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Quantum oscillations in the point-contact magnetoresistance

H. M. Swartjes, A. P. van Gelder,* A. G. M. Jansen, and P. Wyder
Hochfeld-Magnetlabor, Max-Planck-Institut für Festkörperforschung,
25 avenue des Martyrs, Boîte Postale 166X, F-38042 Grenoble CEDEX, France
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The resistance of metallic point contacts at a temperature of 4.2 K shows oscillations as a function of magnetic field. These oscillations are due to the Landau quantization of the conduction electrons as is proven by their frequencies. The oscillations can originate both in the Maxwell and in the Sharvin part of the point-contact resistance. The oscillations in the Maxwell resistance are a direct result of the oscillations in the bulk resistivity of the material (the Shubnikhov–de Haas effect). For a proper understanding of the oscillations in the Sharvin resistance, it is necessary to take diffraction effects of the electron wave functions into account.

I. INTRODUCTION

Metallic microcontacts offer the possibility to study the energy dependence of the electronic scattering processes in a metal by measuring the voltage dependence of the nonlinearity in the current-voltage characteristics at low temperatures.[1,2] An important criterium for the spectroscopic application of point contacts is given by the condition that the mean free path of the electrons is large compared with the dimension of the contact (Sharvin limit). For such a situation, an applied voltage over the contact accelerates the electrons within a mean free path upon passing the contact area. This phenomenon results in a nonequilibrium distribution of the electronic system, where the applied voltage defines the excess kinetic energy of the electrons. Inelastic scattering of the electrons yields corrections to the current which contain spectroscopic information about the interaction mechanisms of the electrons with excitations in a metal. For instance, the second derivative of the voltage with respect to the current is directly proportional to the Eliashberg function for the electron-phonon interaction. In the Sharvin limit the expression for the contact resistance is independent of the electron mean free path and is only determined by the topology of the Fermi surface and the contact area. In the opposite limit (Maxwell limit with mean free path smaller than the contact dimension), the contact resistance depends on the electronic scattering and is proportional to the bulk resistivity.

In the spectroscopic point-contact technique magnetic fields are used occasionally for several purposes. In the first place, material properties sometimes change with the magnetic field, and when these properties have influence on the scattering of electrons, their dependence on the magnetic field can be studied by means of point contacts. In the second place, magnetic fields suppress superconductivity, and in this way it is possible to apply point-contact spectroscopy on superconducting systems that are otherwise inaccessible for a study with point contacts. In the latter case it is tacitly assumed, that the magnetic field has no influence on the point-contact resistance other than the suppression of superconductivity.

Apart from changes in the material properties several other effects occur when a magnetic field is applied, that are inherent to the point-contact geometry and are in principle always to be reckoned with. One of these effects is a classical term in the magnetoresistance of a point contact. This term has it's origin in a resistivity-dependent component of the point-contact resistance.[3] For metals with a large bulk magnetoresistance, the contact resistance contains an additional term, roughly given by the square resistance of the macroscopic sample in the case of a point contact on a flat metallic disk. Moreover, the magnetoresistance of a contact can be enhanced because of the specific Corbino geometry.[3]

A second observed influence of the magnetic field is the presence of an oscillating term in the point-contact resistance. This term, that is also inherent to the geometry of the contact, is the subject of this paper. The origin of these magnetooscillations in the point-contact resistance lies in the Landau quantization of the electrons, as is immediately clear from the frequencies of these oscillations. Via different physical mechanisms this quantization influences both of the two parts in the expression for the resistance of a metallic contact, the "Maxwell" part which depends on the scattering of the electrons and the "Sharvin" part which is independent of the electronic scattering. In this paper we will investigate both these mechanisms, and we will discuss the influence of several parameters on the observed oscillations. The discussion will be based on experiments with point contacts on bismuth, gallium, and aluminum. These materials show a wide range of resistivities and emphasize the different mechanisms that are responsible for the oscillations.

II. THEORY

When a metal is placed in a magnetic field, a redistribution occurs of the conduction electrons over the Fermi sphere. This redistribution is a very general phenomenon and is a direct consequence of the quantization of the electron orbits. For the closed orbit of an electron in a magnetic field, the enclosed magnetic flux is quantized in units of h/e. As a result, the allowed electron states in **k** space now lie on so-called Landau tubes, with a cross section area perpendicular to the magnetic field given by the Onsager formula[4]

$$A_j=(j+\tfrac{1}{2})(2\pi eB/\hbar) \ . \tag{1}$$

The influence of this quantization is particularly manifest whenever a cross section A_j is equal to an extremal area A_{ext} of the Fermi surface. As can be seen from Eq. (1) this happens periodically in $1/B$, with the period $\Delta(1/B)$ given by

$$\Delta(1/B)=2\pi e/\hbar A_{\text{ext}} \ . \tag{2}$$

At an extremal orbit the occupation of the Landau tube j changes rapidly with magnetic field and this change is reflected in all properties that depend on the electron distribution. Among numerous examples we cite the de Haas–van Alphen (dHvA) effect,[5] i.e., the oscillating magnetic susceptibility, which is the best known and from a practical point of view the most versatile, and the Shubnikhov–de Haas (SdH) effect,[6] the oscillating resistivity, which is particularly relevant in the context of the point-contact resistance. Numerous reviews and books have appeared on these effects.[7]

Although the magnetic oscillations in the electrical resistivity were among the first to be discovered experimentally, their theoretical understanding is of much later date and is very complex.[8,9] A very simplified, but transparent model of the Shubnikhov–de Haas effect has been given by Pippard.[10] It can be understood as the oscillation of the electron relaxation time τ. This relaxation time depends on the number of occupied electron states that an electron can scatter from and the number of empty states that an electron can scatter to, and hence it depends on the density of states at the Fermi surface. When because of the Landau quantization this density of states oscillates, the relaxation time will oscillate. Because the resistivity of a metal in the Drude approximation is proportional to τ^{-1}, it will also oscillate as a function of magnetic field.

