Harmonic generation of Alfvén and helicon waves in semiconductors

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The phenomenon of nonlinear second-harmonic generation of elliptically polarized electromagnetic Alfvén waves in compensated germanium and helicon waves in n-type indium antimonide has been studied theoretically. The Boltzmann transport equation has been solved to find the nonlinear response of electrons or holes in the semiconductor plasmas in the presence of external magnetic fields. For microwaves of power density $\sim 200\ \mathrm{kW\,cm^{-2}}$ and magnetic field ~ 10 kG, the power conversion efficiencies of the second-harmonic waves generated turn out to be $\sim 50\%$ and $\sim 2\%$, respectively, for the Alfvén and helicon waves.

I. INTRODUCTION

The use of microwave techniques for the diagnostics and the studies of optical properties in metals, semimetals, and semiconductors is of considerable interest in recent years. In the presence of an external magnetic field the microwaves whose frequencies are less than the electron plasma frequency and the electron cyclotron frequency, may propagate either as an Alfvén wave mode or a helicon wave mode in the semiconductor plasma.[1–7] At large amplitude of these modes, a number of nonlinear mode-coupling interactions may take place modifying the electrical and optical properties of the semiconductors.[8–11] A number of workers have recently studied the nonlinear interactions of the large amplitude electromagnetic waves in semiconductor plasmas.[12–20] However, the phenomenon of harmonic generation where a significant fraction of energy of the incident low-frequency electromagnetic waves, such as Alfvén and helicon waves, may be converted into harmonic generation.[21–24] But, to the best knowledge of the authors there is no such study in the harmonic generation of electromagnetic Alfvén and helicon waves in semiconductors.

In this paper we have studied the second-harmonic generation of the electromagnetic Alfvén and helicon waves in uniformly magnetized plasmas. Since the Larmor radius of electrons or holes may be comparable or even larger than the wavelengths of the Alfvén and helicon waves for the usual plasma parameters for the existence of these waves in the semiconductors, we cannot employ the fluid model of plasmas. We, therefore, use the Boltzmann transport equation to find the nonlinear behavior of electrons and holes in the semiconductor plasmas.

In Sec. II we have studied the nonlinear response of electrons and holes at the second-harmonic frequencies on account of the propagation of elliptically polarized electromagnetic Alfvén and helicon waves in the semiconductors. In Sec. III we derive the power conversion efficiencies of second-harmonic generation for the Alfvén and helicon waves in the magnetized plasmas. The numerical results and discussions are presented in Sec. IV. Finally, a brief conclusion is given in Sec. V.

II. KINETIC ANALYSIS FOR THE SECOND-HARMONIC CURRENT DENSITY

Let us consider the propagation of an elliptically polarized electromagnetic microwave $(\omega_0,\mathbf{k}_0)$ of angular frequency ω_0 and propagation vector $\mathbf{k}_0$ in a semiconductor plasma immersed in an external static magnetic field $(\mathbf{B}_s\|\mathbf{k}_0\|\hat{\mathbf{z}})$. On account of the nonlinear interaction of the incident microwave with itself in the semiconductor plasma, a nonlinear current density is generated at the frequency at twice the frequency of the pump wave.

In the presence of the incident wave and the generated second-harmonic wave, the nonlinear response of electrons of the semiconductor plasma may be described by the Boltzmann transport equation[25]

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla f-\frac{e}{m}\left[\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c}\right]\cdot\nabla_v f=\left[\frac{\partial f}{\partial t}\right]_{\text{collision}}, \tag{1}$$

where $-e$ is the electronic charge, m is the average effective mass of electrons in the semiconductor, and c is the velocity of light in a vacuum.

The velocity distribution function of electrons f can be expanded as

$$f=f_0^0+f_0+f_2\ , \tag{2}$$

where f_0 and f_2 are the distribution functions corresponding to the fundamental and second-harmonic response of electrons, and f_0^0 is the equilibrium Maxwellian distribution function at temperature T_e:

$$f_0^0=n_0^0\left[\frac{m}{2\pi k_B T_e}\right]^{3/2}\exp\left[-\frac{mv^2}{2k_B T_e}\right]. \tag{3}$$

Here, n_0^0 is the equilibrium electron or hole density in the semiconductor, k_B is the Boltzmann constant, and v is the random speed of the electrons.

