Fluctuation conductivity of high-T_c superconductors

Kazumi Maki and R. S. Thompson
Department of Physics, University of Southern California, Los Angeles, California 90089-0484
(Received 26 September 1988)

The leading correction to the normal-state conductivity due to superconducting fluctuations is calculated as a function of temperature, externally applied magnetic field, and internal pair breaking for anisotropic high-T_c superconductors.

Due to the short coherence lengths characteristic of the new high-T_c superconductors the effect of thermodynamic fluctuations on their resistive transitions is readily observable.[1] However, these materials are highly anisotropic, so one needs highly oriented or single-crystal samples to compare the numerical coefficient of the temperature dependence of the fluctuation conductivity with theory.[2] This coefficient depends on the coherence lengths of the material, which are usually measured from the dependence of the upper critical magnetic field H_{c2} on temperature T. However, the electrical resistance drops continuously from the normal-state value to zero without any apparent break.[3] The measurements appear to indicate either the absence of a well-defined flux-flow regime where the magnetoresistance varies almost linearly[4] with the externally applied magnetic field B or that the flux-flow regime merges smoothly with the fluctuation regime.[5] Defining H_{c2} at the point where the resistance is 90% of the normal resistance[2] versus the point where the resistance vanishes[6] results in values of H_{c2} which differ by factors as large as 4 for very similar data.

The fluctuation conductivity is also dependent on B scaled by the coherence lengths. Measurement of the magnetoresistance above the field-dependent transition temperature $T_c(B)$ may allow a more accurate determination of the coherence lengths, which is less dependent on sample inhomogeneity and magnetic flux pinning. Therefore, in the present paper we present a theoretical calculation of the dependence of the fluctuation conductivity on B and T as adapted to anisotropic superconductors with s-wave pairing. The fluctuation conductivity σ' is added to the the normal-state conductivity to get the total conductivity. σ' is the sum of two parts, which we call regular and anomalous, $\sigma'=\sigma_r+\sigma_a$. We first consider a translationally invariant model for the case where the temperature-dependent coherence length is long enough that the layered structure is not important. Later we consider the corrections due to the layered structure.

The regular fluctuation conductivity σ_r was first calculated in the absence of a magnetic field by Aslamazov and Larkin (AL) (Ref. 7) and can also be obtained using only time-dependent Ginzburg-Landau (GL) theory,[8] as shown by Schmidt.[9] This theory is largely independent of the mechanism producing the superconductivity. Aside from the coherence lengths, it contains only two adjustable parameters. One parameter relates to the jump in the heat capacity at T_c in the mean-field theory. This parameter does not affect the leading results for the fluctuation conductivity, just as it does not affect the leading results for the fluctuation contribution to the heat capacity.[10] The other parameter is a dynamic parameter relating to the rate of decay of fluctuations. Relative to the AL value, strong pair breaking increases the fluctuation conductivity, while strong coupling reduces it.[11] For definiteness we take the same value for the decay rate as AL and Schmid.

We use the anisotropic GL theory.[12] The temperature-dependent coherence lengths $\xi_j(T)$ along the principle axes are obtained from constants ξ_j by $\xi_j(T)=\xi_j/\sqrt{|\tau|}$. All temperature dependence is in the parameter $\tau=(T-T_c)/T_c$. Although the two coherence lengths in the ab plane may be equal for Y-Ba-Cu-O, the Bi-Sr-Ca-Cu-O material shows anisotropy in the plane,[13] so we can allow for three different ξ_j. For ease of interpretation, we will only consider applying fields along one of the principle crystal axes. Applying an electric field **E** along the x direction we get

$$\sigma_r=(e^2/32\hbar)(\xi_x/\xi_y\xi_z)/\sqrt{\tau}\,. \tag{1}$$

When **B** is applied perpendicular to **E** in the z direction, it is convenient to introduce the dimensionless parameter $b=2eB\xi_x\xi_y/\hbar c$. The critical field $B=H_{c2}$ is obtained when $b=-\tau$, so the scaled field is $b=B/(-T_c dH_{c2}/dT)$. With this scaling the previous result of Usadel[14] is only modified by a prefactor

$$\sigma_{r\perp}=(e^2/8\hbar)(\xi_x/\xi_y\xi_z)b^{-1/2}S_\perp(\tau/b)\,,$$
$$S_\perp(x)=\sum_{n=0}^{\infty}(n+1)[(x+2n+1)^{-1/2}-2(x+2n+2)^{-1/2}+(x+2n+3)^{-1/2}]\,. \tag{2}$$