In an interpolation model the resistance of a point contact can be thought of as a series resistance of a ballistic component, the Sharvin resistance $R_{\text{Sh}}=4\rho l/3\pi a^2$, and a diffusive component, the Maxwell resistance $R_M=\rho/2a$. The two components are the limiting cases for a pure contact in the Sharvin limit (mean free path $l \gg$ contact radius a) and a dirty contact in the Maxwell limit ($l \ll a$). The diffusive part of the resistance is directly proportional to the bulk resistivity ρ and therefore the Maxwell term will oscillate with magnetic field in the same way as the bulk resistivity does. Whether or not these oscillations are observable depends on various factors. The oscillation amplitude $\delta\rho$ for the resistivity ρ is approximately given by[10]

$$\frac{\delta\rho}{\rho}\approx\frac{\delta D(\varepsilon_F)}{D(\varepsilon_F)} \ , \tag{3}$$

with ε_F the Fermi energy and $D(\varepsilon_F)$ the density of states at the Fermi surface. For a Fermi surface, consisting of one spherical piece, with a quadratic dispersion relation, this yields

$$\frac{\delta\rho}{\rho}\approx q^{-1/2} \ , \tag{4}$$

where $q=\varepsilon_F/\hbar\omega_c$ with $\omega_c=eB/m$ the cyclotron frequency. The integer part of q is equal to the number of Landau tubes that intersect the Fermi surface. Small pockets of electrons would, therefore, have the largest relative oscillation amplitude. In practice, however, such pockets carry only a small part of the current, except for semimetals such as bismuth and poorly conducting metals such as gallium. The amplitude of the oscillations is further weakened by collision broadening of the Landau levels and by a smearing of the Fermi function because of nonzero temperature. For common metals oscillation amplitudes can therefore be expected of no more than one part in 10^4 under typical conditions. In point contacts the observation of oscillations in the contact resistance is further hampered by the fact that the Maxwell part of the resistance is only a small part of the total resistance. In high magnetic fields the expression for the Maxwell part of the contact resistance has to be modified to approximately the square resistance ρ/d, where d is the macroscopic thickness of the sample on which a point contact is placed.[3] In the case of a strong magnetoresistivity the square resistance becomes the dominant contribution to the contact resistance in high magnetic fields. As a rule of thumb we can therefore state that no oscillations in the Maxwell resistance can be observed when they are not observable in the bulk resistance. In practice the observability of Landau quantization in point contacts will be hampered even more because of the mechanical instability that is inherent to point contacts.

For the ballistic part of the point contact resistance, the situation is more complex. At first sight it would appear that there is no oscillation effect, since in the expression for the Sharvin resistance $R_{\text{Sh}}=4\rho l/3\pi a^2$ the product ρl is independent of the scattering length l. This implies that the oscillations in the relaxation time will not lead to oscillations in the Sharvin resistance.

The derivation of the Sharvin expression,[11] however, is only correct in the absence of a magnetic field. For an applied voltage V over the contact the current density J at the orifice is found by integrating the velocity component v_z perpendicular to the contact over all occupied k-states within a distorted Fermi sphere S consisting of two halfspheres with energy difference eV,[12,13]

$$J=2e\left[\frac{1}{2\pi}\right]^3\int_S d^3k\, v_z \ . \tag{5}$$

In the presence of a magnetic field, perpendicular to the contact, the integral over k_x and k_y has to be replaced by a sum over the different Landau tubes,

$$J=2e\left[\frac{1}{2\pi}\right]^3\sum_{j=1}^{[q]}N(B)\int dk_z\, v_z \ , \tag{6}$$

where $N(B)=2\pi eB/\hbar$ is the field-dependent degeneracy factor and $[q]$ is the *entier* function of q. The integral over k_z is independent of magnetic field and given by eV/$\hbar$. Since the extremal area of the Fermi surface is given by $A_{\text{ext}}=(q+1/2)N(B)$, we see that the current, and hence the observed Sharvin resistance, oscillates with a relative amplitude

$$\frac{\delta R}{R} \approx q^{-1} . \quad (7)$$

The relative magnitude is smaller than that of the Shubnikhov–de Haas effect because the electrons on extremal orbits, which in principle are responsible for the oscillations, have a velocity in the z direction $v_z \approx 0$. Oscillations arising solely from this source will therefore be weak although the Sharvin term is usually the major part of the point-contact resistance.

The oscillation amplitude can be considerably enhanced because of diffraction in the point-contact area. In the absence of a magnetic field, no diffraction has to be taken into account, because the size of a typical point contact is large, compared to the de Broglie wavelength of an electron. The electrons can therefore be considered as well-defined particles, following well defined trajectories, and the electron transport through the contact can be treated semiclassically. When on the other hand a magnetic field is applied, the "fuzziness" of the electron is no longer determined by the de Broglie wavelength, but by the magnetic length $\Lambda = (2\hbar/eB)^{1/2}$, the size of the inner Landau tube. For a field of a few tesla this length is of the order of 100 to 200 Å, i.e., comparable to typical point-contact sizes.

A formal treatment of the diffraction problem will be given elsewhere.[14] The main result of diffraction is that an electron on the Fermi surface can no longer be given both a specific velocity component v_z and a specific Landau number j. As a consequence, Eq. (6) is no longer valid and has to be replaced by

$$J = 2e \left[\frac{1}{2\pi}\right]^3 N(B) \sum_j \sum_{j'} \int dk_{z,j}\, v_{z,j'} \Delta_\gamma(j,j') . \quad (8)$$

Here $\Delta_\gamma(j,j')$ is a function that describes the coupling between $k_{z,j}$ and $v_{z,j'}$ for a given value of the parameter γ, which is the size of the point contact in units of the magnetic length Λ. For $\gamma \gg 1$, $\Delta_\gamma(j,j')$ is equal to the Kronecker delta $\delta_{jj'}$ and the result of Eq. (6) is found. For $\gamma \approx 0$, however, $\Delta_\gamma(j,j')$ is independent of j and j' and the sums over j and j' are completely decoupled. Since the electrons on extremal orbits are now no longer weighted by v_z, the current is again proportional to the density of states, which for the case of a spherical Fermi surface gives an oscillation amplitude $\delta R/R \approx q^{-1/2}$ as in the case of the Shubnikhov–de Haas effect. In this limit there is an analogy between the Shubnikhov–de Haas effect and the oscillations of the point-contact resistance. Instead of an ensemble of randomly distributed isotropic scatter centers, we now have one isotropic center, but the simplified picture of Pippard still holds, where the current is determined by the number of occupied states that an electron can come from and the number of empty states that an electron can be diffracted to.

III. EXPERIMENTAL DETAILS

The experiments, described in this paper, were performed with a standard point-contact setup,[2] and resistances were measured phase sensitively. The magnetic fields were provided either by a superconducting coil, capable of producing 11 T at 4.2 K, or by a 20 T Bitter magnet. The measurements were done in the temperature range of 1.5 to 4.2 K. Except where mentioned otherwise, the magnetic field was always oriented parallel to the nominal current direction.

Three different materials were used for the experiments, namely, bismuth, gallium, and aluminum. The bismuth sample was a single crystal, of $5N+$ purity. It was of circular shape with a thickness of 1.8 mm and a diameter of 7.5 mm. The trigonal crystal axis was orientated perpendicular to the sample plane. This sample was etched in a mixture of 50% concentrated nitric acid and 50% concentrated acetic acid. Contacts were made on this sample with 49 °C solder.