By using the Krook model solution the collision term in Eq. (1) can be written as[26]

$$\left[\frac{\partial f}{\partial t}\right]_{\text{collision}} = -\nu(f - f_0^0) , \tag{4}$$

where f is the total distribution function given by Eq. (2) and ν is the average electron-phonon collision frequency in the semiconductor plasma.

In the following we consider the motion of electrons only to obtain the nonlinear current density as the motion of holes will be less important in the contribution of current density. The effect of holes has been included in the dielectric function of the second-harmonic generation. The linear response of electrons due to the incident wave is obtained by solving the Boltzmann transport Eq. (1). To include the first-order effect of the external magnetic field in the distribution function of the incident wave, we take the distribution function in the zero-order approximation of the unmagnetized plasma as

$$-ef_0^0\mathbf{E}_0\cdot\mathbf{v}/k_BT_e(\nu - i\omega_0 + i\mathbf{k}_0\cdot\mathbf{v}) .$$

Thus, the linear distribution function corresponding to the incident wave including the effect of external magnetic field in the plasma can be written from Eq. (1) as

$$f_0^L \simeq \frac{-ef_0^0}{k_BT_e(\nu - i\omega_0 + i\mathbf{k}_0\cdot\mathbf{v})}\left[\mathbf{E}_0\cdot\mathbf{v} + \frac{\mathbf{E}_0\cdot\mathbf{v}\times\boldsymbol{\omega}_c}{\nu - i\omega_0 + i\mathbf{k}_0\cdot\mathbf{v}}\right] , \tag{5}$$

where $\omega_c = eB_s/mc$ is the electron cyclotron frequency.

The nonlinear distribution function of the second-harmonic waves can also be obtained from the Boltzmann transport Eq. (1) as

$$f_2^{\mathrm{NL}} = \frac{e}{2m(\nu - i\omega_2 + i\mathbf{k}_2\cdot\mathbf{v})}\left[\mathbf{E}_0\cdot\nabla_v f_0^L + \frac{1}{c}\mathbf{v}\times\mathbf{B}_0\cdot\nabla_v f_0^L\right] , \tag{6}$$

where ω_2 and $\mathbf{k}_2$ are the angular frequency and propagation vector of the second-harmonic wave, respectively.

The nonlinear current density for the second-harmonic wave is

$$\mathbf{J}_2^{\mathrm{NL}} = -e\int \mathbf{v}f_2^{\mathrm{NL}}\, d^3v . \tag{7}$$

Let us now consider the case when the incident wave is an elliptically polarized electromagnetic Alfvén wave in the semiconductor plasma. The microwave whose frequency ω_0 is less than the electron plasma frequency ω_p and the electron cyclotron frequency ω_c, i.e., $\omega_0 < \omega_p$, ω_c, and which propagates along the direction of the external magnetic field is known as an Alfvén wave. In this case the electric and magnetic fields of the incident homogeneous pump wave are described by[25]

$$\mathbf{E}_0 = \mathbf{E}_0'\exp[-i(\omega_0 t - k_0 z)] ,$$
$$\mathbf{B}_0 = c\mathbf{k}_0\times\mathbf{E}_0/\omega_0 , \tag{8}$$

where

$$E_{0x} = i\beta E_{0y} ,$$
$$k_0 = \frac{\omega_0}{V_A}\left[1 + \frac{V_A^2}{C^2}\right] , \tag{9}$$
$$V_A = B_S/(4\pi n_0^0 m_h)^{1/2} .$$

Here, β is the polarization coefficient, V_A is the Alfvén speed, and m_h is the average effective mass of holes. Under the approximations $\nu > \omega_0 > \mathbf{k}_0\cdot\mathbf{v}$ Eq. (6) simplifies to

$$f_2^{\mathrm{NL}} = \frac{-e^2f_0^0E_{0y}^2}{2m\nu^2k_BT_e}\left[1 + \frac{i\omega_2}{\nu}\right] \times\left[i\beta\left[1 - \frac{k_0v_z}{\omega_0}\right]X + \left[1 - \frac{k_0v_z}{\omega_0}\right]Y + \frac{k_0}{\omega_0}(i\beta v_x + v_y)Z\right] , \tag{10}$$

where

$$X = \frac{-m}{k_BT_e}\left[i\beta v_x + v_y - \frac{\omega_c}{\nu}(v_x - i\beta v_y)\left[1 + \frac{i\omega_0}{\nu}\right]\right] \times\left[\left[1 + \frac{i\omega_0}{\nu}\right]v_x + \left[i\beta + \frac{\omega_c}{\nu}\left[1 + \frac{i\omega_0}{\nu}\right]\right]\right]\left[1 + \frac{i\omega_0}{\nu}\right] , \tag{11}$$