Alternatively, when **B** is applied parallel to **E** in the x direction, b is defined by $b=2eB\xi_y\xi_z/\hbar c$. Again, using Ref. 14 we get

$$\sigma_{r\parallel}=(e^2/8\hbar)(\xi_x/\xi_y\xi_z)b^{-1/2}S_\parallel(\tau/b)\,,$$
$$S_\parallel(x)=\tfrac{1}{4}\sum_{n=0}^{\infty}(x+2n+1)^{-3/2}\,. \tag{3}$$

The anomalous fluctuation conductivity σ_a results from the scattering of the normal excitations by the superconducting fluctuations[15] and is extremely sensitive to pair breaking, which we parametrize by δ. Depending on the microscopic mechanism responsible for the pair breaking, δ may in general vary with field and temperature.[16] The possible strong-coupling corrections to σ_a have not yet been studied. Keeping x as the direction of **E**, for $B=0$ we get

$$\sigma_a=(e^2/8\hbar)(\xi_x/\xi_y\xi_z)/(\sqrt{\tau}+\sqrt{\delta}) . \quad (4)$$

When $B\neq 0$ the form of σ_a does not depend on the direction of **B**, but the parameter b must be defined as in Eq. (2) or (3) according to the chosen direction. Generalizing a previous calculation for the two-dimensional case,[17] we get

$$\sigma_a=(e^2/8\hbar)(\xi_x/\xi_y\xi_z)[b^{1/2}/(\tau-\delta)]\times[(\tau/b)S_a(\tau/b)-(\delta/b)S_a(\delta/b)] ,$$
$$S_a(x)=(1/x)\sum_{n=0}^{\infty}[(2n+1)^{-1/2}-(x+2n+1)^{-1/2}] . \quad (5)$$

The three sums defined above are graphed in Fig. 1. If $\delta=0$ the anomalous contributions to σ' can be read directly off the graph, but for $\delta\neq 0$ a two-dimensional graph would be necessary to illustrate σ'.

Some asymptotic expansions for the sums may be useful. For large $x\gg 0$ we get

$$S_\perp(x)=\tfrac{1}{4}x^{-1/2}(1-\tfrac{3}{16}x^{-2}+\tfrac{35}{128}x^{-4}) ,$$
$$S_\parallel(x)=\tfrac{1}{4}x^{-1/2}(1-\tfrac{1}{8}x^{-2}+\tfrac{49}{384}x^{-4}) , \quad (6)$$
$$S_a(x)=x^{-1/2}(1-0.428x^{-1/2}+\tfrac{1}{24}x^{-2}-\tfrac{7}{384}x^{-4}) .$$

Therefore, for weak magnetic fields when $b\ll\tau$ the leading corrections to σ_r as given in Eq. (1) are of order B^2. If the additional condition $b\ll\delta$ is satisfied, the leading correction to σ_a in Eq. (4) is also of order B^2. However, if this additional condition is not satisfied, then the leading correction to σ_a is of order $\sqrt{B}$, as b plays a pair-breaking role comparable to that of δ in Eq. (4). The experimental data[3] have no strong dependence of σ' on B for $T>T_c$, so δ is not very small in these materials.

For $x\approx 0$, we get

$$S_\perp(x)=0.380-0.302x+0.287x^2 ,$$
$$S_\parallel(x)=0.422-0.414x+0.963x^2 , \quad (7)$$
$$S_a(x)=0.844-0.414x+0.321x^2 .$$

Nothing unusual happens when the denominator $\tau-\delta$ vanishes in Eq. (5) because the numerator which follows in square brackets also vanishes.

