}A_m)$.

Corepresentations	Type	g	$(\mathbf{C}_n^s\|t)$
${}_0\overline{A}_0$	1^*	1	1
${}_0\overline{\overline{A}}_{n/2}$	2^*a	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^sI$
${}_0\overline{\overline{A}}_m$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & e^{-im\alpha}\\ 1 & 0\end{vmatrix}$	$M(m,s)$
${}_{\pi/a}\overline{\overline{A}}_m$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & e^{-im\alpha}\\ 1 & 0\end{vmatrix}$	$(-1)^tM(m,s)$
${}_{\pi/a}\overline{A}_0$	1^*	1	$(-1)^t$
${}_{\pi/a}\overline{\overline{A}}_{n/2}$	2^*a	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{s+t}I$
${}_k\overline{\overline{A}}_m$ $k\in(-\pi/a,\pi/a)$, $m\in(-n/2,n/2)$	2^*b	$\begin{vmatrix}0 & e^{-im\alpha}\\ 1 & 0\end{vmatrix}$	$e^{ikta}M(m,s)$

TABLE V. Irreducible corepresentations of the line groups $\mathbf{L}_c(\bar{2}\bar{n})$, $\mathbf{L}_c\bar{n}$. Here $\mathbf{L}'=\left(\begin{smallmatrix}\mathbf{L}''(\bar{2}\bar{n}) & n\ \text{even}\\ \mathbf{L}''\bar{n} & n\ \text{odd}\end{smallmatrix}\right)=\{(\mathbf{C}_n^s|t),\ (\sigma_h\mathbf{C}_{2n}\mathbf{C}_n^s|t)|s=0,1,\dots,n-1;\ t=0,\pm1,\dots\}$, $g=(E|\frac{1}{2})$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_{n/2}^+,{}_0A_{n/2}^-)$, $({}_0A_m^\pm,{}_0A_{-m}^\pm)$, $({}_{\pi/a}A_m^\pm,{}_{\pi/a}A_{-m})$, $({}_{\pi/a}A_0^+,{}_{\pi/a}A_0^-)$, $({}_k^{-k}E_m,{}_k^{-k}E_{-m})$.

Corepresentations	Type	g	$(\mathbf{C}_n^s\|t)$	$(\sigma_h\mathbf{C}_{2n}\mathbf{C}_n^s\|t)$
${}_0\overline{A}_0^\pm$	1^*	1	1	±1
${}_0\overline{\overline{A}}_m^\pm$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$M(m,s)$	$\pm M(m,s+\frac{1}{2})$
${}_0\overline{\overline{A}}_{n/2}$	2^*b	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$(-1)^sI$	$(-1)^s\begin{vmatrix}i & 0\\ 0 & i\end{vmatrix}$
${}_{\pi/a}\overline{\overline{A}}_0$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^tI$	$(-1)^t\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$
${}_{\pi/a}\overline{\overline{A}}_{n/2}^\pm$	1^*	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{s+t}I$	$\pm(-1)^{s+t}iI$
${}_{\pi/a}\overline{\overline{A}}_m^\pm$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^tM(m,s)$	$\pm(-1)^tM(m,s+\frac{1}{2})\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$
${}_k^{-k}\overline{E}_0$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & e^{ika}\\ 1 & 0\end{vmatrix}$	$K(k,t)$	$K(k,t)\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$
${}_k^{-k}\overline{\overline{E}}_0$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & K(k,1)^*\\ I & 0\end{vmatrix}$	$(-1)^sk(k,t)$	$(-1)^sd\left(\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}\right)c(k,t)$
${}_k^{-k}\overline{\overline{E}}_m$ $k\in(0,\pi/a)$, $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & K(k,1)^*\\ I & 0\end{vmatrix}$	$k(k,t)f(m)$	$d\left(\begin{vmatrix}0 & 1\\ e^{im\alpha} & 0\end{vmatrix}\right)c(k,t)f(m)$

TABLE VI. Irreducible corepresentations of the line groups $\mathbf{L}n/m'$, $\mathbf{L}(\bar{2}\bar{n})'$. Here $\mathbf{L}'=\mathbf{L}n=\{(\mathbf{C}_n^s|t)|s=0,1,\ldots,n-1;\ t=0,1,2,\ldots\}$, $g=(\sigma_h|0)$. The pair of $*$-g-conjugated irreducible representations of L' forming 2^*b corepresentation: $({}_kA_m, {}_kA_{-m})$.

