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New type of superconductivity in very high magnetic fields

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A new superconducting state is proposed that can coexist with very high magnetic fields, provided that Pauli pair breaking is absent. Such a state may arise, for example, in multivalley semiconductors and semimetals, where the valley degrees of freedom play the role of the electron spins in the usual BCS theory. Below the transition temperature, such superconductors show no resistance to a current flowing in the direction parallel to the field; in the plane perpendicular to the field, under suitable conditions, their conducting properties will be reminiscent of a quantum liquid of bosons of charge $2e$. There is no orbital upper critical field for such superconductors.

Recently, one of us[1] has proposed that in multivalley semiconductors or semimetals subjected to a very high magnetic field, a superconductinglike instability may arise at some finite temperature T_c. With all electrons confined to the lowest Landau level and spins fully polarized, the Cooper pairs consist of electrons from different valleys, the valley indices playing the role that spins have in the usual BCS theory. Strictly speaking, such Cooper pairs are spin triplet, and Pauli pair breaking will be absent as the valley indices do not couple to the magnetic field.

Several fundamental questions arise with respect to this observation. We know from standard theory[2] that even if we completely ignore electronic spins the orbital effect of the magnetic field will destroy superconductivity at some critical field strength H_{c2}.[3] How is it possible then that, at fields generally much higher than H_{c2}, when the quantum limit is reached the superconducting instability can again occur at some finite temperature? Does this instability lead to a stable superconducting phase at temperatures below T_c, and what are the properties of this phase?

In this paper we show that indeed a stable superconducting state of a remarkable nature exists below the transition temperature.[4] The equilibrium form of the superconducting order parameter represents a configuration which consists of a collection of "tubes" of radius given by the magnetic length.[5] The density of zeros of the order parameter is always fixed to one per flux quantum. The order parameter is uniform along the axis parallel to the magnetic field (we take this to be the z axis). Correspondingly, the Cooper-pair wave function is highly anisotropic, extending by an ordinary coherence length in the direction of the field, but being compressed to a size given by the magnetic length in the perpendicular plane. The electrical current flows without dissipation along the z axis; in this sense this system behaves like a *quasi-one-dimensional superconductor*. As a consequence of this, as noted by Rasolt,[1] the increase in the magnetic field raises the density of states at the Fermi level and can produce an increase in the transition temperature. There is *no upper critical field* arising from the orbital effect; as long as there is an effective attraction at the Fermi level the transition temperature will be finite. This is a qualitatively new situation in comparison with the behavior of well-known type-I and type-II superconductors. We expect the fluctuations, which are due to the lateral motion of the "guiding" centers of these superconducting tubes and the quantum-mechanical tunneling of Cooper pairs between tubes, to be important in high-magnetic-field superconductors.

We start by considering the system of electrons of mass m in a strong external field B_0 having an attractive δ-function interaction. This interaction can arise through phonons or some other more exotic mechanism which is entirely due to electron-electron interactions. Here we work out in detail the example of a two-valley system. The Hamiltonian of such a system can be written in the following form (we assume that electrons are confined to the lowest Landau level and ignore spin indices):

$$H=\sum_{n,k}(\xi_k+\tfrac{1}{2}\omega_c)c^{\dagger}_{nk\alpha}c_{nk\alpha} - g\int d^3R\,\psi^{\dagger}_{\alpha}(\mathbf{R})\psi^{\dagger}_{\beta}(\mathbf{R})\psi_{\beta}(\mathbf{R})\psi_{\alpha}(\mathbf{R})\,, \qquad (1)$$

where $\xi_k=E(k)-\mu$, with k and $E(k)$ being the wave vector and energy of motion along the z axis, $\omega_c\equiv eB_0/mc$ is the cyclotron frequency, α and β are valley indices, $c^{\dagger}_{nk\alpha}$ is the creation operator for electrons of linear momentum k and angular momentum n in the lowest Landau level, and g is the coupling constant. The field operators $\psi_\alpha(\mathbf{R})$ are given as

$$\psi_\alpha(\mathbf{R})=\sum_{n,k}\frac{1}{\sqrt{L}}c_{nk\alpha}e^{ik\zeta}\phi_n(z)\,,$$

where $\mathbf{R}\equiv(x,y,\zeta)$, $z\equiv x+iy$, and $\phi_n(z)$ form a set of basis functions spanning the lowest Landau level in the symmetric gauge

$$\phi_n(z)=\frac{1}{l\sqrt{2\pi n!2^n}}\left(\frac{z}{l}\right)^n\exp\left(-\frac{z^*z}{4l^2}\right)\,,$$

with magnetic length $l \equiv (e/eB_0)^{1/2}$.