$$Y = \frac{-m}{k_BT_e}\left[i\beta v_x + v_y - \frac{\omega_c}{\nu}(v_x - i\beta v_y)\left[1 + \frac{i\omega_0}{\nu}\right]\right] \times\left[\left[1 + \frac{i\omega_0}{\nu}\right]v_y + \left[1 - \frac{i\beta\omega_c}{\nu}\left[1 + \frac{i\omega_0}{\nu}\right]\right]\right] \times\left[1 + \frac{i\omega_0}{\nu}\right] , \tag{12}$$

$$Z = \frac{-m}{k_BT_e}\left[i\beta v_x + v_y - \frac{\omega_c}{\nu}(v_x - i\beta v_y)\left[1 + \frac{i\omega_0}{\nu}\right]\right] \times\left[\left[1 + \frac{i\omega_0}{\nu}\right]v_z - \frac{ik_0}{\nu}\left[i\beta v_x + v_y - \frac{2\omega_c}{\nu}(v_x - i\beta v_y)\times\left[1 + \frac{i\omega_0}{\nu}\right]\right]\right] . \tag{13}$$

Thus, the nonlinear current density of the generated second-harmonic mode is given by

$$\mathbf{J}_2^{\mathrm{NL}} = \frac{n_0^0e^3k_0E_{0y}^2}{2m^2\nu^2\omega_0}F\hat{\mathbf{z}} , \tag{14}$$

where

$$F = -(1-\beta^2)\left[1 + \frac{i\omega_0}{\nu}\right]\left[1 + \frac{i\omega_2}{\nu}\right] . \tag{15}$$

Similarly, if the incident wave is an elliptically polarized electromagnetic helicon wave, the conditions $\omega_0 < \omega_c$, $\omega_0 \ll \omega_p$ must be satisfied. In this case the electric and magnetic fields of the homogeneous incident wave are described by[17]

$$\mathbf{E}_0=\mathbf{E}_0' \exp[-i(\omega_0 t-k_0 z)] \,, \quad \mathbf{B}_0=c\mathbf{k}_0\times\mathbf{E}_0/\omega_0 \,, \tag{16}$$

where

$$E_{0y}=i\beta E_{0x} \,, \quad k_0=\frac{\omega_0}{c}\left[\epsilon_L+\frac{\omega_p^2}{\omega_c\omega_0}\right]^{1/2} , \quad \omega_p=(4\pi e^2 n_0^0/m)^{1/2} \,, \tag{17}$$

and ϵ_L is the lattice dielectric constant of the semiconductor.

Now, following the derivation of Eq. (14) we obtain the nonlinear current density of the generated second-harmonic mode for the incident helicon wave with approximations $\omega_0>\nu>\mathbf{k}_0\cdot\mathbf{v}$ as

$$\mathbf{J}_2^{\rm NL}=\frac{n_0^0 e^3 k_0 E_{0x}^2}{2m^2\omega_0^2\omega_2}F_1\hat{\mathbf{z}} \,, \tag{18}$$

where

$$F_1=(1-\beta^2)\left[1-\frac{i\nu}{\omega_0}\right]\left[1-\frac{i\nu}{\omega_2}\right] . \tag{19}$$

III. POWER CONVERSION EFFICIENCIES FOR THE SECOND-HARMONIC GENERATION

Substituting the expression for the nonlinear current density in the wave equation for the second-harmonic generation we obtain

$$\underline{\mathbf{D}}_2\cdot\mathbf{E}_2=\frac{4\pi i\omega_2}{c^2}\mathbf{J}_2^{\rm NL} \,, \tag{20}$$

where

$$\underline{\mathbf{D}}_2=k_2^2\underline{\mathbf{I}}-\mathbf{k}_2\mathbf{k}_2-\frac{\omega_2^2}{c^2}\underline{\epsilon}_2 \,, \tag{21}$$