Corepresentations	Type	g	$(\mathbf{C}_n^s|t)$
${}_k\bar{A}_0$ $k\in(-\pi/a,\pi/a)$	1^*	1	e^{ikta}
${}_k\bar{A}_{n/2}$ $k\in(-\pi/a,\pi/a)$	1^*	1	$(-1)^s e^{ikta}$
${}_k\bar{\bar{A}}_m$ $\substack{k\in(-\pi/a,\pi/a)\\ m\in(0,n/2)}$	2^*b	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$e^{ikta}M(m,s)$

The type of the representation is given in the separate column of the tables, and in cases 2^*b the pairs of $*$-g-conjugated representations are listed in the caption of each table. In the next column the matrix of the corepresentation of g is given. The rest of each table contains the matrices representing the elements of $\mathbf{L}'$ in the considered corepresentation.

In order to facilitate the use of the tables, most of the matrices are written explicitly. Only the following abbreviations are introduced:

$$\alpha=2\pi/n\ ,$$

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$M(m,s)=\begin{bmatrix}e^{ims\alpha} & 0\\ 0 & e^{-ims\alpha}\end{bmatrix},$$

$$K(k,t)=\begin{bmatrix}e^{ikta} & 0\\ 0 & e^{-ikta}\end{bmatrix},$$

$$W(k,m)=\begin{bmatrix}0 & e^{ika}\ e^{im\alpha}\\ 1 & 0\end{bmatrix},$$

TABLE VII. Irreducible corepresentations of the line groups $\mathbf{L}n'/m'(n=2p)$. Here $\mathbf{L}'=\left(\substack{\mathbf{L}(\bar{2}\bar{p}),\ p\ \text{even}\\ \mathbf{L}\bar{p},\ p\ \text{odd}}\right)=\{(\mathbf{C}_p^s|t),\ (\sigma_h\mathbf{C}_{2p}\mathbf{C}_p^s|t)|s=0,1,\ldots,p-1;\ t=0,\pm1,\ldots\}$, $g=(\mathbf{C}_n|0)$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_{p/2}^{+}, {}_0A_{p/2}^{-})$, $({}_0A_m^{\pm}, {}_0A_{-m}^{\pm})$, $({}_{\pi/a}A_m^{\pm}, {}_{\pi/a}A_{-m}^{\pm})$, $({}_{\pi/a}A_{p/2}^{+}, {}_{\pi/a}A_{p/2}^{-})$, $({}_{\bar{k}}^{\ k}E_m, {}_{\bar{k}}^{\ k}E_{-m})$.

Corepresentations	Type	g	$(\mathbf{C}_p^s|t)$	$(\sigma_h\mathbf{C}_{2p}\mathbf{C}_p^s|t)$
${}_0\bar{A}_0^t$	1^*	1	1	±1
${}_0\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/4)$	2^*b	$\begin{vmatrix}0 & e^{-im\alpha}\\ 1 & 0\end{vmatrix}$	$M(m,s)$	$\pm M(m,s+1/2)$
${}_0\bar{\bar{A}}_{p/2}$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^s I$	$(-1)^s\begin{vmatrix}i & 0\\ 0 & -i\end{vmatrix}$
${}_{\pi/a}\bar{A}_0^{\pm}$	1^*	1	$(-1)^t$	$\pm(-1)^t$
${}_{\pi/a}\bar{\bar{A}}_{p/2}$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{s+t}I$	$(-1)^{s+t}\begin{vmatrix}i & 0\\ 0 & -i\end{vmatrix}$
${}_{\pi/a}\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/4)$	2^*b	$\begin{vmatrix}0 & e^{-im\alpha}\\ 1 & 0\end{vmatrix}$	$(-1)^t M(m,s)$	$\pm(-1)^t M(m,s+1/2)$
${}_{\bar{k}}^{\ k}\bar{E}_0$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$K(k,t)$	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}K(k,t)$
${}_{\bar{k}}^{\ k}\bar{E}_{p/2}$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$(-1)^s K(k,t)$	$(-1)^s\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}K(k,t)$
${}_{\bar{k}}^{\ k}\bar{\bar{E}}_m$ $\substack{k\in(0,\pi/a)\\ m\in(0,n/4)}$	2^*b	$\begin{vmatrix}0 & e^{-im\alpha}I\\ I & 0\end{vmatrix}$	$k(k,t)f(m)$	$d\left[\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}\right]k(k,t)f(m)$

