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Current-driven plasma instabilities in superconductors

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We examine here the possibility of current-driven plasma instabilities in superconductors in two temperature regimes. At low temperatures ($T \approx 0$) an instability can be generated in a layered system. Near the critical temperature ($T \approx T_c$) an instability can occur in a single superconductor for sufficiently large drifts which might be achievable in the new high-T_c materials. These instabilities offer possibilities for new radiation-source device applications.

Current-driven plasma instabilities are well known in gaseous plasmas.[1,2] In solid-state systems so far only an incoherent, thermal generation of plasma waves has been observed experimentally.[3–7] It has been shown recently, however, that high-mobility semiconductor superlattices provide sufficiently large drift velocities to achieve a coherent generation or amplification in both type-I and type-II superlattices.[8–10] Although this effect is possible for presently achievable drift velocities in these systems, one expects the effectiveness of the current-plasma-wave energy transfer to be limited due to an intrinsic absorption associated with the carrier-phonon or carrier-impurity scatterings. Therefore systems with reduced carrier scatterings are of great interest. In this paper we investigate the possibility of instabilities in a superconducting plasma. We consider two distinct scenarios. (i) At zero temperature the carrier scattering effects are not present in a superconductor, and this regime ($T \approx 0$) may be particularly advantageous from the point of view of generating a plasma instability; a possible candidate is the cold-beam (two-stream) type of instability.[1,2] (ii) Close to T_c, plasma waves of the superconducting electrons are known to exist,[11] damped by the absorption associated with the pair-breaking effects, and a current driven instability due to the usual inverse Landau effect[1,2] might occur.

We first consider the case of $T \approx 0$, and use the dielectric function formalism to study the electromagnetic response of superconductors. The response of a homogeneous superconductor for $T \approx 0$ and at low frequencies is predominantly transverse, since then no bulk plasma waves can propagate in the superconductor.[12] In this case, the transverse dielectric function can be defined as [13]

$$\epsilon(\omega,q) = 1 + (4\pi i/\omega)\sigma_{tt}(\omega,q) ,$$

where σ_{tt} is the transverse component of the conductivity tensor. σ_{tt} at $\omega = 0$ has been calculated by Bardeen, Cooper, and Schrieffer[14] and for arbitrary ω by Abrikosov, Gorkov, and Khalatnikov.[15] An analytical expression for σ_{tt} in the limit $\omega^2 \ll (2\Delta)^2$ is available[15] and leads to

$$\epsilon(\omega,q) \approx -\omega_p^2(q)/\omega^2 , \tag{1a}$$

when $\omega^2 \ll \omega_p^2(q)$. The effective plasma frequency is given by

$$\omega_p^2(q) = 3\omega_p^2\pi^2\Delta k_F/8q\epsilon_F \tag{1b}$$

in the Pippard limit ($qv_F \gg \Delta$), and

$$\omega_p^2(q) = \omega_p^2 \tag{1c}$$

in the London limit ($qv_F \ll \Delta$). In Eqs. (1b) and (1c), $\omega_p^2 = 4\pi ne^2/m$ is the plasma frequency, ϵ_F and k_F are the Fermi energy and Fermi wave vector, respectively, and Δ is the gap. We use for convenience Planck's constant $\hbar = 1$ as well as the Boltzmann constant $k = 1$.

The simple cold-beam instability occurs when there is a relative drift between two components of a uniform plasma, and there are no carrier-carrier and carrier-ion scatterings.[1,2] These conditions can be simulated in a system of two superconducting plasmas: one filling a half space $z > 0$ and drifting with velocity v_{dr} parallel to the $z = 0$ interface, and a second stationary, which occupies the half space $z < 0$. We solve the Maxwell equations in each region separately,[16] with $\epsilon_1 = -\omega_{p1}^2(q)/\omega^2$ for the nondrifting plasma and $\epsilon_2 = -\omega_{p2}^2(q)/(\omega - \mathbf{q}\cdot\mathbf{v}_{\text{dr}})^2$ for the drifting plasma.[17] The effective plasma frequencies $\omega_{pj}(q)$, with $j = 1$ or 2, are given by Eqs. (1) with corresponding material parameters n_j, Δ_j, k_{Fj}, ϵ_{Fj}, and v_{Fj}. q

