Half-integer and integer quantum-flux periods in the magnetoresistance of one-dimensional rings

Jorge L. D'Amato, Horacio M. Pastawski, and Juan F. Weisz
Instituto de Desarrollo Technológico para la Industria Química, Güemes 3450, 3000 Santa Fe, Argentina
(Received 25 April 1988; revised manuscript received 20 October 1988)

A tight-binding model for a disordered ring coupled to two external leads is used to calculate the transmission coefficient T as a function of the magnetic flux ϕ threading through it. We found a dominant $\phi_0=h/e$ period in two cases: (a) strongly disordered rings and (b) arbitrary disorder with weakly coupled branches. We find that the last situation occurs when the Fermi energy lies near the band edge. The phase of T, which may be equal to 0 or π, depends on energy and changes whenever the Fermi energy crosses an eigenvalue of one of the two isolated branches of the ring. Finally, resonances (peaks) and antiresonances (valleys) are found in T as a function of energy. The width and height of the resonances and antiresonances are calculated within this tight-binding model and their shapes are found to be Lorentzian. This represents an extension of the results of Azbel on resonant tunneling in the case of a tight-binding model.

I. INTRODUCTION

In recent years there has been an intense experimental and theoretical interest in Aharonov-Bohm-like effects in solid-state devices that enclose a magnetic flux ϕ. The first results in this field are the magnetoresistance oscillations in metal cylinders. The agreement between theory and experiment was initially successful but shortly lead to a dilemma. Even if quantum-mechanical properties are expected to present a periodicity of a magnetic quantum flux $\phi_0=h/e$, the evaluation of the ensemble-average conductance with weak localization theory predicted magnetoresistance oscillations dominated by a half-integer flux period for both cylinders and rings.[1] The experiments showed that this is true for long cylinders[2-4] and arrays of metal rings.[5] However, for single rings both periods could be present, depending on the sample and the experimental conditions.[6,7] These surprising results motivated the interest of many physicists who were soon aware that an appropriate and simple description of the phenomena comes from the Landauer theory. This relates the zero-temperature dimensionless conductance with the quantum probability T for an electron to tunnel through the system.[8]

The numerical simulations for a single ring at zero temperature showed a conductance of the form $g(E,\phi)=g(E,\phi=0)+\delta g\cos(2\pi\phi/\phi_0+\beta)+\cdots$, where β is either 0 or π, resulting in a positive or negative correction depending on the system and on the value of the Fermi energy E.[9,10] Other values of β in the range $(-\pi,\pi)$ are associated with an asymmetry about $\phi=0$ which is indeed observed in four-probe measurements.[11] The scale of energy ΔE in which the changes of phase occur is characteristic of the system. An important consequence of this is that for finite temperatures, at which all the energy channels in a range of $kT>\Delta E$ are allowed, there is an average which in some cases cancels the integer flux period and retains the half-integer flux period.[12] Although the general ideas of this picture seem to be well understood and explain both the half-integer and integer quantum-flux periods, there is a point which still remains obscure and which we consider in the present paper. This concerns the origin of the different phases in a two-probe device. The problem of the asymmetry was recently clarified by Büttiker[13] and will not be discussed here.

Much of the previous work modeled the two branches of the ring by considering two scattering centers in a free-electron scheme.[9,10,12,14] Although simple, this model does not bring out the relation between the scattering amplitudes with the eigenstates and eigenvalues of the system. No other complex models of finite-area rings were explicit in this respect.[15,16] For this reason, in this paper we consider a simple one-dimensional tight-binding model in which we are able to calculate the exact eigenvalues as well as localization lengths using a decimation scheme. We discuss the model and the basic aspects of its solution in Sec. II. In Sec. III we analyze how the contacts may weaken the coupling between the low-energy states of both branches. In Sec. IV we generalize the idea of resonant tunneling for this tight-binding. Novel results about transmission antiresonances are also presented. Resonances and antiresonances are seen as peaks and valleys, respectively, in T as a function of the energy. In Sec. V we explain how one can understand the different types of transmission curves generally obtained. Section VI presents numerical and theoretical results for the ring model. The most interesting result is the change in phase of $T(\phi)$ which occurs whenever the Fermi energy crosses an eigenvalue of one of the branches of the ring. This happens for a strongly disordered ring when the localization length is much shorter than the length of the ring and in any other case in which the coupling between the ring branches is weak.

II. THE MODEL AND ITS DECIMATION

Consider a tight-binding Hamiltonian which describes the system represented in Fig. 1(a). This is given by

$$\mathcal{H}=\left[E_A|A\rangle\langle A|+(V_{A,1'}|A\rangle\langle 1'|+V_{A,1''}|A\rangle\langle 1''|+\text{H.c.})+\sum_{n'=1}^{N'}[E_{n'}|n'\rangle\langle n'|+(V_{n',n'+1}|n'\rangle\langle n'+1|+\text{H.c.})]\right.$$
$$\left.+\sum_{n''=1}^{N''}[E_{n''}\rangle\langle n''|n''|+(V_{n'',n''+1}|n''\rangle\langle n''+1|+\text{H.c.})]+E_B|B\rangle\langle B|\right]$$
$$+\left[\sum_{n(<0)}(V|n\rangle\langle n+1|+\text{H.c.})+\sum_{n(>0)}(V|n\rangle(n+1|+\text{H.c.})\right] . \tag{2.1}$$

In the first term, sites with position vectors $\mathbf{r}_{n'}$ and $\mathbf{r}_{n''}$ are placed on the ring. Sites A and B are the contacts which are connected to linear chains represented by the last sums. The upper (lower) branch has N' (N'') sites. The length of the ring is $M=N'+N''+2$. Site energies on the ring takes on random values in the range $[-W/2, W/2]$. The hopping parameters are restricted to nearest neighbors and in the ring they have the form

$$V_{n,n+1}=V^*_{n+1,n}=V\exp(i2\pi\phi_{n,n+1}/\phi_0) .$$

The phase factor takes into account the magnetic vector potential in a symmetrical gauge, by means of the path integral

$$\phi_{n,n+1}=\int_{\mathbf{r}_n}^{\mathbf{r}_{n+1}}\mathbf{A}\,d\boldsymbol{l} . \tag{2.2}$$

The Landauer theory of quantum transport at very low temperatures assumes that inelastic processes only occur at the electron reservoirs which are farther from the ring contacts. In this condition the dimensionless resistance is determined from the quantum tunneling probability of an electron with the Fermi energy E.[8] The problem then reduces to the evaluation of $T(E)$. In order to facilitate the calculation, it is convenient to use a real-space decimation procedure[17–19] which was extended to general Hamiltonians.[20,21] The method is exact and allows one to generate an effective Hamiltonian that takes into account the internal sites of the ring through an effective coupling $\tilde{V}_{AB}$ and renormalized site energies to give total site energies $\tilde{E}_A$ and $\tilde{E}_B$ [see Fig. 1(b)]:

