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New growth model: The screened Eden model

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A new and tractable model for the growth of tumors is proposed and studied numerically. We find that the resulting clusters exhibit complicated, nontrivial scaling behaviors, and the Hausdorff dimensions of the growing clusters can be tuned by changing the screening parameters of the model. Our simulation results show that this model can be used to produce both Eden-like clusters and diffusion-limited aggregation-like clusters by choosing the proper values of the control parameters, and may be regarded as a prototype reference to study the relations of the morphology of the aggregate with the growth mechanisms.

INTRODUCTION

The irreversible aggregation of small particles to form large clusters is a quite general phenomenon occurring in many scientific fields such as physics (dendritic growth, electrical breakdown),[1,2] chemistry (flocculation of colloids, formation of gels, polymerization),[3–5] medicine and biology (growth of tumors)[6] etc. It is, therefore, a subject of increasing interest in recent years to study such kinetic growth processes by introducing various simple computer-simulation models such as the DLA (diffusion-limited aggregation),[7,8] ballistic-driven aggregation,[9] and the Eden model.[6] In the DLA process, particles are added to a growing cluster, one at a time, using random-walk trajectories originating from outside of the aggregate of particles. The resulting clusters are fractal objects characterized by a Hausdorff dimension D which is smaller than the Euclidean dimension d of the space in which the clusters are grown. The DLA is considered as a simplified version of those kinetic growth processes in which the structures of the growing clusters strongly depend on their interaction with an outside environment. The Eden model is another simple growth model in which particles are added to one another in a growing cluster with the prescription that each new particle sticks on any point of the surface of the cluster with equal probability. Because there are no external diffusion fields involved in this growth process, the Eden model may be used to describe another kind of typical growth process such as the growth of tumors and many other biological growth processes, in which the constituent subunits are created inside the growing clusters instead of coming from the external surrounds. It is apparent that the Eden cluster is compact and has a fractal dimension equal to the dimension of space, although the scaling properties of its surface are nontrivial.[10–12] A modified Eden model, in which a finite lifetime of the growth sites is assumed, has recently been proposed and studied numerically. The results obtained, however, have not revealed much novel features.[13,14]

In this paper, we propose a new model for the growth of tumors, which is simple in its growth mechanism but has a wide range of behaviors. Using Monte Carlo simulations we find that the resulting clusters have nontrivial scaling behaviors and their Hausdorff dimensions are tunable. By choosing various values of screening parameters of the model we obtain both Eden-like and DLA-like aggregates. Moreover, we find that this model, similar to the DLA, is not universal in the sense that the scaling properties are independent of the lattice on which the clusters are grown.[15] In fact, we find that the anisotropy of the structures, imposed by the underlying lattice, becomes more and more obvious as the strength of screening increases. It is believed that this model does provide a prototype reference to study quantitatively the relations of the morphology of the growing clusters with the growth mechanisms.

MODEL

For specificity, we consider a two-dimensional square lattice. As a starting configuration we occupied one of the central sites of the lattice, and all others are left empty. The perimeter sites are defined as those empty sites which are adjacent to the occupied ones, and are connected to infinity by at least one straight channel (a channel consists of empty sites along a straight line parallel to the X axis or Y axis). We, then, group those growth sites into three different classes as follows: Class I consists of those perimeter sites which are connected to infinity by only one straight channel, and class II by two such channels,

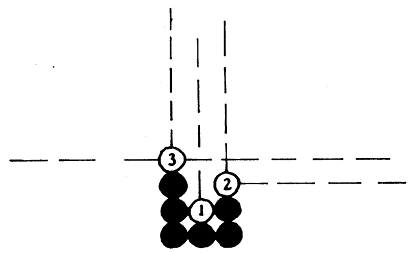

FIG. 1. The schematic illustration of the perimeter of the screened Eden clusters. Open circles denote the perimeter sites, and the numbers inside the circles give the corresponding numbers of the straight channels (denoted by dashed lines).

and class III, by three possible channels. In our two-dimensional case there are three classes defined while in high dimensional space we have, in general, $2d-1$ classes of perimeter sites (see Fig. 1). The growth rule of the model is defined as follows.

(i) Choose with probability P_1 a site from class I, P_2 from class II, and P_3 from class III.

(ii) Occupy the site, where $P_i (i=1,2,3)$ are the control parameters of the model, and each of them changes between 0 and 1 with a normalization condition $\sum_i P_i = 1$.

Our model can be visualized in some practical situations. Imagine, for example, a growing cluster which grows under the stimulation of the light. Suppose that the light incident to the surface of the cluster is parallel to the X axis and Y axis. It is, then, reasonable to assume that the growth takes place only at such sites of the surface which expose to the incident rays, and the growth probability for any of the perimeter sites is proportional to the light flux arriving at the site. Needless to say, the shadowed sites of the surface are no longer active and cease to grow further. This model is evidently a simplified version of the real growth processes in the medical and biological worlds, and may be regarded as a revised Eden model.