We now proceed in the usual BCS fashion. The electrons form spin-triplet Cooper pairs with the valley indices playing the role of the BCS spins. It is important to emphasize here that the spin-triplet structure arises trivially, as we assume that the electrons are fully polarized. The gap parameter is defined through

$$\Delta_{\alpha\beta}(\mathbf{R}) = -gT\sum_{\omega_n}\langle \psi_{\alpha,\omega_n}(\mathbf{R})\psi_{\beta,-\omega_n}(\mathbf{R})\rangle , \quad (2)$$

where the ω_n's are the Matsubara frequencies and $\psi_{\alpha,\omega_n}(\mathbf{R})$ is the Fourier component of the field operator in the thermal representation. We assume that the favored state is the "valley singlet," i.e., antisymmetric in valley indices, and drop valley indices in what follows.

The self-consistent solution of Eq. (2) becomes possible at some temperature T_c given by

$$T_c = 1.14\Omega \exp\left[-\frac{2\pi l^2}{N(0)g(B)} \right] , \quad (3)$$

where Ω is some low-energy cutoff $\Omega \ll E_F$, E_F being the Fermi energy, and $N(0)$ is the density of states at the Fermi level. $g(B)$ is the coupling constant due, for example, to the electron-phonon interaction, which can have a significant magnetic field dependence. Equation (3) suggests that the superconducting transition may be an increasing function of the magnetic field, at least in some region of the parameter space; indeed this is the case for the electron-phonon coupling.[1] Other sources of attractive interaction, arising from some electronic mechanism could also contribute to $g(B)$. To study what happens when the temperature is lowered below T_c, we calculate the free energy in the superconducting state using the Ginsburg-Landau (GL) expansion. One has to be very careful in calculating the GL form of the free energy, as the magnetic field will generally couple to the order parameter.[3] Therefore, the finite superconducting order parameter T_c may change the average magnetic field seen by the electrons forming the Cooper pairs. Here we assume that we can split the total magnetic field into two parts: the constant field B, which is the sum of B_0 and whatever average field arises due to superconductivity; and $\mathbf{b}(\mathbf{R})$, which is the fluctuating part, due entirely to "supercurrents." The corresponding vector potentials are $\mathbf{A}$ and $\mathbf{a}(\mathbf{R})$. We first consider the case when the order parameter is uniform along the z axis. Then both $\mathbf{A}$ and $\mathbf{a}$ will be in the x-y plane and it is useful to define complex quantities A and a. The total free energy can be written as a sum of three parts:

$$F = F_s + F_b + F_{s-b} , \quad (4)$$

where

$$F_s = \alpha(T)\int dr_1\, dr_2\, \Psi^\dagger(r_1)K_2(r_1,r_2)\Psi(r_2) + \frac{\beta(T)}{2}\int dr_1\, dr_2\, dr_3\, dr_4\, \Psi^\dagger(r_1)\Psi(r_2)K_4(r_1,r_2,r_3,r_4)\Psi(r_3)\Psi(r_4) , \quad (5)$$

$$F_b = \int dr \frac{(B+b)^2}{8\pi} , \quad (6)$$

and

$$F_{s-b} = \gamma(T)\int dr_1\, dr_2\, \Psi^\dagger(r_1)K_3(r_1,r_2)\Psi(r_2) . \quad (7)$$

In these equations $\alpha(T)$, $\beta(T)$, and Ψ are all defined in the standard GL fashion,[2] $r \equiv (x,y)$, F_s is the free energy of a superconductor in the average magnetic field B, F_b is the magnetic free energy, F_{s-b} describes the coupling of Ψ to the fluctuating vector potential. One should appreciate that the physical situation here is very different than in Ref. 3 where one expands Ginsburg-Landau free energy in weak magnetic field. In our case, the field is very strong and penetrates everywhere into the superconductor; indeed, the superconductivity is due to such a strong field. Consequently, the relationship between supercurrent and fluctuating part of the vector potential is different from that given by the London equation. The coefficient $\gamma(T)$ is defined as