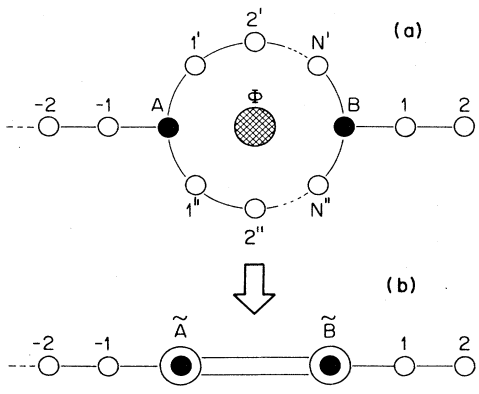

FIG. 1. (a) Representation of the ring and leads. Upper (lower) branch has N' (N'') sites. Sites A and B are the contacts. (b) Effective one-dimensional system after decimation of the internal sites of the ring.

$$\tilde{V}_{AB}=V_U(E)\exp(i2\pi\phi_U/\phi_0)+V_L(E)\exp(i2\pi\phi_L/\phi_0) , \tag{2.3a}$$

$$\tilde{E}_A=E_A+\Delta_A^U(E)+\Delta_A^L(E) , \tag{2.3b}$$

$$\tilde{E}_B=E_B+\Delta_B^U(E)+\Delta_B^L(E) , \tag{2.3c}$$

where V_U (V_L) is the upper (lower) branch contribution to the effective coupling. The dependence on flux is given by the phase factors. $\phi_U=\int_{UB}\mathbf{A}\,d\boldsymbol{l}$ and $\phi_L=\int_{LB}\mathbf{A}\,d\boldsymbol{l}$ are the path integrals of the vector potential on the upper and lower branch, respectively. They are related to the total flux $\phi_U-\phi_L=\phi$. Finally, Δ_A^U and Δ_A^L (Δ_B^U and Δ_B^L) are the self-energy contributions to site A (B) from the upper (U) and lower (L) branches, respectively. These magnitudes can be calculated using a recursive algorithm that eliminates a site in each branch of the ring one at a time. For example, for the upper branch one could begin by eliminating site 1'. This introduces an effective coupling $\tilde{V}_{A2'}$ between the contact A and the site 2' and renormalized site energies $\tilde{E}_A$ and $\tilde{E}_{2'}$ given by

$$\tilde{E}_A=E_A+V_{A1'}\frac{1}{E_{1'}-E}V_{1'A} , \tag{2.4a}$$

$$\tilde{E}_{2'}=E_{2'}+V_{2'1'}\frac{1}{E_{1'}-E}V_{1'2'} , \tag{2.4b}$$

$$\tilde{V}_{A2'}=V_{A1'}\frac{1}{E_{1'}-E}V_{1'2'} . \tag{2.4c}$$

The procedure is continued with site 2', taking into account that these new values must be used in the iteration of Eq. (2.4). After site N' is eliminated the process is stopped. An analogous expression is valid for the lower branch.

The effective coupling and the self-energy are related to the Green's functions of each isolated branch. It is not difficult to obtain the following relations:

$$\Delta_A^U=V_{A1'}G^U_{1'1'}V_{1'A},\quad \Delta_A^L=V_{A1''}G^L_{1''1''}V_{1''A} , \tag{2.5a}$$

$$\Delta_B^U=V_{BN'}G^U_{N'N'}V_{N'B},\quad \Delta_B^L=V_{BN''}G^L_{N''N''}V_{N''B} , \tag{2.5b}$$

$$V_U=V_{A1'}G^U_{1'N'}V_{N'B},\quad V_L=V_{A1''}G^L_{1''N''}V_{N''B} . \tag{2.5c}$$

The transmission probability of the effective one-dimensional system is now evaluated. For this, a procedure described by Stone *et al.*[22] is used. They obtain a general expression for the dimensionless resistance in terms of the promotion matrix $\underline{P}$ which relates the amplitudes at each side of the ring:

$$\begin{bmatrix} a_1 \\ a_B \end{bmatrix} = \underline{P} \begin{bmatrix} a_A \\ a_{-1} \end{bmatrix} . \tag{2.6}$$

The dimensionless resistance is given by

$$\rho = \frac{|P_{11}-P_{22}+(E/2V)(P_{12}-P_{21}|^2}{4-(E/V)^2} + \frac{|P_{12}+P_{21}|^2}{4} , \tag{2.7}$$

where P_{ij} are the matrix elements. Making use of our effective Hamiltonian, the promotion matrix takes the form

$$\underline{P} = \frac{1}{\tilde{V}_{AB} V} \begin{bmatrix} (E-\tilde{E}_A)(E-\tilde{E}_B)-|\tilde{V}_{AB}|^2 & -V(E-\tilde{E}_B) \\ V(E-E_A) & -V^2 \end{bmatrix} . \tag{2.8}$$

From now on V is adopted as unit of energy. Thus energy parameters are dimensionless. Inserting Eq. (2.8) into Eq. (2.7) gives

$$\rho = \frac{A(E)}{|\tilde{V}_{AB}|^2} + B(E)|\tilde{V}_{AB}|^2 + C(E) , \tag{2.9}$$

where

$$A(E) = \frac{[1+\tilde{E}_A\tilde{E}_B+(E/2)(\tilde{E}_A+\tilde{E}_B)]^2}{4-E^2} + \frac{(\tilde{E}_A-\tilde{E}_B)^2}{4} , \tag{2.10a}$$

$$B(E) = \frac{1}{4-E^2} , \tag{2.10b}$$

$$C(E) = \frac{-2[1+\tilde{E}_A\tilde{E}_B+(E/2)(\tilde{E}_A+\tilde{E}_B)]}{4-E^2} . \tag{2.10c}$$

The dimensionless resistance is obtained using the Landauer formula $\rho=(1-T)/T$. In summary, the decimation method allows us to obtain the exact quantum-transport properties of the ring by reducing it to an effective one-dimensional chain with renormalized parameters.