$$k(k,t)=\begin{bmatrix} K(k,t) & 0 \\ 0 & K(k,t)^* \end{bmatrix},$$

$$c(k,t)=\begin{bmatrix} K(k,t) & 0 \\ 0 & K(k,t+1)^* \end{bmatrix},$$

$$f(m)=\begin{bmatrix} e^{ims\alpha}I & 0 \\ 0 & e^{-ims\alpha}I \end{bmatrix},$$

and

$$d(X)=\begin{bmatrix} X & 0 \\ 0 & X^* \end{bmatrix}$$

(note that capital and lower-case letters are used to denote two- and four-dimensional matrices, respectively). Also, Int(x) and Fr(x) stand for the integral and fractional part of the number x.

Among other applications, symmetry is used to predict the degeneracy of the energy bands of the system. In this context it is interesting to analyze the influence of the time reversal to the degeneracy (note that in the cases of the black and white groups, the time reversal is not the symmetry itself, but combined with some spatial transformation). If this symmetry (namely $g\Theta$) is ignored, the remaining group is $\mathbf{L}'$, and the energy bands can be labeled by its irreducible representations. Obviously since $g\Theta$ is the symmetry of the system, the energies labeled by the representations of types 2^*a and 2^*b will be additionally double degenerated, and only the dimensions of 1^* type representations give right degeneracy.

As an example, we consider the energy bands of the magnons[8] in the quasi-one-dimensional crystal of VF_2. It has been shown[3] that the symmetry of the spin subsystem in VF_2 is $\mathbf{L}30'_4$. This group is commutative, which in ordinary case would imply that the corresponding energies are nondegenerate. Nevertheless, in the case of magnetic groups, and corepresentations, this may not be true, and for VF_2 all the energy bands are degenerate (i.e., all the irreducible corepresentations are two dimensional) except at the end points of the Brillouin zone.

APPENDIX

Axial point groups are the point groups consisting of the elements that leave an axis invariant (by definition the z axis). There are seven families of such groups, namely $\mathbf{C}_n$, $\mathbf{S}_{2n}$, $\mathbf{C}_{nh}$, $\mathbf{D}_n$, $\mathbf{C}_{nv}$, $\mathbf{D}_{nd}$, and $\mathbf{D}_{nh}$ $(n=1,2,\ldots)$. The groups $\mathbf{C}_n$ and $\mathbf{S}_{2n}$ are cyclic, with the generators C_n and $\sigma_h C_{2n}$, respectively. The groups of all the other families are semi-direct (Λ) or direct ($\otimes$) products, e.g.,

TABLE VIII. Irreducible corepresentations of the line groups $\mathbf{L}n'/m (n=2p)$. Here $\mathbf{L}'=\left(\begin{smallmatrix}\mathbf{L}p/m, & p \text{ even} \\ \mathbf{L}(\bar{2p}), & p \text{ odd}\end{smallmatrix}\right)=\{(\mathbf{C}_p^s|t), (\sigma_h\mathbf{C}_p^s|-t)|s=0,1,\ldots,p-1;\ t=0,\pm 1,\ldots\}$, $g=(\mathbf{C}_n|0)$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_m^{\pm}, {}_0A_{-m}^{\pm})$, $({}_{\pi/a}A_m^{\pm}, {}_{\pi/a}A_{-m}^{\pm})$, $({}_k^{-k}E_m, {}_k^{-k}E_{-m})$.