is the component of the wave vector tangential to the interface (and parallel to the drift). The denominator of ϵ_2 merely represents the Doppler-shifted frequency due to the drift. Due to the lack of scatterings in a superconductor, the entire electron distribution function acquires a uniform drift, leading to the simple form of ϵ_2. The solutions of the Maxwell equations in the two domains are plane waves given by[13]

$$\mathbf{E}_j(\mathbf{r},t)=\mathbf{E}_j(z)\exp[i(qx-\omega t)],\quad j=1,2, \tag{2a}$$

$$\mathbf{E}_1(z)=\mathbf{E}_1^+\exp(ik_1z)+\mathbf{E}_1^-\exp(-ik_1z),\quad z<0, \tag{2b}$$

$$\mathbf{E}_2(z)=\mathbf{E}_2^+\exp(ik_2z),\quad z>0, \tag{2c}$$

with complex amplitudes $\mathbf{E}_j^\pm=(-k_j/q,0,1)E_j^\pm$ and wave vectors along the z direction

$$k_j=[(\omega^2\epsilon_j/c^2)-q^2]^{1/2}. \tag{2d}$$

At this point it may be noted that one easily recovers the Meissner effect as follows. For a constant field ($\omega=0$, $q=0$) one obtains from Eqs. (1c) and (2d) $k_j=i\omega_{pj}/c$, which substituted into Eq. (2c) describes an exponentially decaying wave with the usual London penetration depth

$$\lambda_{Lj}=c/\omega_{pj}=(mc^2/4\pi n_je^2)^{1/2}.$$

The relations between amplitudes $\mathbf{E}_j^\pm$ are found from Fresnel optics[13] by matching the solutions given by Eq. (2a) across the interface using the standard boundary conditions: the continuity of the tangential (x) components of $\mathbf{E}_j(z)$, and the continuity of the normal (z) components of $\epsilon_j\mathbf{E}_j(z)$. Eigenmodes exist if the reflection amplitude, given by

$$r=E_1^-/E_1^+=(\epsilon_1k_2-\epsilon_2k_1)/(\epsilon_1k_2+\epsilon_2k_1),$$

has a pole, i.e., if $(\epsilon_1k_2+\epsilon_2k_1)=0$, or explicitly,

$$\epsilon_1[(\omega^2\epsilon_2/c^2)-q^2]^{1/2}+\epsilon_2[(\omega^2\epsilon_1/c^2)-q^2]^{1/2}=0. \tag{3}$$

In the nonretarded limit ($c\to\infty$), this reduces to the condition $\epsilon_1+\epsilon_2=0$, which is identical to the corresponding dispersion relation of the cold-stream instability for a sharp-boundary, inhomogeneous velocity profile [Ref. 1, Eq. (1.78), with Eq. (1.71)].

Equation (3) has analytical solutions

$$\omega=qv_{\rm dr}\frac{1\pm i\sqrt{\alpha+\beta}}{1+\alpha+\beta}, \tag{4}$$

where $\beta=\omega_{p2}^2(q)/q^2c^2$ and $\alpha=\omega_{p2}^2(q)/\omega_{p1}^2(q)$. Equation (4) describes two waves with complex-conjugate roots, one damped (minus sign) and the other one amplified (plus sign), indicating an instability. There is no minimum threshold drift for this instability. The properties of this mode inferred from the dispersion relation Eq. (4) are as follows. For large q ($\beta\ll\alpha$, and $qv_F\gg\Delta$), α is independent of q and the amplified mode is an acoustic one with $\omega_R={\rm Re}(\omega)=qv_{\rm dr}/(1+\alpha)$. The growth rate $\gamma={\rm Im}(\omega)=qv_{\rm dr}\alpha^{1/2}/(1+\alpha)$ can be maximized by setting the plasma frequency ratio at $\alpha=1$ to obtain $\gamma=qv_{\rm dr}/2$. The achievable upper limit on $v_{\rm dr}$ is governed by the Cooper-pair-breaking threshold,[18] $v_{\rm dr}=\Delta/k_F=v_F(\Delta/2\epsilon_F)$, leading to the maximum of achievable growth rate $\gamma_{\max}=qv_F(\Delta/4\epsilon_F)$, with corresponding mode frequency $\omega_R=\gamma_{\max}$. The frequency of the mode can be increased up to $\omega_R=qv_F(\Delta/2\epsilon_F)$ by reducing α, but this is at the expense of γ which decreases and disappears as $\alpha\to 0$. It should also be noted that in the lateral direction (z) the mode decays exponentially away from the interface with a scale length q^{-1} [see Eqs. (2), $k_1\approx k_2\approx iq$]. For small q ($\beta\gg\alpha$), the mode becomes strongly subacoustic, with $\omega_R=q^3c^2v_{\rm dr}/(q^2c^2+\omega_{p2}^2)$, $\gamma=v_{\rm dr}\omega_{p2}q^2c/(q^2c^2+\omega_{p2}^2)$.