III. SCATTERING AT THE CONTACT

We want to analyze the nature of the electronic states which appear in the region where the circuit splits [i.e., points A and B in Fig. 1). As a first step we consider a circuit with a single branching point. This T-shaped circuit can be obtained from that of Fig. 1 when the perimeter of the ring becomes infinite ($N'=N''\to\infty$). Since we are interested in the details of the eigenfunctions we come back to the Hamiltonian, Eq. (2.1), and their corresponding equations on differences (with $V=1$) for the eigenvectors

$$(E-E_A)a_A - a_{-1} - a_{1'} - a_{1''} = 0 , \tag{3.1}$$

$$(E-E_n)a_n - a_{n-1} - a_{n+1} = 0 \quad \text{with } n \neq 0 , \tag{3.2}$$

where Eq. (3.1) corresponds to the node and is the only equation which connects one site amplitude to the other three through the hopping (kinetic) parameter V. Equation (3.2) is satisfied by the amplitudes inside each branch. Notice that we may choose $E_n=E_0=0 \forall n$ to represent an ordered system and a phaseless hopping parameter because there is no magnetic field. Equation (3.2) can be written as

$$-\frac{a_{n-1}}{a_n} + E - \frac{a_{n+1}}{a_n} = 0 .$$

The symmetry of the problem imposes the condition a_{n+1}/a_n constant for any branch and position, even for $a_n=a_A$. Therefore

$$\frac{a_{n+1}}{a_n} = \frac{E}{2} \pm \left[\left[\frac{E}{2} \right]^2 - 1 \right]^{1/2} = \exp(i\theta), \quad \theta = k + i\kappa . \tag{3.3}$$

It can be readily verified that the compatibility of Eqs. (3.1) and (3.2) is possible for eigenvalues in the continuous range $|E|<2$ (the band), but also at

$$E_\alpha = \alpha 2\sqrt{9/8}, \quad \alpha = \pm 1 . \tag{3.4}$$

These energies differ from the one-dimensional band edges by an amount which can be interpreted[23] as the extra kinetic energy due to an increase of the effective dimensionality around the branching point, which appear as the three hopping terms in Eq. (3.1). The resulting states are localized around A. Therefore their amplitudes can be chosen to be real ($k=0$ or $k=\pi$) and the symmetry condition becomes $a_{-n}=a_{n'}=a_{n''}=a_n$. From Eqs. (3.4) and (3.3) we find

$$a_n = a_A \exp(-\kappa n) ; \tag{3.5}$$

that is, the wave function decays exponentially with distance to the branching point with a location length given by

$$\lambda = 1/\kappa = \frac{1}{\operatorname{arccosh}(E_\alpha/2)} \simeq 2.88 . \tag{3.6}$$

The value of $a_A^\alpha = \frac{1}{2}$ becomes determined by the normalization condition. This result implies that only $\frac{1}{2}$ of the local density of states at the branching point remains available for the extended states of the continuous spectrum. This must be shown in the transmission probability T for the propagating states.

There are various ways to compute T, such as the promotion matrix presented in the preceding section or a Green's-function formalism.[24] For the present purpose it is enough to impose in Eq. (3.1) an incident wave from the left with its corresponding reflected wave,

$$a_{-n} = \exp(-ikn) + r\exp(ikn) ,$$

where r is the reflexion amplitude, and a transmitted wave at the two equivalent channels of the right,

$$a_{n'} = a_n = a_A \exp(ikn) .$$

With this amplitude inserted into Eqs. (3.3) and (3.1) we get, after some algebra,

$$T=\frac{1-rr^*}{2}=\frac{4}{9}\,\frac{1}{1+\dfrac{E^2}{9(4-E^2)}}\ . \tag{3.7}$$

It is important to note that the transmission probability is zero at the band edges and reaches the maximum value $\frac{4}{9}$ at the band center. For a splitter in the free-electron scheme the transmission results are energy independent ($T=\frac{4}{9}$), which is still lower than the value for an ideal splitter ($T=\frac{1}{2}$). Therefore, the effect of branching a circuit is not only to split the current but also to act as a potential well (barrier) which disturbs the propagation of the electrons (holes).

Now we want to understand the electronic states in a finite ring with leads. For this purpose it is enough to use the effective Hamiltonian approximation presented elsewhere.[21] In the present problem it requires us to decimate the leads, which introduces a third correction in the renormalized energies [Eqs. (2.3b) and (2.3c)]. Thus $E-\tilde{E}_A$ and $E-\tilde{E}_B$ have approximate roots at the values of Eq. (3.4). The next step is to use Eq. (2.3) as an effective (2×2) Hamiltonian matrix, in which the localized states at each branching point interact through the effective hopping $\tilde{V}_{AB}$. For the symmetrical ring ($N'=N''$) the maximum splitting is obtained at $\phi=0$ and becomes

$$E_\alpha^{\pm}\simeq\tilde{E}_\alpha\pm\tilde{V}_{AB}(\tilde{E}_\alpha)\simeq E_\alpha\pm\tfrac{1}{2}\exp(-\kappa N)\ . \tag{3.8}$$

When the branches are long enough ($N\gg\lambda$), both states remain outside the bands and their localization properties are not modified. This is consistent with the result of the infinite ring presented above. Therefore, a propagating wave in a branch will have a probability given by Eq. (3.7) to pass to the other branch. An important consequence which follows from the previous discussion is that in the ring configuration the presence of the leads diminishes the coupling between the branches, the effect being very strong for states near the band edge.

IV. RESONANCES AND ANTIRESONANCES

In a series of papers Azbel[25–27] studied the transmission resonances in a one-dimensional disordered system in the regime of strong localization. He found that resonances are related to the localized eigenstates and eigenvalues of the random system. The height and width of the resonances are functions of the localization length λ and the distance between the center of localization and the middle of the disordered region Λ. In this section we analytically compute the form of the resonances for our tight-binding model. Our results agree with those of Azbel. Moreover, due to the topology of the ring there also exist antiresonances.[21] This situation, which is not possible in a strictly one-dimensional system, is produced by the multiply connected character of the ring. We also calculate the form of these antiresonances. It will be seen that the antiresonances exist for special values of flux but not for others.

To obtain the shape of the resonances it is better to write the dimensionless resistance in terms of the Green's function of the isolated ring:

$$\rho(E)=\frac{1}{|G_{AB}|^2}\left\{\frac{[1-|G_{AB}|^2+(E/2)(G_{AA}+G_{BB})+G_{AA}G_{BB}]^2}{4-E^2}+\frac{G_{AA}-G_{BB}}{4}\right\} \tag{4.1}$$

with

$$G_{ij}=\sum_\alpha\frac{a_i^{*\alpha}a_j^\alpha}{E-E_\alpha}\ , \tag{4.2}$$

where a_i^α is the amplitude of the wave function of the isolated ring on site i with energy E_α. In order to eliminate band-edge effects only the center of the band is studied. Hence the approximation $4-E^2\simeq4$ is made in Eq. (4.1). Also we take $E\simeq E_\alpha$ because we are interested in analyzing the transmission probability T near an eigenstate with energy E_α. For the same reason we use

$$G_{AA}\simeq\frac{|a_A^\alpha|^2}{E-E_\alpha}\ , \tag{4.3a}$$

$$G_{AB}\simeq\frac{a_A^{*\alpha}a_B^\alpha}{E-E_\alpha}\ , \tag{4.3b}$$

$$G_{BB}\simeq\frac{|a_B^\alpha|^2}{E-E_\alpha}\ . \tag{4.3c}$$

This is valid for $|E-E_\alpha|<\overline{\Delta E}$, where $\overline{\Delta E}$ is the mean separation between levels. Inserting these approximations in Eq. (3.1) and using $\rho=(1-T)/T$ one has

$$T(E)=\frac{4|a_A^\alpha|^2\,|a_B^\alpha|^2}{[E-E_\alpha-(E_\alpha/2)(|a_A^\alpha|^2+|a_B^\alpha|^2)]^2+[|a_A^\alpha|^2+|a_A^\alpha|^2]^2}\ . \tag{4.4}$$