$$\gamma(T) \equiv -\frac{16\pi^2 g(B)T^3}{7\zeta(3)n}\sum_{k,\omega_n}\frac{\xi_n}{(\omega_n^2+\xi_k^2)^2} , \quad (8)$$

where n is the density of electrons. Clearly, the structure of kernels K_2, K_3, and K_4 is crucial to the problem; we find

$$K_2(r_1,r_2) = \frac{1}{(2\pi l^2)^2}\exp(-z_1^* z_1/2 - z_2^* z_2/2 + z_1 z_2^*) , \quad (9)$$

$$K_4(r_1,r_2,r_3,r_4) = \frac{1}{(2\pi l^2)^4}\exp[-z_1^* z_1/2 - z_2^* z_2/2 - z_3^* z_3/2 - z_4^* z_4/2 + (z_1+z_3)(z_2^*+z_4^*)/2] , \quad (10)$$

and

$$K_3(r_1,r_2) = \frac{1}{(2\pi l^2)^3}\exp(-z_1^* z_1/2 - z_2^* z_2/2 + z_1 z_2^*/2)\int dr_3 \exp(-z_3^* z_3/4 + z_1 z_3^*/2)h(z_3)\exp(-z_3^* z_3/4 + z_3 z_2^*/2) , \quad (11)$$

where

$$h(r) \equiv \frac{e}{2imc}([a^*,\partial/\partial z^*]_+ + [a,\partial/\partial z]_+) + \frac{e^2}{2mc^2}(A^* a + A a^*) ,$$

and we have now set $z/l \equiv z$.

The role of K_2 is simple; as opposed to the two-particle kernel in standard type-II superconductors, which measure the cost in "kinetic" energy arising from a nonuniform order parameter, our K_2 *projects* $\Psi(r)$ to the "lowest Landau level," i.e., all $\Psi(r)$ of the form $f(z)\exp(-z^*z/2)$, where $f(z)$ is an arbitrary analytic function, have the same T_c given by Eq. (3), while functions orthogonal to this form do not contribute at all. Note that the magnetic length in such a state is $l^* = (1/\sqrt{2})l$ and it corresponds to particles of charge $e^* = 2e$. We expect that K_4 and F_{s-b} will select those particular configurations which have favorably low free energy. Finding such configurations involves, in principle, a minimization of F with respect to $\Psi(r)$ and $a(r)$. This is a very nontrivial problem which we can avoid by simply noting that from (8) follows that the coupling between $\Psi(r)$ and $a(r)$ is of the order T_c/E_F, which translates to order $(T_c/E_F)^2$ for F_{s-b}. Therefore, in the weak-coupling approximation ($T_c \ll E_F$) we can neglect the coupling between the order parameter and the magnetic field and the minimization of the free energy reduces to minimization F_s with respect to $\Psi(r)$ at fixed magnetic field. This is similar to the case $\kappa \gg 1$ in the Abrikosov theory where κ is the Ginsburg-Landau parameter. Setting $\delta F_s/\delta\Psi^\dagger(r_1) = 0$ we obtain

$$\alpha(T)\int dr_2 K_2(r_1,r_2)\Psi(r_2) + \beta(T)\int dr_2 dr_3 dr_4 \Psi(r_2) K_4(r_1,r_2,r_3,r_4)\Psi^\dagger(r_3)\Psi(r_4) = 0\,. \tag{12}$$

We observe here that $\Psi(r) = \Psi\exp(-z^*z/2)$, where Ψ is a constant, is an *exact* solution of Eq. (12). It describes a superconducting order parameter localized to a region of size $\sim l$ around the origin, and extending to infinity with constant magnitude along the z axis. This is our superconducting "tube." This particular solution is not useful since it leads to the order parameter which is not an intensive quantity. But it suggests that we should search for solution of the form

$$\Psi(r) = \Psi \sum_i c_i f(z,R_i)\exp\left(-\frac{|z|^2}{2} - \frac{|R_i|^2}{2}\right),$$

where R_i is a set of the "guiding" centers of superconducting tubes. Function f has to be chosen in such a way that $\Psi(r)$ remains in the form imposed by K_2. This implies that $f(z,R_i)$ has to be an *analytic* function of z.