The gallium samples were less well defined. They were made of high-purity material by melting above 30 °C. The melted drops had a size of a few millimeters. This drop was "etched" by scraping the surface with a scalpel. Thin copper wires were then injected in the drop to provide the electrical contacts and the sample was allowed to solidify again. Because of the low melting point gallium easily anneals at room temperature and a single crystal is obtained.

Several aluminum samples were used of various shapes and sizes. Most experiments were done on small single crystals of known orientation, which were more or less plane, with a thickness of 0.5 to 1 mm and a typical width and length of 3 mm. For Al-Al point contacts wedge-shaped samples were pressed together with known crystalline orientation.[15] The experiments on polycrystalline samples were done with small pieces of aluminum of irregular shape, that were cut from a larger sheet of material. The aluminum samples were etched with the following process.[15] First they were immersed in a mixture of 50% phosphoric acid, 40% sulfuric acid, and 10% nitric acid. After that the samples were transferred to a heated solution of 25% sulfuric acid. Finally, the samples were cleaned and kept in propanol. The electric contacts on the samples were first made with silver paint. Because of the oxide layer on the aluminum, this often resulted in unsatisfactory contacts of rather high resistance. Later, therefore, a method was used, developed by Bruls,[16] by attaching very tiny particles of 49 °C solder on the samples, in a very diluted etching solution, that was warmed to more than 50 °C.

Apart from the aluminum-aluminum contacts, we always used a copper whisker, electrolitically etched to a sharp point, as second electrode. This was necessary, since for contacts between identical materials the orientation of both crystals had to be identical. Even with an insert with an orientable head to control crystal orientation, this remained an unsolved problem. The advantage of copper is that it does not contribute an oscillation signal to the point-contact resistance. The conduction electrons in copper have too high an effective mass and too large Landau numbers to give an observable signal, as will be discussed later.

IV. OSCILLATIONS IN THE MAXWELL RESISTANCE

The semimetal bismuth is known to show very strong Shubnikov–de Haas oscillations, and was the material in which the effect was first discovered.[6] In our experiments the bismuth sample was usually oriented with the magnetic field parallel to the trigonal crystal axis and parallel to the nominal current direction. A typical measurement of the point-contact resistance in this orientation, and at a temperature of 4.2 K is given in Fig. 1. The zero-field resistance of this point contact was 33 Ω. On the basis of the Sharvin expression of Eq. (5) the radius of the contact is $a = 1500$ Å, where we have taken $\rho l \approx 10^{-12}$ $\Omega\,m^2$, as estimated from the size of the Fermi surface. With the value for the resistivity of this sample, $\rho_0 = 13$ nΩ m, the Maxwell contribution $R_M = \rho/2a$ is 40 mΩ, so that the contact is well within the Sharvin limit. Two distinctive features of the measurement of Fig. 1 are immediately obvious. First, a continuously increasing term in the magnetoresistance is observed, which at high fields forms a considerable part of the total resistance. This magnetoresistance term has its origin in the modified Maxwell component of the resistance,[3] which depends on the resistivity and resembles the square resistance ρ/d of the sample. Secondly, large oscillations are visible in the measurement of Fig. 1. That these oscillations are caused by Landau quantization can be seen by comparing the peak positions with those of bulk resistivity measurements,[17,18] which have been tabulated in Table I. In principle, four distinct series are observed with the magnetic field parallel to the trigonal axis. Two of these series are associated with the hole band and two with the electron band, which are both split because of the electron spin. When we compare the values in Table I with the measurements of Fig. 1, it appears that the up-spin electrons give the dominant contribution to the signal, whereas the down-spin electrons are responsible for most of the other structure. The oscillations of Fig. 1 are fairly accurately periodic in $1/B$. Small departures of this behavior can occur at fields above approximately 5 T. Above this field value the Fermi energy begins to rise[19] because the extreme quantum limit is reached, where all electrons are in the lowest Landau levels.

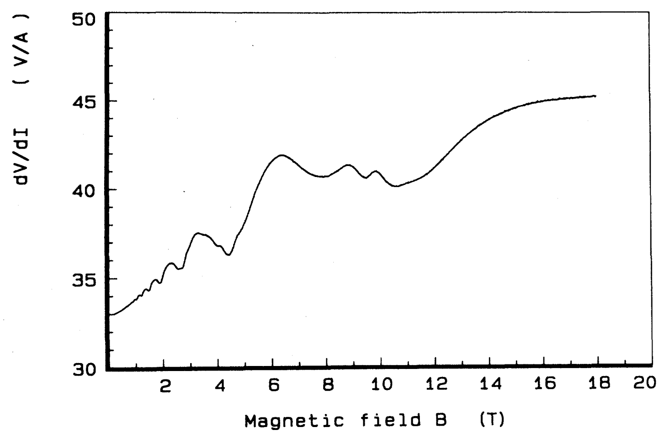

FIG. 1. The resistance dV/dI of a bismuth-copper point contact as a function of magnetic field, for zero bias voltage and at a temperature of 1.8 K. The magnetic field was oriented parallel to the trigonal crystal axis and perpendicular to the plane of the bismuth sample. At zero magnetic field, the resistance of the point contact was 33 Ω.

That the oscillations in the point-contact resistance are a manifestation of the Shubnikov–de Haas effect is seen when the measurement of the point contact is compared with that of a bulk resistivity measurement. Figure 2 shows such a measurement, taken on the same day and under the same circumstances as that of Fig. 1. The similarity between the two measurements is clear, in spite of the fact that the oscillations in the point-contact resistance are sharper. For the case of bismuth the classical magnetoresistance of a point contact is directly proportional to the bulk resistivity.[3] Therefore the point-contact oscillations must also result from resistivity oscillations. The difference in sharpness between the bulk and the point-contact measurements must be ascribed to the rigid sample mounting, necessary for stable contacts. The strain that is hereby introduced is much more likely to affect the bulk measurements than the point contact, because the major part of the point-contact resistance is concentrated in the direct vicinity of the contact.

Shubnikov–de Haas oscillations in the point-contact

TABLE I. Values of the magnetic field, for which the Fermi surface of bismuth touches the Landau tube with level number q, where the magnetic field is taken parallel to the trigonal crystal axis. Values are given for the electron and hole bands series, each of which shows a large spin splitting. The series, corresponding to spin-up and spin-down, have been labeled as $s = +$ and $-$, respectively. The data have been taken from Ref. 17.