Finally, near the superconducting transition for $x\approx -1$ we get

$$S_\perp(x)=(1+x)^{-1/2}-0.985+0.685(1+x) ,$$
$$S_\parallel(x)=\tfrac{1}{4}(1+x)^{-3/2}+0.231-0.089(1+x) , \quad (8)$$
$$S_a(x)=(1+x)^{-1/2}-0.605+(1+x)^{1/2}-1.076(1+x) .$$

The divergence of $S_\parallel$ is much stronger than that of $S_\perp$. Even when both **j** and **B** are in the plane of the layers, it is important to specify their relative orientation.

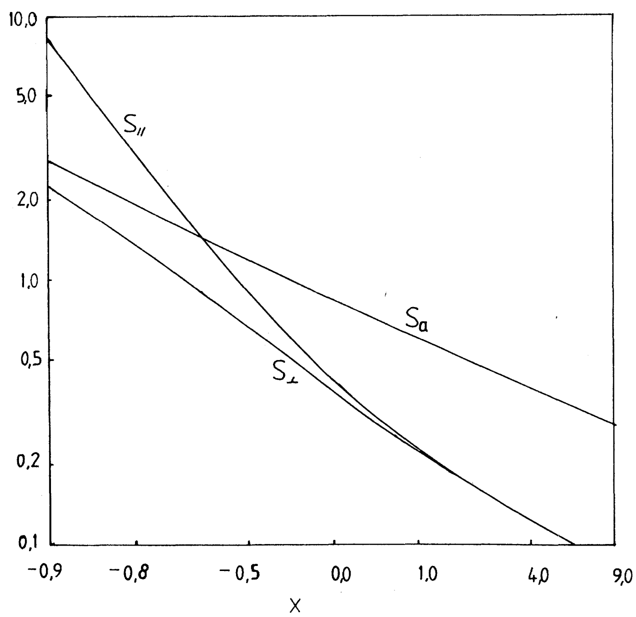

FIG. 1. Three sums S defined in the text are plotted vs a parameter x on a grid of $\log_{10}(S)$ vs $\log_{10}(x+1)$.

The layered structure is important for σ' when the coherence length in the c direction perpendicular to the layers $\xi_c/(\tau+b)^{1/2}$ is less than or equal to the layer spacing. Experiments[2] on Y-Ba-Cu-O were interpreted as showing a dimensional crossover at $\tau\approx 0.1$. However, that analysis, like the present one, is based on a GL approximation and, therefore, is not quantitatively accurate for $\tau\gtrsim 0.1$. Within the GL range, where τ, b, and $\delta<0.1$ (for $B<10$ T the observed[3] $b<0.1$), the correction for layering in this material is apparently unimportant, and one can use our above theory, Eqs. (1) to (8), until the boundary of the critical region where interactions between fluctuations are important (at $\tau\approx 0.003$ for $B=0$) is approached. Nevertheless, for completeness and possible future applications we briefly consider layering.

For a layered superconductor we use a model[18] which assumes coupling between nearest-neighbor layers only. Then for $B=0$ the GL energy of a fluctuation of momentum $\mathbf{q}=(q_x,q_y,q_c)$ is proportional to $\tau+(\xi_xq_x)^2+(\xi_yq_y)^2+4K\sin^2(q_cs/2)$, where s is the layer spacing in the c direction, and ξ_c is related to the coupling between layers K by the small q_c expansion, $K=(\xi_c/s)^2$. The fluctuation conductivity along the x direction in the layer is

$$\sigma_r=(e^2/16\hbar)(\xi_x/\xi_ys)\tau^{-1/2}(\tau+4K)^{-1/2} ,$$
$$\sigma_a=(e^2/4\hbar)(\xi_x/\xi_ys)(\tau-\delta)^{-1}\times\ln\{[\tau^{1/2}+(\tau+4K)^{1/2}]/[\delta^{1/2}+(\delta+4K)^{1/2}]\} . \quad (9)$$