Thus T has a Lorentzian shape. The position of the maximum E_M, the maximum value T_M, and the width W_R are given by

$$E_M=E_\alpha+(E_\alpha/2)(|a_A^\alpha|^2+|a_B^\alpha|^2)\ , \tag{4.5a}$$

$$T_M=\frac{4|a_A^\alpha|^2\,|a_B^\alpha|^2}{(|a_A^\alpha|^2+|a_B^\alpha|^2)^2}\ , \tag{4.5b}$$

$$W_R=|a_A^\alpha|^2+|a_B^\alpha|^2\ . \tag{4.5c}$$

Note that the width is proportional to the inverse of the lifetime of the eigenstate of the ring. The center of the resonances are shifted with respect to the eigenvalues of the ring by a quantity which is proportional to the eigenvalue and to the inverse of the lifetime (or width). The validity of this result depends on the width of the resonances, which must be smaller than the mean separation between levels ΔE. That is the region of strong localization. Following Azbel the amplitude on the contacts are approximated by

$$|a_A^\alpha| \propto \exp[-(M/2+\Lambda)/\lambda] ,$$
$$|a_B^\alpha| \propto \exp[-(M/2-\Lambda)/\lambda] , \tag{4.6}$$

where Λ is the smallest distance between the center of localization of the eigenstate and the middle of the corresponding branch. Then the width of the resonances is exponentially small and the resonances are sharp. This also happens when one has highly reflective contacts, even when the branches of the rings are ordered, because the amplitudes of the eigenstates on contacts A and B are small. Using Eq. (4.5) the maximum takes the form

$$T_M = \frac{1}{\cosh^2(2\Lambda/\lambda)} . \tag{4.7}$$

If $\Lambda \gg \lambda$, Eq. (4.7) reduces to $T_M \simeq 4\exp(-4\Lambda/\lambda)$, which agrees with Azbel's proposal.

Proceeding now with the evaluation of the antiresonances, consider an interval around the energy E_w at which $|\tilde{V}_{AB}|=0$. This situation, which is not possible in a linear chain, can occur in rings and other multiply connected systems when the path contributions to the total effective coupling cancel each other. If that situation occurs, the transmission probability T goes to zero and antiresonances are produced near E_w. Taking a linear approximation near this energy we get $|\tilde{V}_{AB}| = m(E-E_w)$ and using Eq. (2.1) we find that the resistance goes to infinity as

$$\rho \simeq \frac{A(E_w)}{m^2(E-E_w)^2} . \tag{4.8}$$

Thus the reflection probability $R=1-T$ may be written as

$$R = \frac{1}{1+(E-E_w)^2 m^2/A(E_w)} . \tag{4.9}$$

The antiresonances occur as Lorentzian peaks of R as a function of E. The width is $W_A = A(E_w)^{1/2}/m$. Now, let us remark that there are only two values of ϕ for which $|\tilde{V}_{AB}|$ can be zero, that is for $\phi=0$ and $\phi=\phi_0/2$. This is because V_U and V_L are real numbers so that $\tilde{V}_{AB}$ is then a complex number, making it necessary that $V_U=V_L=0$ in order to have $\tilde{V}_{AB}=0$. But in a linear chain neither V_U nor V_L are ever zero. However, for $\phi=0$, $|\tilde{V}_{AB}| = |V_U+V_L|$, and for $\phi=\phi_0/2$, $|\tilde{V}_{AB}| = |V_U-V_L|$. Then one may have antiresonances only for these two values of ϕ. For $\phi=0$ or ϕ_0 an antiresonance can occur between two neighboring eigenvalues, E_α of the upper branch and E_β of the lower one, with identical parities. In contrast, for $\phi=\phi_0/2$ they must have opposites parities. The width of the antiresonances can be related to the localization length in the strongly disordered regime. The approximations

$$V_U = G^U_{1'N'} \simeq \frac{a^\alpha_{1'} a^\alpha_{N'}}{E-E_\alpha} , \quad V_L = G^L_{1''N''} \simeq \frac{a^\beta_{1''} a^\beta_{N''}}{E-E_\beta} , \tag{4.10a}$$

$$\tilde{E}_A = E_A + G^U_{1'1'} + G^L_{1''1''} \simeq E_A + \frac{|a^\alpha_{1'}|^2}{E-E_\alpha} + \frac{|a^\alpha_{1''}|^2}{E-E_\beta} , \tag{4.10b}$$

$$\tilde{E}_B = E_B + G^U_{N'N'} + G^L_{N''N''} \simeq E_B + \frac{|a^\alpha_{N'}|^2}{E-E_\alpha} + \frac{|a^\beta_{N''}|^2}{E-E_\beta} \tag{4.10c}$$

are valid between E_α and E_β. In this regime

$$a^\alpha_{1'} a^\alpha_{N'} \simeq a^\beta_{1''} a^\beta_{N''} \simeq \pm \exp(-M/\lambda) ,$$

where the sign is related to the parity of the eigenstates, that is, to the number of nodes. To go further we assume that E_α and E_β have equal parities and that $\phi=0$ or $\phi=\phi_0$. Then the linear approximation takes the form

$$|\tilde{V}_{AB}| \simeq \exp(-M/\lambda)\left[E - \frac{E_\alpha+E_\beta}{2}\right] , \tag{4.11}$$

where one can identify $m=\exp(-M/\lambda)$ and $E_w = (E_\alpha+E_\beta)/2$. Now, in the strongly localized regime one can approximate $\tilde{E}_\alpha \simeq E_A$ and $\tilde{E}_B \simeq E_B$ for $E=E_w$ because of the exponential localization of the eigenstates. Thus $A(w)^{1/2}$ is a finite number of order unity [see Eq. (2.10)] and the width of the antiresonance is

$$W_A \propto \exp(M/\lambda) . \tag{4.12}$$

Note that a smaller localization length gives a larger width. We finally want to emphasize that antiresonances are very much influenced by the flux. An antiresonance for $\phi=\phi_0$ ($\phi=\phi_0/2$) disappears for $\phi=\phi_0/2$ ($\phi=\phi_0$). However, this is not the case for resonances. That is because to first order in perturbation theory the eigenstates of the ring do not change and their energies only oscillate with an exponentially small amplitude

$$E_\alpha \simeq E^0_\alpha + (-1)^n 2\exp(-M/\lambda)\cos(2\pi\phi/\phi_0) , \tag{4.13}$$

where n indicates the number of nodes of the eigenstate of the ring ($n=0$ for the ground state). Therefore the flux only moves the resonances an infinitesimally small amount without modifying their shape.