		Electrons		Holes	
q	$s=$	$-$	$+$	$-$	$+$
0					3.95 T
1			5.85 T		2.41 T
2		4.09 T	3.38 T	4.33 T	1.766 T
3		2.817 T	2.406 T	2.60 T	
4		2.146 T	1.896 T	1.83 T	
5		1.724 T	1.566 T		

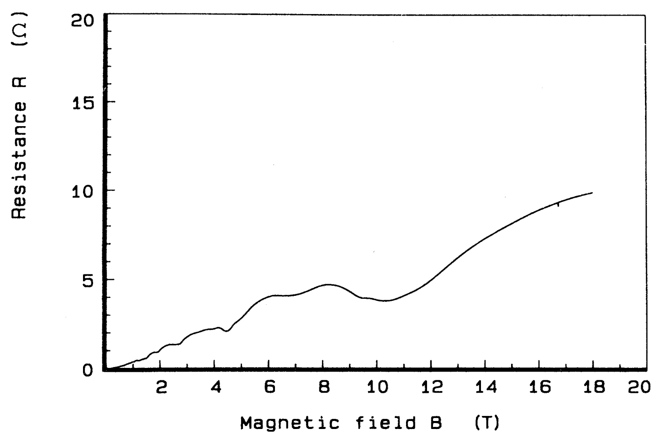

FIG. 2. Bulk resistance vs magnetic field, for the same bismuth single crystal and under the same circumstances as Fig. 1. At zero magnetic field, the measured resistance was 4.85 $\mu\Omega$.

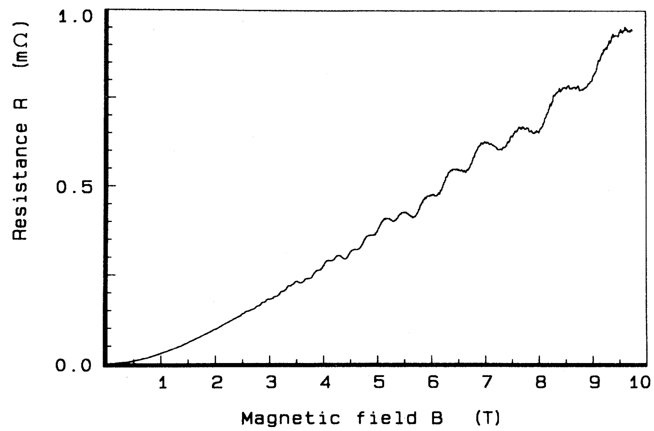

FIG. 4. Bulk resistance vs magnetic field, for the same gallium single crystal and under the same circumstances as Fig. 3.

resistance are not limited to semimetals like bismuth. This is illustrated in the measurements on gallium, that are presented in Figs. 3 and 4 for a point-contact resistance and a bulk resistance, respectively. These figures were also taken both under the same conditions and on the same day and are therefore directly comparable. Gallium is a metal in which Shubnikhov–de Haas oscillations have been demonstrated in the past.[20] It is a compensated metal and thus has a large, nonsaturating quadratic magnetoresistance as is shown in the bulk measurement of Fig. 4. This magnetoresistance term is also observed in the point-contact measurement of Fig. 3. The sharp rise in resistance below 3 teslas (only partly drawn) is no ordinary magnetoresistance, but arises because of the suppression of superconductivity of gallium. The observation of the superconducting β phase ($T_c = 6$ K) of gallium is quite common in point contacts.[21] In general,

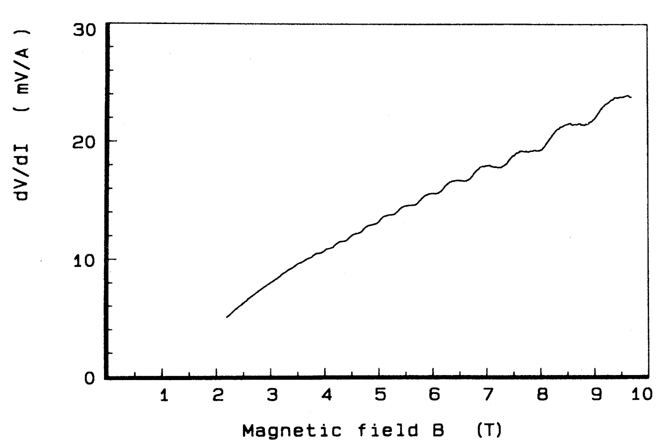

FIG. 3. The resistance dV/dI of a gallium-copper point contact as a function of magnetic field, for zero bias voltage and at a temperature of 1.8 K. The orientation of the crystal itself was unknown. At zero magnetic field, the point-contact resistance was 0.52 Ω.

in order to suppress superconductivity in point-contact experiments, magnetic fields are required that are much higher than the critical fields of the bulk materials. In the work of Shklyarevskii *et al.*[21] a field of 3 T was used, which in Fig. 3 also is the field where the suppression can be considered complete.

The observed ratio between the point-contact magnetoresistance term and that of the bulk measurement is approximately 15. This is much higher than the ratio of approximately 2 that we found for bismuth. The discrepancy between these two values must be ascribed to the irregular size of the gallium sample, which in no way resembled a disc of uniform thickness, and to the method of contact preparation by inserting copper wires rather deeply in the material in its liquid phase, which is not a good method for reliable resistivity measurements.

The frequencies that are observed in de Haas–van Alphen experiments of gallium fall in several groups, that cover almost the complete frequency range between 50 and 5000 T.[22] It is therefore impossible to give a definite assignment for the dominant oscillation that is present in Figs. 3 and 4 with a frequency of 75 T. An important point is that the relative magnitude of the oscillations in the point-contact measurements, with respect to it's continuously rising magnetoresistance term, is equal to that of the bulk measurements, which justifies the conclusion that the observed oscillations in the gallium point contact are caused by the Shubnikhov–de Haas effect.

V. OSCILLATIONS IN THE SHARVIN RESISTANCE

The third material that was used in the experiments was aluminum. Contrary to bismuth and gallium no reliable observation of the Shubnikhov–de Haas effect in this metal is known. Large oscillations in the resistivity have been observed in certain symmetry directions,[23] but these are caused by magnetic breakdown and not by the Shubnikhov–de Haas effect. Through the de Haas–van Alphen effect the Fermi surface of aluminum is known very well.[24,25] The second Brillouin zone of aluminum is almost completely filled and gives rise to high-frequency oscillations, of approximately 40 000 T. The third zone contains small, tubular pockets of electrons, where the tubes lie on the sides of a small square. There are three nonequivalent pieces, with the normal to the square along each of the (100) axes of the crystal, and the sides along a (110) axis. The fourth zone is empty. For two symmetry directions the frequencies corresponding to the third zone pockets are listed in Table II, together with their effective masses.