The generalization of this result when **B** is applied perpendicular to the layers is easy:

$$\sigma_{r\perp}=(e^2/4\hbar)(\xi_x/\xi_y s)\sum_{n=0}^{\infty}(n+1)\{[\tau+(2n+1)b]^{-1/2}[\tau+(2n+1)b+4K]^{-1/2}$$
$$-2[\tau+(2n+2)b]^{-1/2}[\tau+(2n+2)b+4K]^{-1/2}$$
$$+[\tau+(2n+3)b]^{-1/2}[\tau+(2n+3)b+4K]^{-1/2}\}\,,$$
$$\sigma_a=(e^2/4\hbar)(\xi_x/\xi_y s)[b/(\tau-\delta)]\sum_{n=0}^{\infty}\{[\delta+(2n+1)b]^{-1/2}[\delta+(2n+1)b+4K]^{-1/2}$$
$$-[\tau+(2n+1)b]^{-1/2}[\tau+(2n+1)b+4K]^{-1/2}\}\,. \tag{10}$$

If **B** is applied parallel to the layers the generalization of Eq. (9) is more difficult and requires study of solutions to the Mathieu equation.[19] We have not studied this case. Instead, we wish to discuss what happens if the layering is more complicated with some layers more tightly coupled than others. The simplest example is when alternative layers are coupled by alternate coupling constants K and G. Then, there are two modes for the GL energy:

$$\tau+(\xi_x q_x)^2+(\xi_y q_y)^2+K+G \pm[K^2+G^2+2KG\cos(q_c a)]^{1/2}\,,$$

where a is the periodicity distance.[20] The perpendicular coherence length is determined by the small q_c expansion of the lower-energy solution $(\xi_c/a)^2=KG/[2(K+G)]$. For $B=0$ we get

$$\sigma_r=(e^2/8\hbar)(\xi_x/\xi_y a)(\tau+K+G)\{\tau[\tau+2(K+G)](\tau+2K)(\tau+2G)\}^{-1/2}\,,$$
$$\sigma_a=(e^2/4\hbar)(\xi_x/\xi_y a)(\tau-\delta)^{-1}(\ln\{\tau^{1/2}[\tau+2(K+G)]^{1/2}+(\tau+2K)^{1/2}(\tau+2G)^{1/2}\}$$
$$-\ln\{\delta^{1/2}[\delta+2(K+G)]^{1/2}+(\delta+2K)^{1/2}(\delta+2G)^{1/2}\})\,. \tag{11}$$

If $K=G$ Eq. (11) is the same as Eq. (9) with $s=a/2$. However, if $G\ll K$ the dimensional crossover is controlled by the weaker coupling constant G and occurs as a function of T when $\tau\approx G$. For δ and $\tau\ll K$ Eq. (11) is the same as Eq. (9) when expressed in terms of ξ_c, except that s is replaced by a. The more tightly coupled pairs of layers flucuate like single layers, and the relevant length scale is the periodicity length a. In the extreme two-dimensional limit, where δ and $\tau\gg K$, the relevant length scale is the average layer spacing, $s=a/2$, and each layer fluctuates separately. The same results are expected in these limits for a more complicated multilayer unit cell.

In summary, we have worked out expressions for the magnetoconductivity σ above the superconducting transition for anisotropic and layered superconductors as functions of temperature T, external magnetic field B, and pair breaking δ. Our results are consistent with the lack of a strong dependence of σ on B for $T>T_c$ and indicate that the critical field H_{c2} should be associated with a point near the top of the transition curve, perhaps near the point where the derivative of the resistance is maximal, rather than with the point where the resistance vanishes. Closer analysis of the small changes in σ on the upper part of the transition curves as functions of T and B are necessary for quantitative comparison of theory and experiment.

After completion of this paper an article appeared by Hikami and Larkin,[21] which uses the same basic model for **B** perpendicular to a layered structure and obtains results which are equivalent to our Eq. (10), although expressed in a different form which is less convenient for numerical evaluation. They also give an explicit expansion of these results for small B. Our work is complementary to theirs by considering some additional cases and giving an explicit numerical evaluation of the summations in Fig. 1, which we hope will be of use in comparing theory with experiment.