Corepresentations	Type	g	$(\mathbf{C}_p^s\|t)$	$(\sigma_h\mathbf{C}_p^s\|-t)$
${}_0\bar{A}_0^{\pm}$	1^*	1	1	± 1
${}_0\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/4)$	2^*b	$\begin{vmatrix} 0 & e^{-im\alpha} \\ 1 & 0 \end{vmatrix}$	$M(m,s)$	$\pm M(m,s)$
${}_0\bar{\bar{A}}_{p/2}^{\pm}$	2^*a	$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$	$(-1)^s I$	$\pm(-1)^s I$
${}_{\pi/a}\bar{A}_0^{\pm}$	1^*	1	$(-1)^t$	$\pm(-1)^t$
${}_{\pi/a}\bar{\bar{A}}_{p/2}$	2^*a	$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$	$(-1)^{s+t} I$	$\pm(-1)^{s+t} I$
${}_{\pi/a}\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/4)$	2^*b	$\begin{vmatrix} 0 & e^{-im\alpha} \\ 1 & 0 \end{vmatrix}$	$(-1)^t M(m,s)$	$\pm(-1)^t M(m,s)$
${}_k^{-k}\bar{E}_0$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$	$K(k,t)$	$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} K(k,t)$
${}_k^{-k}\bar{\bar{E}}_{p/2}$ $k\in(0,\pi/a)$	2^*a	$\begin{vmatrix} 0 & -I \\ I & 0 \end{vmatrix}$	$(-1)^s k(k,t)$	$(-1)^s d\left(\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}\right) k(k,t)$
${}_k^{-k}\bar{\bar{E}}_m$ $k\in(0,\pi/a)$, $m\in(0,n/4)$	2^*b	$\begin{vmatrix} 0 & e^{-im\alpha}I \\ I & 0 \end{vmatrix}$	$k(k,t)f(m)$	$d\left(\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}\right) k(k,t)f(m)$

TABLE IX. Irreducible corepresentations of the line groups $\mathbf{L}_c n/m, \mathbf{L}_c(\bar{2}\bar{n})$. Here $\mathbf{L}'\left(\begin{smallmatrix}\mathbf{L}''n/m, & n \text{ even}\\ \mathbf{L}''(\bar{2}\bar{n}), & n \text{ odd}\end{smallmatrix}\right)=\{(\mathbf{C}_n^s|t), (\sigma_n \mathbf{C}_n^s|-t)|s=0,1,\ldots,n-1;\ t=0,\pm1,\ldots\}$, $g=(E|\frac{1}{2})$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_{\pi/a}A_{n/2}^{+}, {}_{\pi/a}A_{n/2}^{-})$, $({}_0A_m^{\pm}, {}_0A_{-m}^{\pm 0})$, $({}_{\pi/a}A_m^{\pm}, {}_{\pi/a}A_{-m}^{\mp})$, $({}_{\pi/a}A_0^{+}, {}_{\pi/a}A_0^{-})$, $({}_{\bar{k}}^{-k}E_m, {}_{\bar{k}}^{-k}E_{-m})$.

Corepresentations	Type	g	$(\mathbf{C}_n^s\|t)$	$(\sigma_h \mathbf{C}_n^s\|-t)$
${}_0\bar{A}_0^{\pm}$	1^*	1	1	±1
${}_0\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$M(m,s)$	$\pm M(m,s)$
${}_0\bar{A}_{n/2}^{\pm}$	1^*	1	$(-1)^s$	$\pm(-1)^s$
${}_{\pi/a}\bar{\bar{A}}_0$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^t I$	$\pm(-1)^t\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$
${}_{\pi/a}\bar{\bar{A}}_{n/2}$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t}\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$
${}_{\pi/a}\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^t M(m,s)$	$\pm(-1)^t M(m,s)$
${}_{\bar{k}}^{-k}\bar{E}_0$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & e^{ika}\\ 1 & 0\end{vmatrix}$	$K(k,t)$	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix} K(k,t)$
${}_{\bar{k}}^{-k}\bar{E}_{n/2}$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & e^{ika}\\ 1 & 0\end{vmatrix}$	$(-1)^s K(k,t)$	$(-1)^s\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix} K(k,t)$
${}_{\bar{k}}^{-k}\bar{\bar{E}}_m$ $\begin{smallmatrix}k\in(0,\pi/a]\\ m\in(0,n/2)\end{smallmatrix}$	2^*b	$\begin{vmatrix}0 & K(k,t)^*\\ I & 0\end{vmatrix}$	$k(k,t)f(m)$	$d\left[\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}\right] k(k,t)f(m)$

TABLE X. Irreducible corepresentations of the line groups $\mathbf{L}_s n/m (n=2p)$. Here $\mathbf{L}'=\mathbf{L}''(2p)_p/m=\{(\mathbf{C}_n^s|Fr(s/2)+t), (\sigma_h \mathbf{C}_n^s|-Fr(s/2)-t)|s=0,\ldots n-1; t=0,\pm1,\ldots)$, $g=(E|\frac{1}{2})$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_m^{\pm}, {}_0A_{-m}^{\pm})$, $({}_{\pi/a}E_m^{m-p}, {}_{\pi/a}E_{p-m}^{-m})$, $({}_{\bar{k}}^{-k}E_m, {}_{-k}^{k}E_{-m})$.