Because of the nearly-free-electron character of aluminum, we will use the free electron value for the parameter ρl, i.e., $\rho l = 3.99\times10^{-16}$ $\Omega\,m^2$. Our samples typically had a residual resistance ratio of 10 000, and hence a mean free path can be estimated of the order of 150 μm, and a resistivity of approximately 2.5 pΩ m at low temperatures. Aluminum is not compensated and only a small, linear magnetoresistance term is usually observed in the bulk resistivity. Because of the Corbino geometry in point contacts an enhanced magnetoresistance is observed[3] proportional to the bulk magnetoresistivity. In Fig. 5 a point-contact measurement is shown for an aluminum-aluminum contact of 0.7 Ω, which was measured at a temperature of 1.5 K. Here the classical magnetoresistance term has a magnitude of a few mΩ.

Also present in Fig. 5 is a relatively large oscillation signal. When this signal is plotted against $1/B$ and the Fourier transform is consecutively taken, Fig. 6 is obtained. In this figure, as in the other Fourier transforms in this paper, a linear term was usually subtracted from the measured resistance in order to correct for the magnetoresistance term, and an antialiasing function was applied, but no other signal processing prior to the transform was performed. Upon comparing the peak positions in Fig. 6 with the known de Haas–van Alphen frequencies from Table II, it is clear that the origin of the signal must lie in the Landau quantization. Both the low-frequency α and β oscillation are observed, as are the somewhat higher γ frequencies. (The observation of multiple peaks between 500 and 600 T can be ascribed to misalignment, since these oscillations are multiply degenerate and the point contact consists of two crystals, which may not be perfectly aligned.) The Shubnikhov–de Haas effect as a possible cause of the oscillations can be ruled out by giving an order of magnitude estimate. The dominant frequency in Fig. 5 is 289 T, arising from one of the γ orbits. At a magnetic field of 7 T the number of Landau levels that are present in the corresponding branch of Fermi surface is therefore $q=41$. The relative contribution of these pockets of electrons to the total current can not easily be calculated, since they are not very ellipsoidal. A rough estimate can however be given by comparing the extremal areas of these pieces with that of the main, second zone part of the Fermi surface. The ratio between these extremal areas is the same as that of the corresponding de Haas–van Alphen frequencies, which are 289 T for the γ piece and 43 150 T for the second zone ψ piece.[25] Since there are two equivalent γ pieces, their contribution to the total current is approximately 1.5%. With a dependence of the amplitude on q given by Eq. (4) we can expect a Shubnikhov-de Haas effect of 2.3×10^{-3} of the classical magnetoresistance term or only approximately 10 $\mu\Omega$, even when no damping of the signal because of finite temperature T or finite relaxation time τ is taken into account.

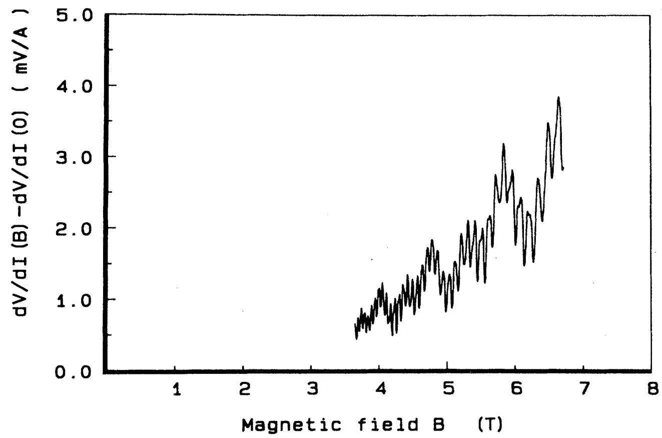

FIG. 5. $dV/dI(B)-dV/dI(0)$ of an aluminum-aluminum point contact as a function of magnetic field, for zero bias voltage and at a temperature of 1.8 K. The magnetic field was oriented parallel to a (110) axis and perpendicular to the nominal plane of the contact. At zero magnetic field the resistance was 0.70 Ω.

Another possible source of resistivity oscillations is magnetic breakdown. This is known to cause large signals in aluminum,[23] when the magnetic field is oriented along certain symmetry directions. The oscillations that are observed in point contacts, however, are not limited

TABLE II. The observed de Haas–van Alphen frequencies F for the third zone of aluminum, for the magnetic field along two symmetry directions, together with their effective masses m^*. The values for the frequencies have been taken from Ref. 25 and for the effective masses from Ref. 24. The β mass in the ⟨110⟩ direction is mentioned in Ref. 24 as unreliable, because of difficulties in separating the α and β oscillations.

100	F (T)	m^*/m_0	110	F (T)	m^*/m_0
α	28.2	0.091	α	26.1	
β	46.8	0.102	β	50.7	0.119(?)
γ	391.4	0.180	γ	289.4	0.130
			γ	508	0.227

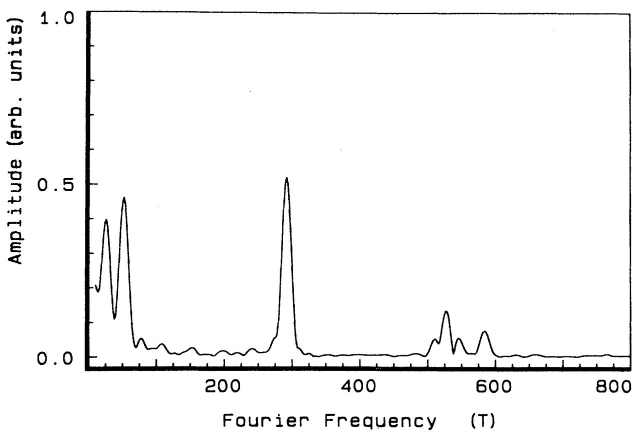

FIG. 6. Fourier transform of Fig. 5, when plotted against $1/B$, showing the frequencies from electrons in the third zone of aluminum.

to these symmetry directions. Furthermore, all frequencies that can be expected for the third Brillouin zone appear in the resistance, not just those that are found from magnetic breakdown, and hence this can also be ruled out as a possible cause.

On the basis of the amplitude of the observed signal it must be concluded that the total point-contact resistance is responsible for the oscillations, i.e., it's main source is found in the Sharvin resistance component. The amplitude of the 289 T γ oscillations is 0.5 mΩ at 7 T, or 7×10^{-4} of the total point-contact resistance. An estimate, based on the semiclassical relation of Eq. (8), gives an expected amplitude of 0.25 mΩ. Here we again have assumed that the contribution of the γ electrons to the total current is given by the ratio between the de Haas–van Alphen frequencies. Since damping effects diminish the signal by roughly a factor of 3 to 5, as will be seen later, the observed amplitude is still too large to be accounted for by the pure semiclassical model. In order to fully explain the signal it is necessary to assume some enhancement because of diffraction effects. That this must indeed be expected is seen by comparing the magnetic length $\Lambda=(2\hbar/eB)^{1/2}$ with the contact size, which can be estimated with Eq. (5). With $\rho l=3.99\times10^{-16}$ $\Omega\,m^2$ and a resistance of 0.7 Ω we find a contact radius $a=150$ Å, whereas $\Lambda=135$ Å at a field of 7 T. Although the contact is not fully in the quantum limit, the resulting spread in wave functions is sufficient to explain the observed amplitude, and the observed variation of the amplitude with magnetic field will be somewhere between a q^{-1} and a $q^{-1/2}$ law.