V. DETERMINATION OF THE PHASES

Before introducing the numerical results for $T(\phi)$, we show the different types of curves that can be seen. $T(\phi)$ typically has one of the forms shown in Fig. 2. The double-peaked function (c,d) is the more likely for weakly disordered rings. They have Fourier components with period ϕ_0 and $\phi_0/2$ of the same order of magnitude. The

principal difference between these two curves is the initial phase of the ϕ_0 component. Figure 2(c) [2(d)] has a phase $\beta=0$ ($\beta=\pi$). For numerical purposes we assign a parameter $P=0$ to this kind of curve. On the other hand, single-peaked functions (a) and (b) are characteristic of strongly disordered rings. They have a dominant ϕ_0 period with a initial phase $\beta=0$ [(a)] and $\beta=\pi$ [(b)]. The parameter $P=-1$ ($P=1$) defines a curve of type (a) [(b)].

To understand why only these three cases are possible it is necessary to examine Eq. (2.9). The dimensionless resistance depends on $|\tilde{V}_{AB}|^2$ because the Fermi energy is fixed. This function has an absolute minimum at

$$|\tilde{V}_{AB}|^2=\bar{V}^2\equiv\left[\frac{A(E)}{B(E)}\right]^{1/2}. \tag{5.1}$$

The oscillation in $T(\phi)$ is produced because $|\tilde{V}_{AB}|^2$ oscillates with flux with period ϕ_0,

$$|\tilde{V}_{AB}|^2=V_U^2+V_L^2+2V_UV_L\cos(2\pi\phi/\phi_0)\ , \tag{5.2}$$

so that it moves from $|\tilde{V}_{AB}(\phi=0)|^2=V_I^2=(V_U+V_L)^2$ to $|\tilde{V}_{AB}(\phi=\phi_0/2)|^2=V_M^2=(V_U-V_L)^2$ in a monotonic way (rising or decreasing) and then goes back. It is not difficult to see that a double-peaked function ($P=0$) occurs if $V_I^2<\bar{V}^2<V_M^2$ or $V_M^2<\bar{V}^2<V_I^2$. The condition for a single-peaked function with $P=1$ requires that $V_I^2<V_M^2<\bar{V}^2$ or $\bar{V}^2<V_M^2<V_I^2$. Finally, the last case $P=-1$ occurs whenever $V_M^2<V_I^2<\bar{V}^2$ or $\bar{V}^2<V_I^2<V_M^2$. This exhausts all the possible relations between the parameters V_I^2, V_M^2, and $\bar{V}^2$.

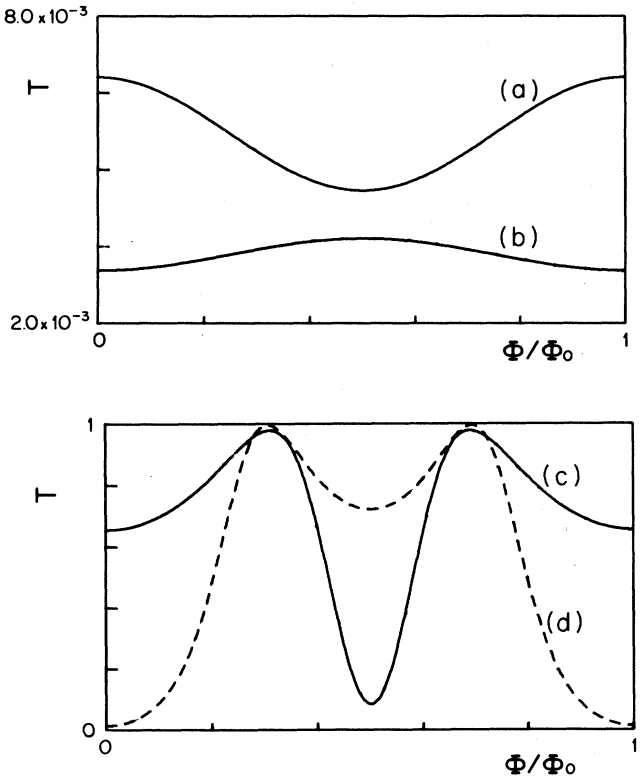

FIG. 2. (a) and (b) Typical transmittance T for a strongly disordered ring. Both curves have a dominant ϕ_0 period. (c) and (d) Typical transmittance T for a newly ordered ring. These curves have both ϕ_0 and $\phi_0/2$ periods of equal order of magnitude. Curves of type (a) and (b) are characterized by a parameter P equal to -1 or 1, respectively. Curves (c) and (d) correspond to $P=0$.

VI. FLUX DEPENDENCE OF T FOR THE RING

In this section numerical results are presented for $T(\phi)$ and the data are interpreted by means of an analytical approach. First, the discussion is made for the case of strong disorder ($\lambda\ll M$). For this case it can be verified that the results do not depend strongly on the position of the contacts so that the values $N'=N''=M/2-1$ were selected. Figure 3 shows the numerical results for $N'=N''=99$ and a disorder parameter of $W=2$. Notice that for a strongly disordered ring almost any energy gives $P=\pm1$, while $P=0$ occurs for energy windows of infinitesimal measure around the points at which P