Corepresentations	Type	g	$(\mathbf{C}_n^s\|Fr(s/2)+t)$	$(\sigma_n \mathbf{C}_n^s\|-Fr(s/2)-t)$
${}_0\bar{A}_0^{\pm}$	1^*	1	1	±1
${}_0\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/2)$	2^*b	$\begin{vmatrix}0 & e^{-ima}\\ 1 & 0\end{vmatrix}$	$M(m,s)$	$\pm M(m,s)$
${}_0\bar{A}_p^{\pm}$	1^*	1	$(-1)^s$	$\pm(-1)^s$
${}_{\pi/a}\bar{E}_p^{0}$	1^*	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^{s+t}K(\pi/a,Fr(s/2))$	$(-1)^{s+t}\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix} K(\pi/a,Fr(s/2))$
${}_{\pi/a}\bar{\bar{\bar{E}}}_{p/2}^{-p/2}$	2^*a	$\begin{vmatrix}0 & -I\\ I & 0\end{vmatrix}$	$(-1)^{t+\text{Int}(s/2)} d\left[\begin{vmatrix}(-1)^s & 0\\ 0 & 1\end{vmatrix}\right]$	$(-1)^{t+\text{Int}(s/2)} d\left[\begin{vmatrix}0 & 1\\ (-1)^s & 0\end{vmatrix}\right]$
${}_{\pi/a}\bar{\bar{\bar{E}}}_m^{m-p}$ $m\in(0,n/4)$	2^*b	$\begin{vmatrix}0 & -I\\ I & 0\end{vmatrix}$	$(-1)^t k(\pi/a,Fr(s/2))f(m)$	$\pm(-1)^t d\left[\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}\right] k(\pi/a,Fr(s/2))f(m)$
${}_{\bar{k}}^{-k}\bar{E}_0$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & e^{ika}\\ 1 & 0\end{vmatrix}$	$K(k,Fr(s/2)+t)$	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix} K(k,Fr(s/2)+t)$
${}_{\bar{k}}^{-k}\bar{E}_p$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & e^{ika}\\ 1 & 0\end{vmatrix}$	$(-1)^s K(k,Fr(s/2)+t)$	$(-1)^s\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix} K(Fr(s/2)+t)$
${}_{\bar{k}}^{-k}\bar{\bar{E}}_m$ $\begin{smallmatrix}k\in(0,\pi/a)\\ m\in(0,n/2)\end{smallmatrix}$	2^*b	$\begin{vmatrix}0 & K(k,t)^*\\ I & 0\end{vmatrix}$	$k(k,t+Fr(s/2))f(m)$	$d\left[\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}\right] c(k,t+Fr(s/2))f(m)$

TABLE XI. Irreducible corepresentations of the line groups $\mathbf{L}(2p)_p/m'$. Here $\mathbf{L}'=\mathbf{L}(2p)_p=\{(C_n^s|Fr(s/2)+t)|s=0,1,\ldots,n-1;\ t=0,1,2,\ldots\}$, $g=(\sigma_h|0)$. The pair of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentation: $({}_kA_m,{}_kA_{-m})$.

Corepresentation	Type	g	$(\mathbf{C}_n^s\|Fr(s/2)+t)$
${}_k\overline{A}_0\quad k\in(-\pi/a,\pi/a)$	1^*	1	$e^{ik(Fr(s/2)+t)a}$
${}_k\overline{A}_p\quad k\in(-\pi/a,\pi/a)$	1^*	1	$(-1)^s\, e^{ik(Fr(s/2)+t)a}$
${}_k\overline{\overline{A}}_m\quad {}^{k\in(-\pi/a,\pi/a)}_{m\in(0,n/2)}$	2^*b	$\begin{vmatrix}0&1\\1&0\end{vmatrix}$	$e^{ika(Fr(s/2)+t)}M(m,s)$

$\mathbf{C}_{nh}=\mathbf{C}_n\otimes\{e,\sigma_h\}$.