According to the definition of the model we see that the growth sites of class III are the most exposed ends of the growing clusters; those of class II are less exposed and even less the sites of class I. It is easy to note that the definition of the perimeter of our model is different than that of the Eden model in that the empty sites, which are adjacent to the occupied ones but not connected to infinity by any of the straight channels defined above, are not perimeter sites any more in our case. In view of that the P_i's represent the growth probabilities of different portions of the growing cluster, the physical relevant cases are obviously that $P_1 \le P_2 \le P_3$ due to the fact that the most exposed sites have the largest growth probability. Thus it is possible for our model to simulate the growth processes with various screening effects by suitably tuning the values of P_i's. For example, in the limiting case of $P_1 = P_2 = P_3$ the model is equivalent to the ballistic aggregation model producing a compact cluster, and in the case of the $P_1 = P_2 = 0$ and $P_3 = 1$ one finds a cross-shaped object with its Hausdorff dimension being $D=1$. What is in between those two limiting cases is of considerable interest. Our numerical results do reveal a rich source of nontrivial scaling behaviors of the model.

SIMULATION RESULTS

Monte Carlo simulations on this simple model have been carried out for some special values of parameter P_i. In Figs. 2(a), 2(b), and 2(c) we show three typical clusters grown on a $2D$ square lattice with $P_1=0$, $P_2=P_3=0.5$; $P_1=0$, $P_2=0.2$, $P_3=0.8$; and $P_1=0$, $P_2=0.1$, $P_3=0.9$. The results indicate that the morphology of the aggregate varies with different values of P_i's and DLA-like clusters are formed. We also calculate the Hausdorff dimensions for the corresponding clusters. Our results are obtained from the radius of gyration R_g:

$$R_g = \left[\left[\sum_{i<j} (\mathbf{r}_i - \mathbf{r}_j)^2 \right] \Big/ N(N-1) \right]^{1/2} , \qquad (1)$$

i and j running over all sites in the cluster and $\mathbf{r}_i$ being the vector from the origin to the ith site. R_g is a characteristic length of a cluster of N occupied sites and has a power-law dependence on the number of the particles for sufficiently large N: $R_g \sim N^{1/D}$. Table I shows the results obtained for six clusters grown on a square lattice for $P_1=0$, $P_2=P_3=0.5$. The Hausdorff dimension D shown in the tables of this presentation are all obtained by using a "least-squares" procedure to fit the values of $\ln(R_g)$ and the corresponding values of $\ln(N)$. We find a mean value

TABLE I. Results obtained in two-dimensional simulations on a square lattice with $P_1=0$, and $P_2=P_3=0.5$.

Number of particles for cluster	Hausdorff dimension (D) 50%	65%	80%	95%
1634	1.854	1.848	1.867	1.792
1640	1.868	1.822	1.836	1.817
1730	1.794	1.831	1.842	1.830
1780	1.770	1.827	1.818	1.859
2034	1.785	1.878	1.892	1.761
2500	1.748	1.798	1.782	1.755
Avg. 1886	1.803 ± 0.034	1.834 ± 0.028	1.821 ± 0.046	1.802 ± 0.051