The contact of Fig. 5 shows an oscillation signal that is of more than average amplitude, although still larger amplitudes have been obtained in other point contacts. Over a large number of contacts, measured under comparable circumstances, the magnitude of the oscillations varies over more than an order of magnitude, with no apparent relation to experimental parameters as, e.g., the contact resistance. The best cases that have been observed showed an amplitude at 7 T for the 289-T γ orbit of 1 part in 10^3 of the total resistance. For the contacts with low amplitude the observability was usually limited by noise because of mechanical instability, which was typically 10^{-4} to 10^{-5} of the total signal. Several causes are plausible for this large spread in amplitude. One of the main problems with aluminum point contacts is the oxide layer that covers the aluminum. As a result of this layer, the point contact may in some cases not be purely metallic but possibly also has a tunneling character. In that case, the contact dimensions will actually be larger than estimated on the basis of the Sharvin expression of Eq. (5), and hence diffraction effects will be less important. A further complication that has been neglected in our diffraction theory is the shape of the contact which we for simplicity assumed circular. A real point contact will likely show a more irregular shape and, e.g., a slit-shaped contact may show a larger diffraction effect.

Another problem concerns the orientation of the point contact itself. The plane of the contact was assumed to be perpendicular to the applied magnetic field. In reality, though, the contact is of microscopic size and it's actual orientation is determined for a large part by irregularities on the surface of the samples and could be different from the perpendicular one. The magnetic length Λ only sets the length scale for the direction perpendicular to the magnetic field, whereas in the parallel direction the de Broglie wavelength is still the determining factor. Thus, the actual importance of diffraction may differ for both orientations. Experiments, that were performed with the field parallel to the nominal plane of contact showed no conclusive evidence. The resistance curves as a function of the magnetic field for this orientation were both qualitatively and quantitatively similar to those for the perpendicular orientation, and showed the same spread in oscillation amplitudes, but these contacts suffer from the same uncertainty in the real orientation.

Although the theory was derived for contacts between identical metals and for spherical Fermi surfaces, heterogeneous point contacts between aluminum and other materials also gave good results. Experimentally, this even was to be preferred, since this eliminates the problem of alignment between the two electrodes that form the contact. In most of our measurements a copper whisker was used as the second electrode. The oscillations of copper are much too feeble to be observable, both because of the larger q and because of damping effects, that will be discussed in the next paragraph. An example of an aluminum-copper contact is shown in Fig. 7, with the corresponding Fourier transform in Fig. 8. The orientation of the aluminum sample in this case was the same as for the point contact of Fig. 5, i.e., with the magnetic field parallel to a (110) axis. On the whole, qualitatively the same oscillation behavior is observed for both Al-Al and Al-Cu contacts, with the oscillations in the aluminum-copper contact in general somewhat lower in amplitude. The spread in oscillation amplitude for the aluminum-copper contacts was very large but on the average their signal strength was roughly a factor of 3 smaller than for the aluminum-aluminum contacts. The frequencies, observed in both contacts are nominally the same. The multiple peaks between 500 and 600 T in Fig.

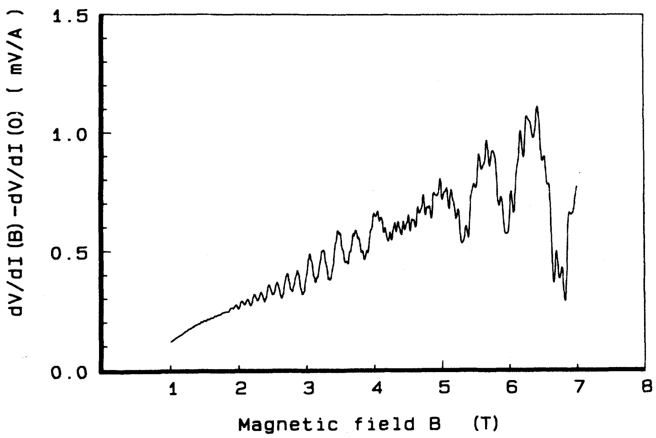

FIG. 7. $dV/dI(B) - dV/dI(0)$ of an aluminum-copper point contact as a function of magnetic field, for zero bias voltage and at a temperature of 1.8 K. The magnetic field was oriented parallel to a (110) axis of the aluminum single crystal and perpendicular to the nominal plane of the contact. The resistance at zero magnetic field was 0.74 Ω.

6 are reduced to one peak in Fig. 8, whereas the peak at 100 T in Fig. 8 is a higher harmonic of the 50-T signal.

Contacts between two polycrystalline samples were deliberately made in order to check that the oscillation effect is really a local effect, and occurs in the direct vicinity of the contact. For this purpose, samples were cut with scissors out of a sheet of high purity, polycrystalline aluminum. The samples had a size of a few millimeters and were of irregular shape. The size of the crystallites, that were visible in the crystals after etching, was of the order of tenths of millimeters, although it can be assumed that along the border of the sample, where the cutting heavily deformed the material, the crystallites were smaller. The contact was then made by pressing together two samples and an example of a measured resistance

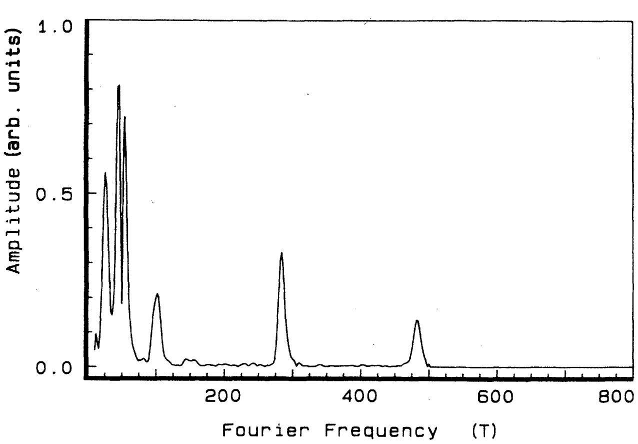

FIG. 8. Fourier transform of Fig. 7, when plotted against $1/B$. Only the part of Fig. 7 between 3.5 and 7 T has been used in the transformation.

versus magnetic field is shown in Fig. 9. This contact shows all the features of a properly oriented aluminum-aluminum contact, although the oscillation pattern is more complex. Apparently, the very rough treatment through the cutting process did not seriously affect the observability of the oscillations.