$\underline{\mathbf{I}}$ is the unity tensor of rank two, and the linear dielectric tensor $\underline{\epsilon}_2$ for the second harmonic in the case of Alfvén wave where $\omega_0<\omega_{ch}$ is given by[11]

$$\underline{\epsilon}_2=\begin{bmatrix} \epsilon_L+\frac{\omega_p^2}{\omega_c^2}\left[1+\frac{m_h}{m}\right] & \frac{2i\omega_p^2}{\omega_c\omega_2} & 0 \\ -\frac{2i\omega_p^2}{\omega_c\omega_2} & \epsilon_L+\frac{\omega_p^2}{\omega_c^2}\left[1+\frac{m_h}{m}\right] & 0 \\ 0 & 0 & \epsilon_L-\frac{\omega_p^2}{\omega_2^2}\left[1+\frac{m}{m_h}\right] \end{bmatrix} . \tag{22}$$

The solution of Eq. (20) gives the electric field at second harmonic as

$$\mathbf{E}_2=\frac{-iek_0\omega_p^2E_{0y}^2F}{2m\nu^2\omega_0\omega_2\left[\epsilon_L-\frac{\omega_p^2}{\omega_2^2}\left[1+\frac{m}{m_h}\right]\right]}\hat{\mathbf{z}} \,. \tag{23}$$

Since the direction of the electric vector of the generated second-harmonic wave is along the direction of its propagation, i.e., $\mathbf{k}_2\|\mathbf{E}_2\|\hat{\mathbf{z}}$, the second-harmonic wave is purely an electrostatic electron plasma wave whose dispersion relation is given by[27]

$$\omega_2^2=\omega_p^2+\tfrac{3}{2}k_2^2v_e^2 \,, \tag{24}$$

where $v_e=(2k_BT_e/m)^{1/2}$ is the thermal speed of electrons in the semiconductors.

The power density of the generated second-harmonic wave propagating along the incident Alfvén wave is given by[25,26]

$$P_2=\frac{1}{8\pi}\left[\frac{\partial\omega_2}{\partial k_2}\right]\frac{\partial(\omega_2\epsilon_{2zz})}{\partial\omega_2}(\mathbf{E}_2\cdot\mathbf{E}_2^*) =\frac{3k_2v_e^2}{16\pi\omega_2}\left[\epsilon_L+\frac{\omega_p^2}{\omega_2^2}\left[1+\frac{m}{m_h}\right]\right](\mathbf{E}_2\cdot\mathbf{E}_2^*) \,. \tag{25}$$

The power density of the incident Alfvén wave is given by

$$P_0=\frac{c}{8\pi}\mathbf{E}_0\cdot\mathbf{E}_0^* =\frac{c}{8\pi}E_{0y}E_{0y}^*(1+\beta^2) \,. \tag{26}$$

Thus, the power conversion efficiency of the generated second-harmonic wave due to the propagation of the Alfvén wave in a semiconductor is given by

$$\frac{P_2}{P_0}=\frac{3\pi e^2k_0^2k_2v_e^2\omega_p^4P_0FF^*\left[\epsilon_L+\frac{\omega_p^2}{\omega_2^2}\left[1+\frac{m}{m_h}\right]\right]}{m^2\nu^4\omega_0^2\omega_2^3c^2(1+\beta^2)^2\left[\epsilon_L-\frac{\omega_p^2}{\omega_2^2}\left[1+\frac{m}{m_h}\right]\right]^2} \,, \tag{27}$$

where F is given by Eq. (15).

Similarly, for the helicon wave we obtain the power conversion efficiency as

$$\frac{P_2'}{P_0'}=\frac{3\pi e^2 k_0^2 k_2 v_e^2 \omega_p^4 P_0' F_1 F_1^* \left[\epsilon_L+\frac{\omega_p^2(\omega_2^2-\nu^2)}{(\omega_2^2+\nu^2)^2}\right]}{m^2\omega_0^4\omega_2^5 c^2(1+\beta^2)^2\left[\epsilon_L^2-\frac{2\epsilon_L\omega_p^2}{\omega_2^2+\nu^2}+\frac{\omega_p^4}{\omega_2^2(\omega_2^2+\nu^2)}\right]}, \quad (28)$$