Generalized translations are three classes of groups generated by $(C_n^r|1/n)$, $(\sigma_v|\frac{1}{2})$, and $(E|1)$ and denoted by $\mathbf{n}/r$, $\mathbf{T}_c$, and $\mathbf{T}$, respectively. [We use Coster's symbol $(R|v+t)$ for these transformations, to denote that an orthogonal transformation R, leaving the z axis invariant, is followed by a nonzero translation along the z axis for a distance $(v+t)a$, a being a length unit, while v and t determine the fractional and the primitive translation.] Obviously, $\mathbf{n}/r$ is related to the occurrence of a screw axis, $\mathbf{T}_c$ indicates the existence of a glide plane, and $\mathbf{T}$ is a group of pure translations.

Line groups describe the symmetry of the systems that are translationally periodical in one direction. Every line group $\mathbf{L}$ has an invariant subgroup $\mathbf{T}$ of pure translations. The factor group $\mathbf{L}/\mathbf{T}$ is isomorphic to axial point group $\mathbf{Q}$, which is called the isogonal point group of $\mathbf{L}$. More precisely, every line group $\mathbf{L}$ can be expressed as

TABLE XII. Irreducible corepresentations of the line groups $L(2p)'_p/m'$. Here, $\mathbf{L}'=\left({}^{\mathbf{L}(\bar{2}\bar{p}),\ \ p\ \text{even}}_{\mathbf{L}\bar{p},\ \ p\ \text{odd}}\right)=\{(\mathbf{C}_p^s|t),(\sigma_h\mathbf{C}_{2p}\mathbf{C}_p^s|t)|s=0,1,\ldots,p-1;\ t=0,\pm1,\ldots\}$, $g=(\mathbf{C}_{2p}|\frac{1}{2})$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A^+_{p/2},{}_0A^-_{p/2})$, $({}_0A^\pm_m,{}_0A^\pm_{-m})$, $({}_{\pi/a}A^\pm_m,{}_{\pi/a}A^\pm_{-m})$, $({}_{\pi/a}A^+_0,{}_{\pi/a}A^-_0)$, $({}_k^{-k}E_m,{}_k^{-k}E_{-m})$.

Corepresentations	Type	g	$(\mathbf{C}_p^s\|t)$	$(\sigma_h\mathbf{C}_{2p}\mathbf{C}_p^2\|t)$
${}_0\overline{A}{}_0^t$	1^*	1	1	±1
${}_0\overline{\overline{A}}{}^\pm_m\quad m\in(0,n/4)$	2^*b	$\begin{vmatrix}0&e^{-im\alpha}\\1&0\end{vmatrix}$	$M(m,s)$	$\pm M(m,s+\frac{1}{2})$
${}_0\overline{\overline{A}}_{p/2}$	2^*b	$\begin{vmatrix}0&-1\\1&0\end{vmatrix}$	$(-1)^sI$	$(-1)^s\begin{vmatrix}i&0\\0&-i\end{vmatrix}$
${}_{\pi/a}\overline{\overline{A}}_0$	2^*b	$\begin{vmatrix}0&-1\\1&0\end{vmatrix}$	$(-1)^tI$	$(-1)^t\begin{vmatrix}i&0\\0&-i\end{vmatrix}$
${}_{\pi/a}\overline{A}{}^\pm_{p/2}$	1^*	1	$(-1)^{s+t}$	$\pm(-1)^{s+t}i$
${}_{\pi/a}\overline{\overline{A}}{}^\pm_m\quad m\in(0,n/4)$	2^*b	$\begin{vmatrix}0&e^{-im\alpha}\\-1&0\end{vmatrix}$	$(-1)^tM(m,s)$	$\pm(-1)^tM(m,s+\frac{1}{2})$
${}_k^{-k}\overline{E}_0\quad k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0&e^{ika}\\1&0\end{vmatrix}$	$K(k,t)$	$\begin{vmatrix}0&1\\1&0\end{vmatrix}K(k,t)$
${}_k^{-k}\overline{E}_{p/2}\quad k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0&e^{ika}\\1&0\end{vmatrix}$	$(-1)^sK(k,t)$	$(-1)^s\begin{vmatrix}0&1\\1&0\end{vmatrix}K(k,t)$
${}_k^{-k}\overline{\overline{E}}_m\quad {}^{k\in(0,\pi/a)}_{m\in(0,n/4)}$	2^*b	$\begin{vmatrix}0&e^{-im\alpha}K(k,t)^*\\I&0\end{vmatrix}$	$k(k,t)f(m)$	$d\left(\begin{vmatrix}0&1\\1&0\end{vmatrix}\right)k(k,t)f(m)$