In acknowledgment, one author (K.M.) would like to thank P. Monceau and O. Laborde for useful discussion of their experimental results prior to publication. The work of K.M. is supported by the National Science Foundation under Grant No. DMR 86-11829.

[1] P. P. Freitas, C. C. Tseui, and T. S. Plaskett, Phys. Rev. B **36**, 833 (1987).

[2] B. Oh, K. Char, A. D. Kent, M. Naito, M. R. Beasley, T. H. Geballe, R. H. Hammond, A. Kapitulnik, and J. M. Graybeal, Phys. Rev. B **37**, 7861 (1988).

[3] O. Laborde, P. Monceau, M. Potel, P. Gougeon, J. Padiou, J. C. Levet, and H. Noel, Physica C **153-155**, 1467 (1988); Solid State Commun. **67**, 609 (1988); R. J. Wijngaarden, K. Heeck, R. Griessen, A. A. Menovsky, and M. J. V. Menken, Physica C **153-155**, 1329 (1988); Y. Iye, T. Tamagai, H. Takeya, and H. Takei, Jpn. J. Appl. Phys. **26**, L1057 (1987); T. K. Worthington, W. J. Gallagher, and T. R. Dinger, Phys. Rev. Lett. **59**, 1160 (1987).

[4] See, for example, J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965) or K. Maki, Physica **55**, 124 (1971).

[5] Perhaps it is noteworthy that very similar pretransition magnetoresistance is observed in some organic superconductors, which have quasi-two-dimensional Fermi surfaces: see K.

Oshima, T. Mori, H. Inokuchi, H. Urayama, H. Yamochi, and G. Saito, Phys. Rev. B **36**, 938 (1988); K. Oshima, H. Urayama, H. Yamochi, and G. Saito, J. Phys. Soc. Jpn. **59**, 730 (1988).

[6]M. Oda, Y. Hidaka, M. Suzuki, and T. Murakami, Phys. Rev. B **38**, 252 (1988); Y. Hidaka, Y. Enomoto, M. Suzuki, M. Oda, A. Katsui, and T. Murakami, Jpn. J. Appl. Phys. **26**, L726 (1987).

[7]L. G. Aslamazov and A. I. Larkin, Fiz. Tverd, Tela **10**, 1104 (1968) [Sov. Phys. Solid State **10**, 875 (1968)].

[8]A. Schmid, Phys. Kondens. Mater. **5**, 302 (1966); E. Abrahams and T. Tsuneto, Phys. Rev. **152**, 416 (1966).

[9]H. Schmidt, Z. Phys. **216**, 336 (1968).

[10]R. S. Thompson and V. Z. Kresin, Mod. Phys. Lett. B **2**, 1159 (1988).

[11]P. Fulde and K. Maki, Phys. Kondens. Mater. **8**, 371 (1969).

[12]V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **23**, 236 (1952).

[13]S. Martin, A. T. Fiory, R. M. Fleming, L. F. Schneemeyer, and J. V. Waszczak, Phys. Rev. Lett. **60**, 2194 (1988).

[14]K. D. Usadel, Z. Phys. **227**, 260 (1969).

[15]K. Maki, Prog. Theor. Phys. (Kyoto) **39**, 897 (1968); R. S. Thompson, Phys. Rev. B **1**, 327 (1970).

[16]For an analysis of the two-dimensional system, see W. Brenig, M. C. Chang, E. Abrahams, and P. Wolfle, Phys. Rev. B **31**, 7001 (1985).

[17]R. S. Thompson, Physica **55**, 296 (1971).

[18]W. E. Lawrence and S. Doniach, in *Proceedings of the Twelfth International Conference on Low Temperature Physics, Kyoto, 1970,* edited by Eizo Kanda (Keigaku, Tokyo, 1971), p. 361.

[19]R. A. Klemm, A. Luther, and M. R. Beasley, Phys. Rev. B **12**, 897 (1975).

[20]See a similar textbook treatment of phonons: N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart, and Winston, New York, 1976), p. 433.

[21]S. Hikami and A. I. Larkin, Mod. Phys. Lett. B **2**, 693 (1988).