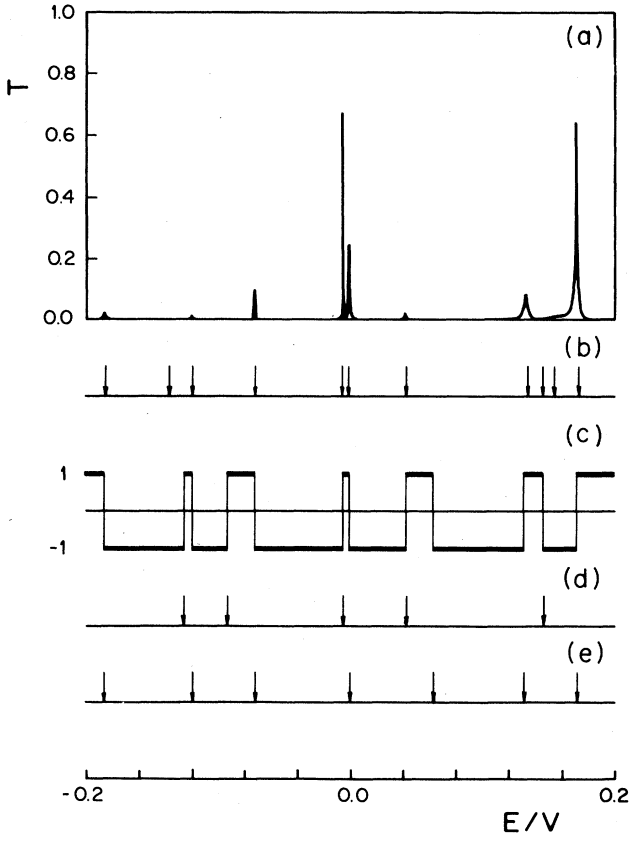

FIG. 3. Data for a ring with $N'=N''=99$ and disorder parameter $W=2$. (a) Transmittance as a function of energy (scale at bottom) in the middle of the band for $\phi=0$. (b) Arrows over the axis indicate the eigenvalues of the isolated ring. (c) Parameter P defined in the text (see Fig. 2) as a function of energy. (d) and (e) Arrows indicate the eigenvalues of the isolated upper and lower branch, respectively. Observe the correlation of (c) with (d) and (e).

changes from the value -1 to 1 and vice versa [see Fig. 3(c)].

In order to understand the origin of the different phases it is convenient to analyze T as a function of the energy. A typical situation of this is shown in Fig. 3(a) for $\phi=0$. Notice that T is exponentially small for almost any energy except small ranges, at which it has large values. These are just the resonances related to the "symmetrical" eigenstates of the isolated ring (Λ is approximately zero), which were discussed in Sec. IV. Note that the resonances are related to the eigenvalues of the isolated ring [see Fig. 3(b)]. The position of the resonances has a slight dependence on the magnetic flux as determined by Eq. (4.13). Then one could be tempted to conclude that the observed phases and oscillations are due to the movement of the resonances, as previous authors have pointed out.[14] However, for strong disordered systems this is not the case as one can see comparing Fig. 3(a) to Fig. 3(c). At the resonances there occurs a change in phase, but the inverse is not true. There exist changes of phase for energies where there are no resonances (nor eigenvalues of the isolated ring). Also, there is not an exact correlation between the infinitesimal movement of the resonance and the oscillation of T for a Fermi energy near to it.

In Figs. 4(a) and 4(b) we plotted $-\ln(T)$ as a function of energy for $\phi=0$ and $\phi=\phi_0/2$, respectively. Resonances appear as valleys while antiresonances correspond to the peaks. Note that the positions of the valleys are practically unaffected. The purpose of the logarithmic plot is to show the strong dependence of antiresonances on magnetic flux. If we had an antiresonance when $\phi=0$ ($\phi=\phi_0/2$) it disappears at $\phi=\phi_0/2$ ($\phi=0$). This influence gives us a reason to expect that the increment (decrement) of the magnitude of the effective coupling $|\tilde{V}_{AB}|$ with flux will cause $P=1$ ($P=-1$). With these ideas we calculated P using values of $|V_U+V_L|/|V_U-V_L|$ both larger and less than 1. A curve is obtained in complete agreement with Fig. 3(c). Therefore, the change from one phase to the other one is associated with a change from a situation in which $|V_U+V_L|>V_U-V_L|$ to the opposite one or vice versa. Since both V_U and V_L are real numbers, this requires that one of them modify its sign. This change in sign occurs exactly at the eigenvalues of the disordered chains, because

$$V_U=G^U_{1'N'}=\sum_\alpha \frac{a^\alpha_{1'}a^\alpha_{N'}}{E-E_\alpha}\ , \tag{6.1a}$$

$$V_L=G^L_{1''N''}=\sum_\beta \frac{a^\beta_{1''}a^\beta_{N''}}{E-E_\beta}\ , \tag{6.1b}$$

where E_α and E_β are the eigenvalues of the upper and lower branch, respectively. Given these results one can find a spectral interpretation for the energies at which the change of phase occurs: *We conclude that the changes in phase for $T(\phi)$ occur at the eigenvalues of the isolated branches.* Figures 3(d) and 3(e) show these exact coincidences. Therefore, if there are states with the same parity in both branches, just below the Fermi energy E, one has $P=-1$. On the contrary, for states of different parities one must assign $P=+1$. In the strongly disordered regime an asymmetry in the position of the contacts cannot introduce any new fact, since the relative parity is a random variable. This was verified numerically.

The above discussion implies that for almost every energy and flux the inequality $|\tilde{V}_{AB}|<\bar{V}^2$ holds. The reason lies on the strong localization of the branch eigenstates. While $\bar{V}^2$ is a number of order unity or larger, $|\tilde{V}_{AB}|\simeq\exp(-M/\lambda)\ll 1$. Only when E is very close to a branch eigenvalue (for instance, an upper value E_α), $|\tilde{V}_{AB}|^2$ increases as

$$V_U^2=(G^U_{1'N'})^2\simeq(a^\alpha_{1'})^2(a^\alpha_{N'})^2/(E-E_\alpha)^2\ .$$

But at this energy

$$\bar{V}^2=\tilde{E}_A\tilde{E}_B\simeq G^U_{1'1'}G^U_{N'N'}\simeq(a^\alpha_{1'})^2(a^\alpha_{N'})^2/(E-E_\alpha)^2\ .$$

Therefore, $|\tilde{V}_{AB}|^2\simeq\bar{V}^2$ and one has a double-peaked function for $T(\phi)$.