TABLE XIII. Irreducible corepresentations of the line groups $\mathbf{L}(2p)'_p/m$. Here $\mathbf{L}'=\left(\begin{smallmatrix}\mathbf{L}p/m, & p \text{ even} \\ \mathbf{L}(\bar{2p}), & p \text{ odd}\end{smallmatrix}\right)=\{(\mathbf{C}_p^s|t), (\sigma_h\mathbf{C}_p^s|t)|s=0,1,\ldots,p-1;\ t=0,\pm1,\ldots\}$, $g=(\mathbf{C}_{2p}|\frac{1}{2})$. The pairs of $*$-g-conjugated irreducible representations of $\mathbf{L}'$ forming 2^*b corepresentations: $({}_0A_m^{\pm},{}_0A_{-m}^{\pm})$, $({}_{\pi/a}A_0^{+},{}_{\pi/a}A_0^{-})$, $({}_{\pi/a}A_{p/2}^{+},{}_{\pi/a}A_{p/2}^{-})$, $({}_{\pi/a}A_m^{+},{}_{\pi/a}A_{-m}^{-})$, $({}_k^{-k}E_m,{}_k^{-k}E_{-m})$.

Corepresentations	Type	g	$(\mathbf{C}_p^s\vert t)$	$(\sigma_h\mathbf{C}_p^s\vert -t)$
${}_0\bar{A}_0^{\pm}$	1^*	1	1	±1
${}_0\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/4)$	2^*b	$\begin{vmatrix}0 & e^{-im\alpha}\\ 1 & 0\end{vmatrix}$	$M(m,s)$	$\pm M(m,s)$
${}_0\bar{\bar{A}}_{p/2}^{\pm}$	2^*a	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^s I$	$\pm(-1)^s\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$
${}_{\pi/a}\bar{\bar{A}}_0$	2^*b	$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}$	$(-1)^t I$	$(-1)^t\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$
${}_{\pi/a}\bar{\bar{A}}_{p/2}$	2^*b	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t}\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$
${}_{\pi/a}\bar{\bar{A}}_m^{\pm}$ $m\in(0,n/4)$	2^*b	$\begin{vmatrix}0 & -e^{-im\alpha}\\ 1 & 0\end{vmatrix}$	$(-1)^t M(m,s)$	$\pm(-1)^t\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}M(m,s)$
${}_k^{-k}\bar{E}_0$ $k\in(0,\pi/a)$	1^*	$\begin{vmatrix}0 & e^{ika}\\ 1 & 0\end{vmatrix}$	$K(k,t)$	$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}K(k,t)$
${}_k^{-k}\bar{\bar{E}}_{p/2}$ $k\in(0,\pi/a)$	2^*a	$\begin{vmatrix}0 & -K(k,1)^*\\ I & 0\end{vmatrix}$	$(-1)^s k(k,t)$	$(-1)^s d\left(\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}\right)c(k,t)$
${}_k^{-k}\bar{\bar{E}}_m$ $k\in(0,\pi/a)$, $m\in(0,n/4)$	2^*b	$\begin{vmatrix}0 & e^{-im\alpha}K(k,1)^*\\ I & 0\end{vmatrix}$	$k(k,t)f(m)$	$d\left(\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}\right)c(k,t)f(m)$

the sum of $|\mathbf{Q}|$ cosets of $\mathbf{T}$:

$$\mathbf{L}=\sum_{i=1}^{|\mathbf{Q}|}(R_i|v_i)\mathbf{T}\ ,$$

where all v_i are fractional translations and R_i belongs to $\mathbf{Q}$.

An ordinary line group is denoted by the letter $\mathbf{L}$ and the international symbol of the isogonal point group, which is slightly changed when a glide plane or a screw axis exists, e.g., instead of n the symbol n_p is used in the latter case.