In the Fourier transform, presented in Fig. 10, a limited number of peaks can be observed. The observed spectrum can be compared with known de Haas–van Alphen data. We used the measurements of Larson and Gordon,[24] who give data for a wide range of crystal orientations. The frequencies below 1000 T fall in two groups, one group of very low frequencies, containing the α and β oscillations, and one group of intermediate frequencies, associated with the γ electrons. The low-frequency group covers the range from 25 T to approximately 70 T, with a high probability around 50 T, where oscillations are found over a wide range of orientations. The 50 T peak that is found in Fig. 10 is due to these branches. Furthermore, eight clearly distinguished peaks are visible between 280 and 600 T. No attempt was made to reconstruct the crystal orientations from the values of these frequencies. This would require more knowledge about the frequencies for directions out of the planes of symmetry and even then it would be a very difficult task. Some general observations can be made, however. When the Fermi surface of aluminum is considered, it turns out that at most six different γ frequencies can be found for each crystal orientation. In practice, at least one of these oscillations will usually not be observable, because the corresponding tube is oriented too closely perpendicular to the magnetic field. The lowest γ oscillation that can occur is the 289 T oscillation in the $\langle 110 \rangle$ direction, whereas the main part of the oscillations for different orientations falls below 500 T. This is also observed in the Fourier transform of Fig. 10. the total number of

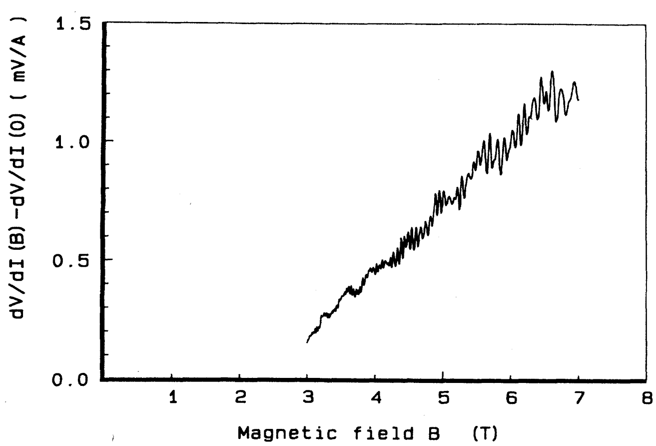

FIG. 9. $dV/dI(B) - dV/dI(0)$ of an aluminum-aluminum point contact as a function of magnetic field, for zero bias voltage and at a temperature of 1.8 K. Both electrodes of the contact consisted of polycrystalline material. The magnetic field was oriented perpendicular to the nominal plane of the contact. The resistance at zero magnetic field was 0.42 Ω.

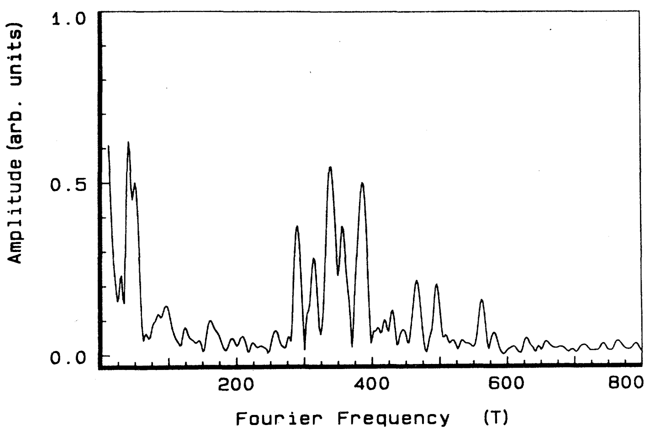

FIG. 10. Fourier transform of Fig. 9, when plotted against $1/B$.

eight different peaks in this figure supports the assumption that only the two crystallites forming the actual point contact contribute to the observed signal. Furthermore, it can be observed, that both crystallites contribute signals of comparable strength. The amplitude distribution over the peaks agrees well with that of single-crystal measurements, as in Figs. 6 and 8. The lower amplitude for the higher frequencies can be attributed to the higher value of the quantum number q and to the higher effective masses that cause a larger damping of the oscillations, as will be discussed in the next paragraph.

VI. DAMPING MECHANISMS

Quite a number of mechanisms act to reduce the observed amplitude of the various quantum oscillations that are observed in metals. Shoenberg[7] cites as the most important ones inhomogeneity of the sample, inhomogeneity of the applied magnetic field, damping because of interference between the two different spins and damping because of finite temperature and relaxation time. All of these mechanisms can be treated with the general concept of phase smearing[26] and the resulting damping factors have a very general character, irrespective of the oscillation phenomenon that is studied.

Of the several sources, cited above, inhomogeneity of the magnetic field clearly will not be very relevant in our experiments, because of the very small size of the actual point-contact area. Sample inhomogeneity would also appear to be unimportant, although it is difficult to determine whether damage, that occurs in the process of making the contact, in the first place acts to reduce the mean free path or also causes mosaic spread in the contact region. In general, this distinction is experimentally hard to make[7] as sample inhomogeneities are also accompanied by an increased scattering. Phenomenologically we will assume it to be included in the reduction factor because of finite relaxation time and here we will not consider it further. Damping because of spin is a constant factor for a given field direction. In our present experiments it cannot be studied because it is impossible to keep a contact stable upon changing the orientation in a magnetic field.

The other two damping effects are very similar in appearance, although their physical origin is different. A finite temperature causes smearing of the Fermi function and gives a vaguer transition between occupied and unoccupied electron states, but keeps the Landau tubes well defined. A finite relaxation time on the other hand blurs the Landau levels but the transition between occupied and unoccupied states remains sharp. It is obvious that the observable effects are quite similar, and for historical reasons the effect of a finite relaxation time τ is expressed in a phenomenological parameter, the Dingle temperature T_D.[27] Still, the factors that describe the two damping sources are not quite the same. For a given oscillation the amplitude of the pth harmonic at a finite temperature T is reduced with a factor of

$$R_T = \frac{X}{\sinh(X)} \tag{9}$$

with $X = 2\pi^2 p k_B T/\hbar\omega_c$, where k_B is the Boltzmann constant. A finite relaxation time τ causes a similar reduction, given by the Dingle reduction factor

$$R_D = \exp(-2\pi^2 k_B T_D/\hbar\omega_c) , \tag{10}$$

expressed in the Dingle temperature $T_D = \hbar/\pi\tau$.