Note that the above considerations are based on the smallness of the amplitudes of the branch eigenstates at the contacts. Thus a similer description holds for an ordered ring with a highly reflective contact, for which this statement is valid. According to Eq. (3.7) this situation occurs if the Fermi energy lies near the band edge. However, E at the band center is more convenient for the correlation between the resonance maxima and eigenstates. Therefore, we maintained $E\simeq 0$ and simulated the strong scattering at the contacts by taking lower site en-

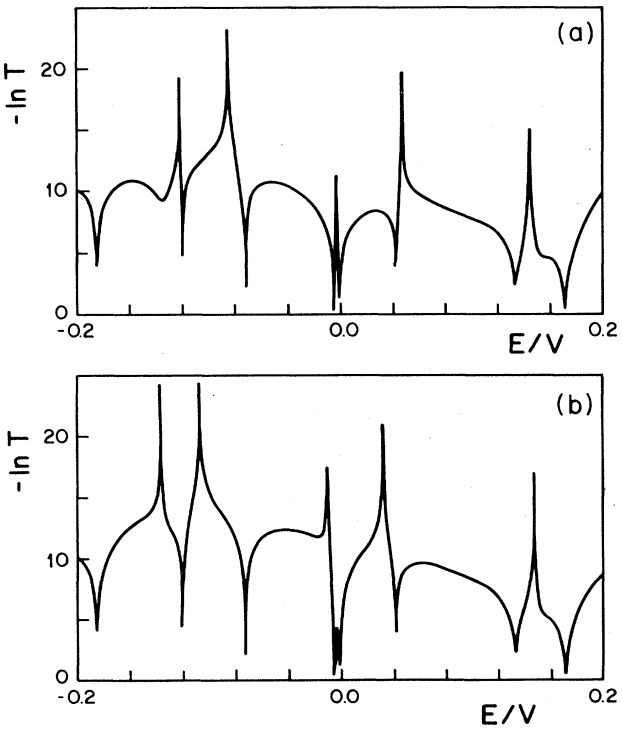

FIG. 4. Plot of $-\ln(T)$ as a function of energy for (a) $\phi=0$ and (b) $\phi=\phi_0/2$, respectively, with parameters of Fig. 3.

ergies at the contacts than on the rest of the ring. Figure 5 shows the extreme case of two ordered branches with $N'=99$, $N''=100$, and $E_A=E_B=E_{1'}=E_{1''}=E_{N'}=E_{N''}=-2$, with the other site energies equal strength. An interesting fact occurs in this regime. For very weak disorder the eigenvalues of each branch are approximately equal to zero. Our arguments, were completely confirmed. For almost every energy, one observes a phase $P=\pm 1$ [see Fig. 5(d)]. The change from one phase to the other occurs close to the eigenvalue of each branch of the ring [compare Fig. 5(d) with Figs. 5(e) and 5(f)]. This plot makes it evident that the positions of the resonances are related to the eigenvalues of the isolated ring, and that they are only weakly displaced by the magnetic flux. One can then attribute to this movement the double-peaked transmittance ($P=0$) near the eigenvalues, recovering the seminal ideas of Ref. 14. However, for most energies (where $P=-1$ and 1) the principal effect is the decrease (increase) of T due to the formation (collapse) of the antiresonant regions with the magnetic flux.

There remains to be considered the weak and intermediate disordered regimes. For the first case, the behavior of $T(\phi)$ is described by a double-peaked function for almost every energy. T takes on appreciable values and the amplitudes of the oscillations are large. As we already mentioned, $T(\phi)$ has a ϕ_0 and $\phi_0/2$ period of approximately equal strength. An interesting fact occurs in this regime. For very weak disorder the eigenvalues of each branch are approximately equal to those of the ordered case. Hence, correlation among energy positions and their relative parities is very important in determining the phase of the ϕ_0 oscillation. We will discuss the typical situations.

For equal branch lengths, say $N'=N''=99$, T is large for $\phi=0$ and small for $\phi=\phi=\phi_0/2$. The reason resides in the almost one-to-one correspondence between the equal-parity eigenvalues of both branches. Thus for $\phi=\phi_0/2$ there exist destructive interference for most energies. Then the ϕ_0 component of T has phase $\beta=0$ [see Fig. 2(c)]. On the other hand, different numbers of sites can invert this situation. For example, for $N'=99$ and $N''=101$ there is a one-to-one correspondence, but between eigenstates of different parities. Destructive interference now happens at $\phi=0$. Hence the phase of the ϕ_0 component is $\beta=\pi$ [see Fig. 2(d)]. A more symmetrical situation happens for $N'=99$ and $N''=100$. The eigenvalues of one of the branches are between the eigenvalues of the other one. Then, choosing random energies, one sometimes observes a double-peaked function as in Fig. 2(c) and sometimes as in Fig. 2(d).

The transmittance of a real many-channel ordered system may be conceived as a superposition of all the situations described above. As disorder turns up, the correlation between eigenstates is lost and one begins to see both types of single-peaked functions [see Figs. 2(a) and 2(b)], and the ϕ_0 component of the double-peaked curve has a random phase. The results of Mural *et al.*[12] are applicable to this case. Averaging over energy produced by the finite temperature leads to the subsistence of only the $\phi_0/2$ component. Finally, more disorder causes the double-peaked structure to disappear. T has a dominant ϕ_0 period and is small. This also happens for every disorder with highly reflective contacts. In that case, temperature will not cause the elimination of the ϕ_0 component, as Murat *et al.*[12] pointed out. The only effect would be to diminish the amplitude of the oscillations and to change the phase as the temperature is raised.

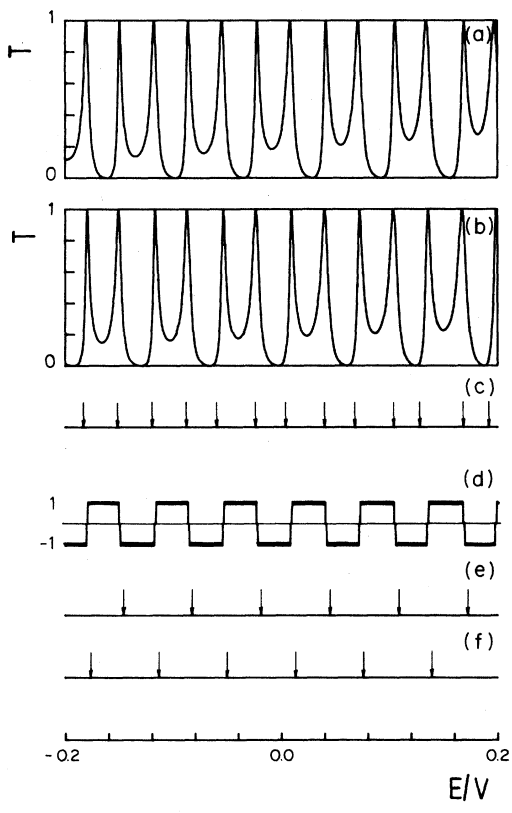

FIG. 5. Data for a ring with $N'=99$ and $N''=100$ with all sites energies equal to zero and $E_A=E_{1'}=E_{N'}=E_B=N_{N''}=E_{1''}=-2$. (a) and (b) Transmittance as a function of energy for $\phi=0$ and $\phi=\phi_0/2$, respectively. The scale of energy is shown at bottom. (c) Arrows indicate the eigenvalues of the isolated ring. (d) Plot of the P parameter as defined in the text. (e) and (f) Arrows indicate the eigenvalues of the isolated upper and lower branches of the ring, respectively. Observe that (d) correlates with (c) as well as with (e) and (f).