Let us now briefly discuss the practical implications of this instability. It is well known in gaseous-plasma physics that an unstable plasma wave can radiatively decay, and therefore be a source of an electromagnetic radiation.[19] In fact, devices exist in which this principle has been utilized.[19] We can expect similar device applications of the superconducting system discussed above. Here we are interested in the case of large q which allows for maximum amplification gain as discussed above. In order to couple a surface plasma wave to an external radiation an extra parallel momentum is necessary,[13] which can be provided by a grating,[6,7] parallel to and near the interface.

While we have analyzed the simple two-region model with a sharp boundary, it should be noted that in reality there are three regions: the stationary plasma, the uniformly moving plasma, and an interface layer of thickness of the order of the London penetration depth, in which the drift velocity smoothly drops from $v_{\rm dr}$ to 0. However, the essential reason for the instability is that one part of the plasma is moving relative to the other, and that feature is correctly included in our model. Similar modeling of soft-interface problems in terms of sharp-boundary models has been successfully used in metal optics,[13] as well as for superconductors.[16]

Next we consider another domain, T very close to, but below T_c. It is known that in this case longitudinal plasma waves can propagate.[11] They are damped by the usual single-particle absorption associated with the normal electrons. A possible mechanism for generating an instability is to introduce a relative drift between the normal and superconducting components of the plasma. Without the drift, the longitudinal dielectric function of a weakly coupled superconducting plasma in the clean limit was given by Dinter[20] for $\omega\ll v_Fq\ll T$, $\omega\lesssim 2\Delta$, and $T\approx T_c$:

$$\epsilon(q,\omega)\approx\frac{\kappa^2}{q^2}\left[\frac{\frac{1}{3}\rho_s}{\eta^2-(\omega/qv_F)^2}+P\left(\frac{\Delta}{2T},\frac{\omega}{qv_F}\right)\right], \tag{5}$$

with

$$\eta^2=A^2\rho_s^{1/2},\quad A=[2\sqrt{7\zeta(3)}/3\pi^2]^{1/2}\approx 0.44,\quad \kappa^2=3\omega_p^2/v_F^2,$$

$$P\left(\frac{\Delta}{2T},\frac{\omega}{qv_F}\right)\approx 1-\frac{\pi}{2}\left(\frac{\Delta}{2T}\right)+i\frac{\pi}{2}\left(\frac{\omega}{qv_F}\right)\left(1-\frac{\Delta}{2T}\right)\quad(\Delta\ll T). \tag{6}$$

$\rho_s=n_s/n$ is the fraction of superconducting electrons, which near T_c can be expressed as [20]

$$\rho_s = \frac{7\zeta(3)\Delta^2}{4\pi^2 T^2} \approx 0.21 \left(\frac{\Delta}{T}\right)^2, \ \Delta = 3.06 T_c \left(1 - \frac{T}{T_c}\right)^{1/2}. \quad (7)$$

The first term in the square brackets in Eq. (5) describes the response of the superconducting electrons in the plasma, while the second term represents the contribution due to the normal electrons. The latter, which in general appear in the system as a result of pair-breaking effects, are dominant in the vicinity of T_c. A plasma mode exists when $\epsilon=0$, which occurs very close to, and in fact is due to, the sharp pole structure of the superconducting part. Then Eq. (5), in the frequency domain of interest, leads to the following dispersion relation of the plasma mode (to order ρ_s):

$$\frac{\omega}{qv_F} \approx \eta + \frac{\rho_s}{6\eta} - i\pi\frac{\rho_s}{12} \approx A\rho_s^{1/4} + \frac{1}{6A}\rho_s^{3/4} - i\frac{\pi}{12}\rho_s . \quad (8)$$

This is an acoustic mode with a weak Landau damping due to the normal electrons.