where F_1 is given by Eq. (19) and

$$P_0'=\frac{c}{8\pi}E_{0x}E_{0x}^*(1+\beta^2) . \quad (29)$$

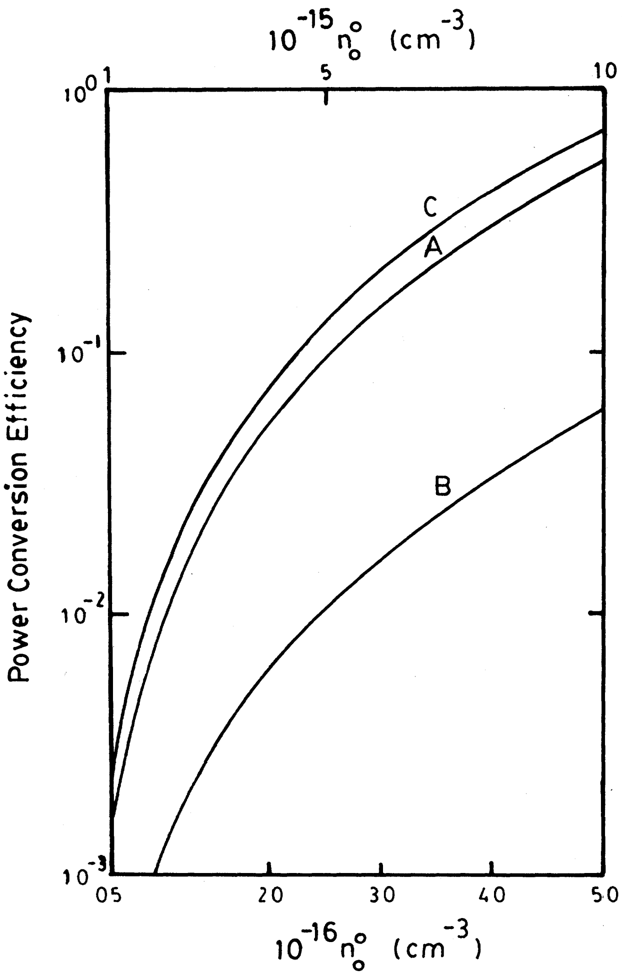

FIG. 1. The variation of power conversion efficiency with n_0^0 for the Alfvén and helicon waves. Curves A and B correspond to the Alfvén wave in compensated Ge where $\epsilon_L=16$ at 77 K, $B_s=10$ kG, $\omega_0=5\times10^{10}$ rad sec^{-1}, $P_0=200$ kW cm^{-2}, and $\beta=1.2$. The curve A is for $\nu=3\times10^{11}$ rad sec^{-1}, and the curve B is for $\nu=5\times10^{11}$ rad sec^{-1}. The curve C and the upper scale correspond to the helicon wave in n-InSb where $\epsilon_L=16$ at 77 K, $B_s=1$ kG, $\omega_0=2.5\times10^{11}$ rad sec^{-1}, $\nu=5\times10^{10}$ rad sec^{-1}, $P_0'=200$ kW cm^{-2}, and $\beta=1.2$.

IV. NUMERICAL RESULTS AND DISCUSSIONS

To have some numerical appreciation of the results, we have made calculations for the second-harmonic power conversion efficiencies for the Alfvén and helicon waves for parallel propagation with respect to the external static magnetic field. The calculations are made for the following typical plasma parameters for Alfvén wave in compensated Ge: $\epsilon_L=16$ at 77 K, $n_0^0=5\times10^{17}$ cm^{-3}, $m=0.1m_e$ (m_e is the mass of a free electron), $m_h=3m$, $\omega_0=5\times10^{10}$ rad sec^{-1}, $\nu=3\times10^{11}$ rad sec^{-1}, $B_s=10$ kG, $P_0=200$ kW cm^{-2}, and $\beta=1.2$; and for the helicon wave in n-InSb: $\epsilon_L=16$ at 77 K, $n_0^0=1\times10^{16}$ cm^{-3}, $m=0.014m_e$, $\omega_0=2.5\times10^{11}$ rad sec^{-1}, $\nu=5\times10^{10}$ rad sec^{-1}, $B_s=1$ kG, $P_0'=200$ kW cm^{-2}, and $\beta=1.2$, respectively.

The results of calculations of the power conversion efficiencies as functions of the electron density n_0^0 and external static magnetic field B_s are displayed in the form of curves in Figs. 1 and 2 for both the Alfvén and helicon waves.