VII. DISCUSSION

The microscopic meaning of the Aharonov-Bohm effect was described using a simple model. This allowed us to clear up some basic questions, such as which are the parameters that control the dominant period and phase of the low-temperature magnetoresistance oscillations? The importance of antiresonant and resonant regions in the observed oscillations becomes clear in this work. Other notable aspects of our work are the consideration of branching effects and a formal extension of the tight-binding model of the resonant tunneling theory due to Azbel. In the case of a loop this theory was further extended in order to include antiresonances.

In the regime of intermediate and strong disorder, our model presents a transmittance whose main contribution has a ϕ_0 period, which is originated by the formation or collapse of the antiresonances. Their phase depends of the position of the eigenvalues of each branch relative to the Fermi energy. In this regime, the connection of the quantum transmittance with the finite-temperature magnetoresistance of metal rings was already clearly discussed by other authors.[12,13,15]

It is in the regime of weak disorder where a new ingredient shows up. According to the results of Sec. III, a branched circuit presents localized states in the region of the contacts. The main consequence of this is that contacts produce the backscattering of the incoming waves. This effect is weak for energies near the band center while it is very strong near the band edges. In the first situation both ring branches are almost perfectly coupled. Then, the magnetoresistance presents a strong $\phi_0/2$ period. This is originated on the displacement of the resonance maxima, associated with the eigenvalues of the ring, which crosses the Fermi energy two times as flux varies in the range $(0,\phi_0)$. This spectral argument, which applies also to cylinders,[28] is consistent with the traditional idea that the $\phi_0/2$ period is due to the interference of electronic paths of opposite directions that go around the annulus. The amplitude of this oscillation is of the same order as that corresponding to the ϕ_0 period, provided that the branches are almost perfectly coupled. This condition breaks down when the contact represents a strong scattering center. We then come to the second situation, where backscattering at the contacts occurs when the Fermi energy lies near the band edge. We have shown both analytically and numerically that in this regime it appears to be a dominant ϕ_0 component associated, as in the case of strong disorder, with the formation or collapse of the antiresonances. We believe that this situation could have its experimental correlate in the observed dominant ϕ_0 period of the magnetoresistance of high-mobility rings.[29] These circuits are made using a quantum-well heterostructure of $GaAs/GaAs_{1-x}Al_x$. Since the number of occupied channels (or transverse subbands) is very low, the Fermi energy lies near the band edge and the above condition applies. In this case the localized states can be thought of as having originated as a consequence of an increase of the well size (effective dimensionality[23]) in the neighborhood of the contact. Notice that the branching effect should not be as critical for metals because they have a Fermi energy far away from the band edge.

The appeal of our tight-binding model is that, in spite of being simple, it allows for the retention of much of the physics of the Aharonov-Bohm effect in solid-state devices. Therefore, it can be used in the quantitative discussion of predictive value in different experimental conditions.

ACKNOWLEDGMENTS

H.M.P. acknowledges discussion with Professor M. Brodsky in the early stages of this work, as well as the correspondence of Dr. Washburn.

[1]B. L. Al'tshuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. 33, 101 (1981) [JETP Lett. 33, 94 (1981)].
[2]D. Yu Sharvin and Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. 34, 285 (1981). [JETP Lett. **34**, 273 (1981)].
[3]B. L. Al'tshuler, A. G. Aronov, B. Z. Spivak, D. Yu Sharvin, and Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 476 (1982) [JETP Lett. **35**, 589 (1982)].
[4]M. Gijs, C. Van Haesendonck, and Y. Bruynseraede, Phys. Rev. Lett. **52**, 2069 (1984).
[5]B. Pannetier, J. Chausay, R. Rammal, and P. Gandit, Phys. Rev. Lett. **53**, 718 (1984).
[6]R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985); S. Washburn, C. P. Umbach, R. B. Laibowitz, and R. A. Webb, Phys. Rev. B **32**, 4789 (1985).
[7]V. Chandrasekhar, M. J. Rooks, S. Wind, and D. E. Prober, Phys. Rev. Lett. **55**, 1610 (1985).
[8]R. Landauer, Philos. Mag. **21**, 863 (1970).
[9]Y. Gefen, Y. Imry, and M. Ya Azbel, Phys. Rev. Lett. **52**, 129 (1984).
[10]Y. Gefen, Y. Imry, and M. Ya Azbel, Surf. Sci. **142**, 203 (1984).
[11]A. D. Benoit, S. Washburn, C. P. Umbach, R. B. Laibowitz, and R. A. Webb, Phys. Rev. Lett. **57**, 1765 (1986).
[12]M. Murat, Y. Gefen, and Y. Imry, Phys. Rev. B **34**, 659 (1986).
[13]M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).
[14]M. Büttiker, Y. Imry, and M. Ya Azbel, Phys. Rev. A **30**, 1982 (1984).
[15]A. D. Stone, and Y. Imry, Phys. Lett. **56**, 189 (1986).
[16]Q. Li and C. M. Soukoulis, Phys. Rev. Lett. **57**, 3105 (1986).
[17]J. Jose, in *Proceedings of the XIXth Latin American School of Physics, Cali, Columbia, 1982,* edited by A. Rueda (World Scientific, Singapore, 1983).
[18]E. Domany, S. Alexander, D. Bensimon, and L. P. Kadanoff, Phys. Rev. B **28**, 3110 (1983).
[19]C. Wiecko and E. Roman, Phys. Rev. B **30**, 1603 (1984).
[20]H. M. Pastawski, C. Slutsky, and J. F. Weisz, Phys. Rev. B **32**, 3642 (1985).
[21]P. Levstein, H. M. Pastawski, and J. L. D'Amato (unpublished).
[22]A. D. Stone, J. D. Joannopoulos, and D. J. Chadi, Phys. Rev. B **24**, 5583 (1981).
[23]H. M. Pastawski and C. Wiecko, Phys. Rev. A **36**, 5854 (1987).
[24]F. Guinea and J. A. Verges, Phys. Rev. B **35**, 979 (1987).
[25]M. Ya Azbel and P. Soven, Phys. Rev. B **27**, 831 (1983).
[26]M. Ya Azbel, Solid State Commun. **45**, 527 (1983).
[27]M. Ya Azbel, Phys. Rev. B **28**, 4106 (1983).
[28]H. M. Pastawski, A. Rojo, and C. A. Balseiro, Phys. Rev. B **37**, 6246 (1988).
[29]G. Timp, A. M. Chang, J. E. Cunningham, T. Y. Chang, P. Mankiewich, R. Behringer, and R. E. Howard, Phys. Rev. Lett. **58**, 2814 (1987).