We now introduce a dc current. The superconducting electrons will drift much faster than the normal ones, and in fact, the drift motion of the latter can be neglected. The frequency of the superconducting part in ϵ is thus Doppler shifted to $\omega - \mathbf{q}\cdot\mathbf{v}_{dr}$, where $\mathbf{v}_{dr}$ is the drift velocity, while the contribution of the normal electrons remains unaffected. Equation (5) now becomes

$$\epsilon(q,\omega) \approx \frac{\kappa^2}{q^2}\left[\frac{\frac{1}{3}\rho_s}{\eta^2 - [(\omega - \mathbf{q}\cdot\mathbf{v}_{dr})/qv_F]^2} + P\left(\frac{\Delta}{2T}, \frac{\omega}{qv_F}\right)\right]. \quad (9)$$

It is obvious from Eq. (9) that the plasma mode which is associated with the pole structure of the superconducting electrons still exists, but is Doppler shifted. If the direction of propagation of the plasma mode is chosen to be opposite to the direction of the drift ($\mathbf{q}\cdot\mathbf{v}_{dr} < 0$), the mode shifts toward $\omega_R = 0$, at which point [see Eq. (6)] the imaginary part of $\epsilon = \mathrm{Im}(P)$, which is a measure of the absorption associated with normal electrons, changes sign. Further increase in v_{dr} leads to a growth of the plasma mode due to the inverse Landau damping mechanism. It should be noted that this is an example of the so-called "negative energy wave,"[21] with wave growth accompanied by upward single-particle excitations of normal electrons.

The minimum threshold drift velocity needed for this mode to arise can be easily calculated from $\epsilon=0$ in Eq. (9) with $\omega=0$. This leads to

$$v_{dr} = v_F \left(\eta^2 + \frac{\frac{1}{3}\rho_s}{1 - \pi\Delta/4T}\right)^{1/2} \approx \eta v_F . \quad (10)$$

This v_{dr} is approximately the phase velocity of the driftless mode in Eq. (8). Just as in the usual inverse Landau damping condition,[1,2] the amplification is possible if the drift velocity exceeds the phase velocity of the original plasma mode. Contrary to the previous case of $T \approx 0$, a minimum drift velocity is required for this instability. Whether it is achievable in practice depends on the pair-breaking threshold[18] $v_{dr} = \Delta/k_F = v_F(\Delta/2\epsilon_F)$ for any given material. Thus we require $\eta v_F < v_F(\Delta/2\epsilon_F)$ or approximately $(T_c T/\epsilon_F^2)(1 - T/T_c)^{1/2} > 0.11$, which is not fulfilled in standard superconductors where $T_c/\epsilon_F \ll 1$. However, new developments in the high-T_c superconductors can be expected to considerably increase the ratio T_c/ϵ_F, where besides the higher T_c, one can obtain a strongly reduced ϵ_F as well because of reduced electron densities and increased effective electron masses.[22]

In summary, we have shown in this paper that (i) near $T \approx 0$ a cold-beam type of plasma instability can be generated in a system of two superconductors by introducing a relative drift, and (ii) in the other temperature domain ($T \approx T_c$) an inverse Landau damping related instability arises in a single superconductor for a sufficiently large drift.[23] An experimental verification of the existence of the low-temperature instability should now be undertaken. We expect this instability to survive well into the intermediate-temperature range since the normal electron population (and its associated single-particle absorption) will remain small until T is close to T_c. Appropriate extensions of the theory into the intermediate-temperature regime as well as higher frequencies should be carried out. Also, multilayered systems (superconducting superlattices), as in semiconductors, may offer significant advantages over the simple two-layer configuration, and therefore should be investigated.

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