Figure 1 shows the variation of the second-harmonic

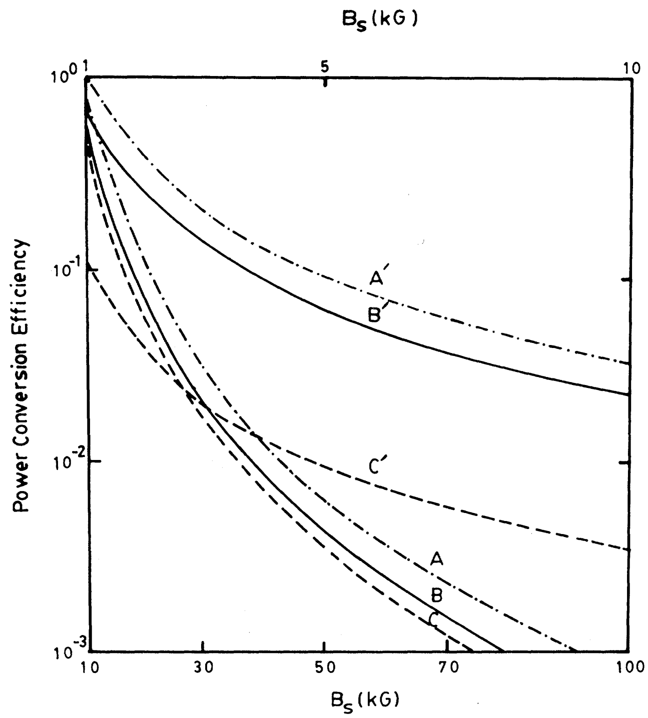

FIG. 2. The variation of power conversion efficiency with B_s for the Alfvén and helicon waves. Curves A, B, and C correspond to the Alfvén and helicon waves. Curves A, B, and C correspond to the Alfvén wave for $n_0^0=5\times10^{17}$ cm^{-3} and $\omega_0=5\times10^{10}$ rad sec^{-1}, $\beta=0.8$; $\omega_0=5\times10^{10}$ rad sec^{-1}, $\beta=1.2$; and $\omega_0=5\times10^{9}$ rad sec^{-1}, $\beta=1.2$, respectively. The upper scale and curves A', B', and C' correspond to the helicon wave for $n_0^0=1\times10^{16}$ cm^{-3} and $\omega_0=2.5\times10^{11}$ rad sec^{-1}, $\beta=0.8$; $\omega_0=2.5\times10^{11}$ rad sec^{-1}, $\beta=1.2$; and $\omega_0=3.5\times10^{11}$ rad sec^{-1}, $\beta=1.2$, respectively. The other parameters are the same as in Fig. 1.

power conversion efficiency as a function of the electron density n_0^0. It is observed that the power conversion efficiency increases rapidly with the increasing density of electrons of the semiconductor. It is a highly sensitive function of average electron-phonon collision frequency for the Alfvén wave, while for the helicon wave the power conversion efficiency does not vary appreciably with the variation of the average electron-phonon collision frequency. The variation of the power conversion efficiency is quite similar for both the Alfvén and helicon waves.

Figure 2 represents the variation of the second-harmonic power conversion efficiency as a function of the external static magnetic field B_s. For the Alfvén wave the power conversion efficiency decreases rapidly as the external static magnetic field increases (curves A, B, and C) and for the helicon wave it decreases slowly (curves A', B', and C'). From the figure it is seen that the power conversion efficiency is almost insensitive to the angular frequency of the external incident Alfvén wave, while for the helicon wave it depends strongly on the angular frequency.

V. CONCLUSION

The large amplitude low-frequency elliptically polarized electromagnetic microwaves, viz., Alfvén and helicon waves propagating through the semiconductors give rise to high yields of the second-harmonic generation. We employ the Boltzmann transport equation to obtain the nonlinear response of electrons in a semiconductor plasma in the presence of an external magnetic field. For the usual parameters in compensated Ge the second-harmonic power conversion efficiency turns out to be $\sim 50\%$ for the Alfvén wave of power density ~ 200 kW cm^{-2} and for the magnetic field ~ 10 kG. It is $\sim 2\%$ for the helicon wave in n-InSb for the same incident power density and external magnetic field. It is observed that the power conversion efficiency for the incident Alfvén wave is larger than that for the helicon wave for a similar set of parameters. The power conversion efficiency increases rapidly with increasing density of electrons or holes and decreases drastically with increasing external magnetic field for both the Alfvén and helicon waves in the semiconductors. It may be mentioned here that the second-harmonic generation vanishes for the case of circularly polarized microwaves in semiconductors. It may be added further that the generation of higher harmonics may also be important for microwaves of large amplitudes in the semiconductors. This remains to be investigated. The results of the present investigation also suggest that the phenomenon of harmonic generation of semiconductor plasmas can be easily verified in the laboratory where the plasma parameters can be varied over a wide range of values without much difficulty.

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