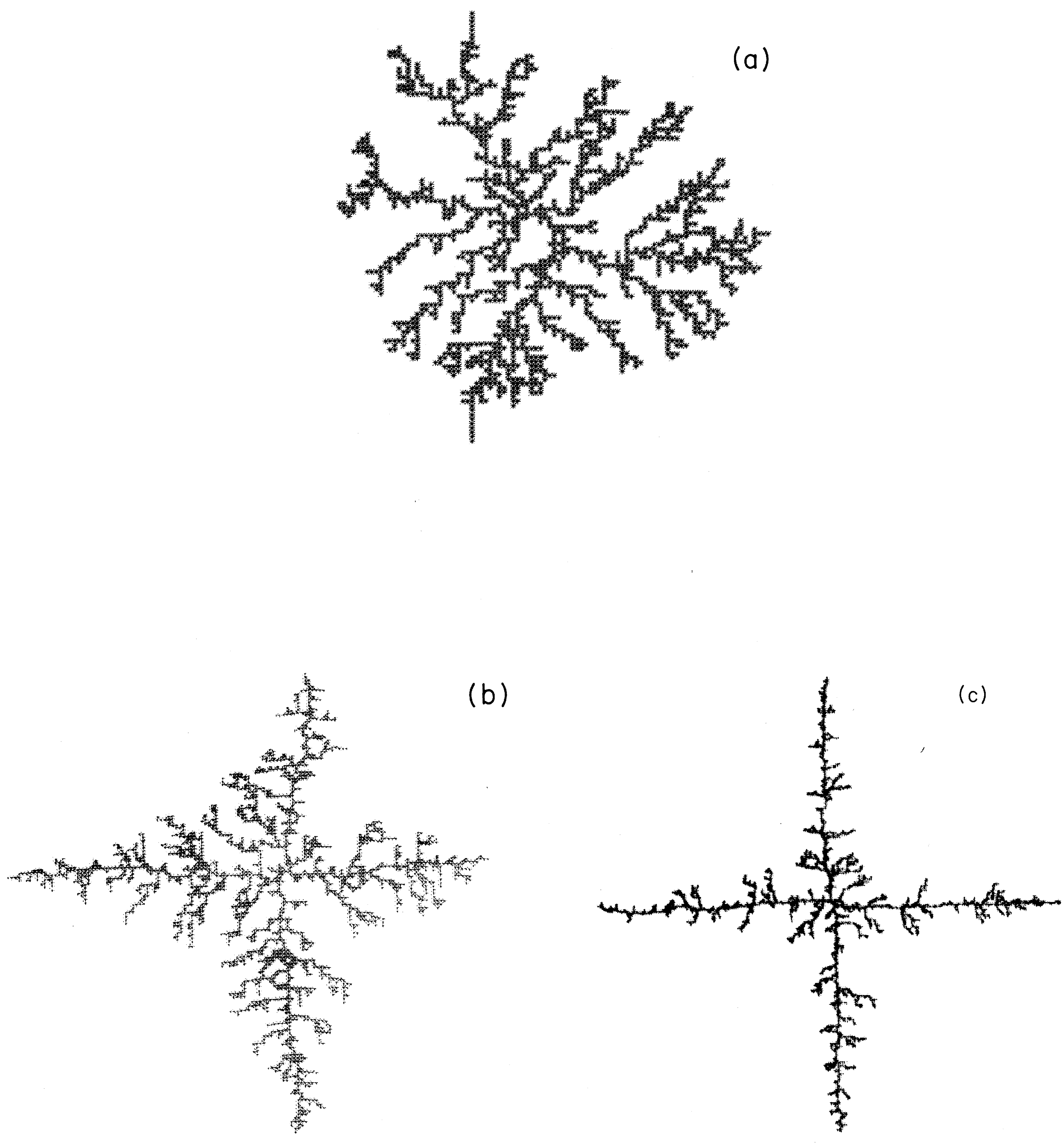

FIG. 2. Two-dimensional clusters obtained using the screened Eden model: (a) The cluster of 2032 particles with $P_1=0$, $P_2=P_3=0.5$; (b) the cluster of 1805 particles with $P_1=0$, $P_2=0.2$, $P_3=0.8$; (c) the cluster of 1814 particles with $P_1=0$, $P_2=0.1$, $P_3=0.9$.

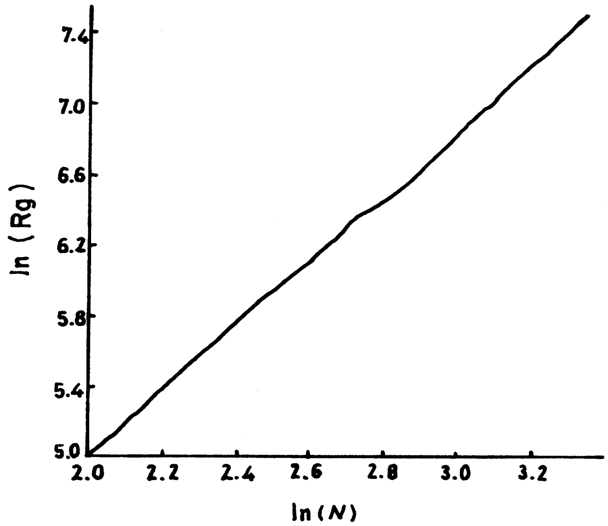

FIG. 3. Dependence of radius of gyration on cluster size during the growth of the cluster shown in Fig. 2(a).

of $D=1.803+0.043$ from clusters obtained during the last 50% of the cluster formation. Figure 3 shows how the radius of gyration increases with increasing cluster size during the growth of a cluster of 2034 particles under the conditions used to obtain Fig. 2(a). To investigate how the structures of the cluster are related with the screening parameters P_i's, we also calculate the Hausdorff dimensions of the growing clusters for $P_1=0$, $P_2=0.05$, $P_3=0.95$; $P_1=0$, $P_2=0.1$, $P_3=0.9$; $P_1=0$, $P_2=0.2$, $P_3=0.8$; $P_1=0$, $P_2=0.3$, $P_3=0.7$; and $P_1=0$, $P_2=0.4$, $P_3=0.6$ as well. In Table II, we show the results obtained from four clusters for $P_1=0$, $P_2=0.1$, and $P_3=0.9$. Table III shows the results obtained for various values of ratio of P_2/P_3, indicating that, with $P_1=0$ fixed, the fractal (Hausdorff) dimensions of growing clusters tend to decrease as the values of ratio P_2/P_3 decrease. However, to discuss the asymptotic property more accurately, one should use clusters of larger size that require a more capable computer or effective algorithm. It will be our future work.