Every line group $\mathbf{L}$ is expressed as a *weak direct product* of two of its subgroups: $\mathbf{L}=\mathbf{Z}\circ\mathbf{P}$, where $\mathbf{Z}$ is one of the groups of the generalized translations, and $\mathbf{P}$ is an axial point group. This means that $\mathbf{Z}$ and $\mathbf{P}$ commute and that each element of $\mathbf{L}$ is a unique product of one element of $\mathbf{Z}$ and one element of $\mathbf{P}$. Hence, to construct all the line groups in the factorized form one multiplies each $\mathbf{Z}$ and each $\mathbf{P}$ and verifies whether they commute. In the cases when $\mathbf{Z}$ is $\mathbf{n}/r$ or $\mathbf{T}_c$ some restrictions on $\mathbf{P}$ appear. For some line groups several pairs $\mathbf{Z}$ and $\mathbf{P}$ can be found. For the line groups that are considered in this paper these factorizations are (see the beginning of Sec. III): $\mathbf{L}n_p=\mathbf{n}/r\otimes\mathbf{C}_q$ (isogonal to $\mathbf{C}_n$; q is the greatest common divisor of n and p, and r is the solution of the equation $nFr(rp/n)=q$ which is a coprime relative to n, but less than n); $\mathbf{L}(\bar{2}\bar{n})=\mathbf{T}\wedge\mathbf{S}_{2n}$, $\mathbf{L}\bar{n}=\mathbf{T}\wedge\mathbf{S}_{2n}$ (isogonal to S_{2n}); $\mathbf{L}n/m=\mathbf{T}\wedge\mathbf{C}_{nh}$, $\mathbf{L}(\bar{2}\bar{n})=\mathbf{T}\wedge\mathbf{C}_{nh}$, $\mathbf{L}(2k)_k/m=(2\mathbf{k})/1\circ\mathbf{C}_{kh}=(2\mathbf{k})/1\circ\mathbf{S}_{2k}$ (isogonal to $\mathbf{C}_{nh}$).

The axial point groups $\mathbf{P}$, occurring in the above factorizations, should not be confused with the isogonal point groups. These groups are equal only when $\mathbf{Z}=\mathbf{T}$ (symmorphic line groups); otherwise, $\mathbf{P}$ is a subgroup of the isogonal point group $\mathbf{Q}$.

Construction of all the *magnetic line groups* was performed in the following way: first, all ordinary line groups were factorized as weak direct products of generalized translations $\mathbf{Z}$ and axial point groups $\mathbf{P}$. Utilizing this factorization all halving subgroups $\mathbf{L}'$ of line groups $\mathbf{L}$ were found: $\mathbf{L}=\mathbf{L}'+g\mathbf{L}'$. To this purpose the halving subgroups of $\mathbf{Z}$ and $\mathbf{P}$ are combined.

The class of magnetic line groups associated with ordinary line group $\mathbf{L}$ consists of $\mathbf{L}$ itself, the grey group $\mathbf{L}+\Theta\mathbf{L}$, where Θ is the time reversal, and black and white magnetic line groups which are of the form

$$\mathbf{L}(\mathbf{L}')=\mathbf{L}'+g\Theta\mathbf{L}'\ .$$

Linear-antilinear representation appear when a magnetic group is homomorphically mapped onto a group of unitary and antiunitary operators in the state space of a given quantum system:

$$\hat{D}(\mathbf{L}')+\hat{D}_a(g\Theta)\hat{D}(\mathbf{L}') .$$

A choice of an orthonormal basis $\{u_1, \ldots, u_n\}$ in the state space provides an isomorphic mapping of the above operator group onto a group of matrices and antimatrices in the space $\mathbf{C}^n$. *Antimatrices* are antilinear operators in $\mathbf{C}^n$, and they are of the form $A_a = AK_0$, where A is the matrix factor with the elements $a_{ij}=(e_i, A_a e_j)$ in the standard basis

$$\{e_1=(1,0,\ldots,0)^T, \ldots, e_n=(0,\ldots,0,1)^T\} ,$$

while K_0 is the unique antilinear operator in $\mathbf{C}^n$ (it is a generalization of the complex conjugation in $\mathbf{C}$) satisfying $K_0 e_i = e_i$, $i=1,\ldots,n$. and therefore $K_0 x = x^*$ for each x from $\mathbf{C}^n$. The operator K_0 is unitary, Hermitian and involutive. Also $K_0 A = A^* K_0$ for each matrix A.

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