The temperature dependence, described by Eq. (10), was checked as follows. The oscillation pattern was observed over a limited field range for different temperatures, but on one and the same contact. For the oscillation that was observed, we chose the 391.4-T γ oscillation, that is found when the field is parallel to a (100) axis. Although this frequency is fourfold degenerate and hence can cause a rather complicated beat pattern, the four oscillations all have identical mass and will vary with temperature in the same way. The advantage in choosing this frequency is that no other oscillations above 50 T are present in the observed signal. For every period between 6.5 and 7.0 T the amplitude ratio between the signal at 4.2 K and at 2.5 K was determined. This, via Eq. (10), resulted in a value for $\hbar\omega_c$ and hence for the effective mass. Averaging over several periods yielded an effective-mass value of $m^*/m_0 = 0.176 \pm 0.010$, expressed in free-electron masses. This compares very well with the value of Larson and Gordon,[24] who give $m^*/m_0 = 0.180$.

A reliable determination of the Dingle temperature T_D poses more problems, because it can only be determined from the dependence of the amplitude on the magnetic field. This field dependence also comprises other factors, among which is the dependence on the Landau number q. Furthermore, care must be taken to eliminate the effect of a slow beat between two oscillations that are close in frequency. In order to prevent such beat problems, we chose the frequency of 289 T in the $\langle 110 \rangle$ direction, which is fairly isolated in frequency and is nondegenerate. For a large number of aluminum-copper contacts, we determined the ratio of the amplitude of this frequency between 3.5 and 7 T. It is not expected that aluminum-aluminum contacts differ too much from the aluminum-copper contacts with respect to the Dingle temperature, since the data are very similar in appearance. As average dependence of the amplitude on the Landau number q we took $q^{-0.75}$. The mass of the electrons, associated with the 289 T frequency, is $m^*/m_0=0.130$. Taking into account the temperature damping factor, we determined an average Dingle temperature $T_D=3.5$ K or $\omega_C\tau=3.5$ at a magnetic field of 7 T. In de Haas–van Alphen measurements this is a rather high value for the used purity of aluminum. Using other exponents in the q dependence, slightly different values are found for the Dingle temperature, with values between 3.0 and 4.0 K for a q^{-1} and a $q^{-1/2}$ dependence, respectively. Still, such a high value for the Dingle temperature must be seen in view of the rather violent procedure with which the contacts are made. The mean free path, that corresponds to a value of 3.5 for $\omega_c\tau$, is approximately 2 μm. This value does not vary too much from contact to contact as the Dingle temperatures mostly fall in the range of 3 to 4 K for different contacts.

When apart from the small modulation voltage over the contact (typically 1 mV or less) a dc bias voltage was applied over the contact, an additional damping of the signal was observed. This is illustrated in Fig. 11, where the open circles give the relative amplitude of the 289-T frequency at a field of 6 T. The Eliashberg function $\alpha^2F(\omega)$ for aluminum[15] that is also drawn in the figure, suggests that this damping has a relation to the electron-phonon interaction. For the precise nature of this relation two simple models are possible. The hot electrons, generated in the point-contact region, have a reduced mean free path when their energy increases to typical phonon energies and hence a larger Dingle reduction may result. It is equally well possible, that a small increase in temperature, caused by Joule heating, gives a larger temperature reduction. This effect will be most important when the typical electron energy becomes comparable to phonon energies. At a starting temperature of 1.8 K, the temperature increase, necessary to give a reduction with 30%, is 3.5 K. Since this is also a very acceptable value, it is difficult to decide which mechanism is valid for the observed amplitude reduction.

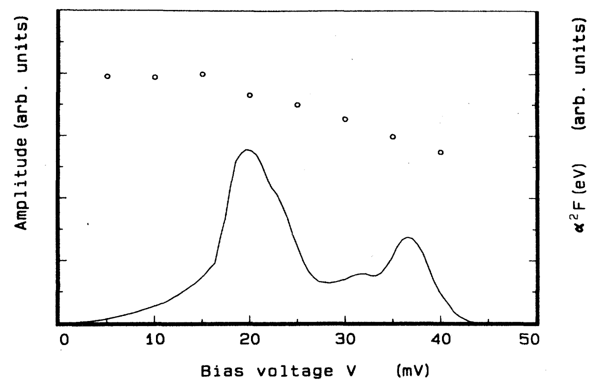

FIG. 11. Relative amplitude of the point-contact oscillation amplitude vs bias voltage for an aluminum-copper point contact of 0.7 Ω, at a temperature of 1.8 K. The open dots give the amplitude of the 289-T oscillation at a magnetic field of 6 T parallel to a (110) axis. The solid line gives the Ehliashberg function $\alpha^2F(\omega)$ for aluminum, as given in Ref. 15.

VII. CONCLUSIONS

In this paper we have shown that Landau quantization can cause oscillations in both the Maxwell and the Sharvin part of the point-contact resistance. Which of the two is most important is determined mainly by material parameters. For metals like bismuth and gallium, where the bulk resistivity shows a considerable Shubnikhov–de Haas effect, the oscillations in the point-contact resistance result from the resistivity-dependent Maxwell resistance. Due to the strong and anisotropic bulk magnetoresistivity the Maxwell contact resistance becomes important in a magnetic field and is given by the square resistance of the macroscopic sample. However, for good conductors such as aluminum with no appreciable Shubnikhov–de Haas effect, the point-contact resistance still shows oscillations. Since magnetic breakdown can be ruled out as a possible cause, the origin of the effect must lie in the Sharvin resistance. Some enhancement is usually necessary to explain the observed oscillation amplitudes. When diffraction at the metallic constriction is taken into account such an enhancement must indeed be expected and in the extreme diffraction limit ($\Lambda > a$) the dependence of the oscillation amplitude on the magnetic field is similar to that of the Shubnikhov–de Haas effect, but now related to the resistivity-independent Sharvin resistance.

No oscillations have been observed in our experiments, corresponding to electrons with effective masses higher than 0.3 free-electron masses or with Landau numbers q higher than 150. In view of the importance of the damping, due to a finite relaxation time in the contact region, this can easily be understood. With an effective mass of 0.5 and a Dingle temperature of 3 K, the reduction factor at, e.g., 10 T is $R_D=0.11$, and this value decreases exponentially with the effective mass. In combination with the large Landau number q that is usually associated with the heavier electrons, this makes the amplitude of the effect too low to be observed. Unfortunately, this limits the observability of quantum oscillations in the Sharvin resistance to materials with small pockets of light electrons.

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*Also at Research Institute for Materials, University of Nijmegen, Toernooiveld 1, NL-6525 ED Nijmegen, The Netherlands.

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