Nevertheless, we have investigated the critical behaviors of the model for different values of ratio P_2/P_3, and found a rich source of ramified structures which are observed in DLA and other aggregating processes. The following points can be concluded. (1) This model indeed produces DLA-like clusters with their fractal dimensions less than the space dimension. The resulting clusters also show strong screening effect, which result in the randomly branched structures of the growing clusters. (2) By varying the control parameters P_i's, one can obtain various aggregates with different Hausdorff dimensions. It seems that the Hausdorff dimension of the growing cluster may decrease continuously with decreasing P_2/P_3, and as P_2/P_3 goes to zero the corresponding Hausdorff dimension seems to approach the limiting case of $D=1.5$.

It should be noted that our results depend on the detail of the lattice structure, which is similar to the DLA clusters. Since there are no complicated diffusion fields involved in our model, one may expect that this model can be dealt with more easily than the existing aggregation models such as DLA. For example, in our model one may readily calculate the generalized dimensions defined by[15–18]

$$D_q=\lim_{\epsilon\to 0}(q-1)^{-1}\ln\left[\sum_{i=1}^{N(\epsilon)}p_i(\epsilon)^q\right]/\ln(\epsilon)\ , \tag{2}$$

where the growth probabilities of the perimeter sites, p_i, are related to the parameters P_j by

$$p_i=P_j/\left[\sum_k N_k P_k\right]\ , \tag{3}$$

the ith site is in class j where N_i is the number of growth sites in ith class. We find

$$D_q=\lim_{N\to\infty}(q-1)^{-1}\ln \times\left[\frac{N_1P_1^q+N_2P_2^q+N_3P_3^q}{(N_1P_1+N_2P_2+N_3P_3)^q}\right]/\ln(1/N)\ . \tag{4}$$

Here we have made use of $\epsilon=1/N$ and $N=N_1+N_2+N_3$, which is the number of unscreened surface sites. Thus, as long as the scaling behaviors of the distributions N are known one may calculate the generalized dimension D_q from Eq. (4). This is in contrast to the DLA processes in which both N_i and p_i are unknown.

TABLE II. Hausdorff dimensions obtained for $P_1=0$, $P_2=0.1$, and $P_3=0.9$.

Number of particles for cluster	Hausdorff dimension (D) 50%	65%	80%	95%
1604	1.615	1.592	1.598	1.643
1800	1.639	1.631	1.629	1.611
1980	1.645	1.662	1.630	1.634
2250	1.624	1.637	1.622	1.661
3000	1.665	1.623	1.617	1.633
3600	1.654	1.644	1.620	1.628
Avg. 2372	1.640±0.025	1.631±0.036	1.619±0.022	1.635±0.029

TABLE III. Hausdorff dimensions obtained from two-dimensional simulations for different values of the control parameters, P_i. The result shows that D varies with P_i.

$N=2000$ P_2/P_3	Hausdorff dimension 50%[a]
$\frac{1}{19}$	1.558 ± 0.042
$\frac{1}{9}$	1.640 ± 0.025
$\frac{1}{4}$	1.693 ± 0.031
$\frac{3}{7}$	1.727 ± 0.033
$\frac{2}{3}$	1.755 ± 0.041
1	1.803 ± 0.034

[a]Last 50% of clusters formed.

In conclusion, we have presented a new computer-simulation model for the growth of tumors. Using this model we are able to study the role of screening in the development of the ramified structure of the growing cluster. We have performed numerical investigation on the connection of the morphology with the growth mechanism, and found that the Hausdorff dimension of the cluster is tunable.

The revised Eden model does present complicated DLA-like structures which are important and practical in nature, although its growth mechanism is essentially different from that of DLA. Moreover, the new model is more efficient in producing growing clusters than DLA, and is, thus, much easier to treat numerically. It is worthwhile to point out that the present model can also be regarded as a revised ballistic aggregation model in some sense. In fact, one can obtain a growing cluster similar to those obtained in this paper, by "shooting" particles with a probability distribution from four directions in $2D$ square lattice. This suggests that a mean-field theory may be possible for describing the complicated behaviors exhibited in our model. These ideas are planned to be discussed in a separate paper.

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