With this form of the order parameter we will have to resort to an approximate calculation. We first choose the magnitude Ψ so that $\delta F_s/\delta\Psi = 0$ without varying f. The overall magnitude Ψ is now eliminated from the problem, and the free energy becomes simply $F_s = -\alpha^2(T)/2\beta(T)K$, where K is given by

$$K = \frac{\langle ffK_4ff\rangle}{\langle fK_2f\rangle^2}\,. \tag{13}$$

The problem is now reduced to finding the set of guiding centers R_i, the set of coefficients c_i, and the form of f which give the smallest K.

Even within the above approximation, the minimization of K is a very complex problem. However, we can show that K_4 in (13) can be transformed in the form analogous to the fourth-order kernel of the Abrikosov theory. This is due to the fact that K_4 acts as a δ function on the part of Hilbert space consisting of analytic functions. Therefore, the problem of minimization of K is identical to the corresponding problem in the Abrikosov theory near H_{c2}, although the form of the free energy and the physics are quite different. Having made this observation, we expect that the lowest free-energy state arises for the choice of real vectors $\mathbf{R}_i$'s forming a hexagonal lattice in the x-y plane. For the choice of c_i's and $f(z,R_i)$ we rely on the work of Kleiner, Roth, and Autler,[6] who have determined the minimum free-energy configuration of the order parameter in the case of the Abrikosov flux lattice. We can adopt their results, *but the scale of variation of the order parameter has to be changed from the coherence length in their paper to the magnetic length in present work.* The zeros in the order parameter form a triangular lattice dual to the hexagonal lattice of tubes. The area of the elementary hexagon is $2\pi l^{*2}$; it is very important to emphasize that this quantity, at least in principle, can be arbitrarily small. In type-II superconductor the unit cell of flux lattice diverges near T_c and decreases as one lowers the temperature but it cannot be "compressed" beyond the value $2\pi\xi_0^2$, where ξ_0 is the coherence length at zero temperature. This smallest unit cell defines the maximum upper critical field through $H_{c2}(0) = \Phi_0/2\pi\xi_0^2$, Φ_0 being the elementary flux. In our case, the area of unit cell scales with the magnetic field, decreasing and becoming very small with increasing B, always containing the elementary flux, and being basically *independent* of temperature.[7] Therefore, there is *no limit* on the strength of magnetic field arising from nonuniformity of order parameter. If one ignores the effect of magnetic field on the coupling constant, the transition temperature would increase very strongly with magnetic field, as long as one remains within the limits of weak-coupling approximation.

We have now obtained a stable, stationary solution for the order parameter which represent the local minimum of the free energy (of course, one cannot exclude the possibility that the solution exists which has a lower free energy). Making use of the results in Ref. 6, we have $K = 1.16$. How do we interpret these results? The free energy is minimized by the configuration describing a Wigner lattice of superconducting tubes. It is important to note, however, that the lateral size of these tubes is essentially the same as the separation between the electrons. The extremely anisotropic Cooper-pair wave function reaches its *molecular* limit in the x-y plane. We expect, therefore, that quantum fluctuations will be important for the ultimate form of the order parameter. These fluctuations will lead to tunneling of the Cooper pairs between superconducting tubes and the system could gain further energy by them hopping from site to site. While the quantum fluctuations will have a quantitative effect on our results (for example, they will renormalize the mean-field transition temperature) we do not expect any qualitative changes in properties obtained from the mean-field theory. In this respect, the role of quantum fluctuations in our problem is similar to that in the system of *coupled* superconducting chains. Qualitatively new effects may arise, however, if the motion of electrons is restricted along the direction of magnetic field. They would be particularly pronounced in a film geometry, with **B** perpendicular to the film and the thickness comparable to coherence length. The situation is then somewhat similar to the one suggested by Kivelson, Kallin, Arovas, and Schrieffer[8] for the quantum Hall effect. There the Wigner lattice of electrons melts into the incompressible quantum liquid via the cooperative ring exchanges. In our case, we can consider the Cooper pairs to behave like bosons of charge $2e$; R_i are the coordinates describing the positions of these bosons in the x-y plane. By analogy, we would expect that our Wigner lattice of superconducting tubes may melt via cooperative ring exchanges into the highly correlated quantum liquid, once the superconducting density in the tubes is sufficiently large. One, however, does not expect such a liquid to be strictly incompressible, as it is the case for purely two-dimensional fermionic system. Whether such a state would be realized in some portion of the T-B phase diagram is clearly a question that deserves further study. We also expect that near T_c the thermal fluctuations of the Wigner lattice will be very important. As pointed out in Ref. 9, the fluctuations drive the transition to the Abrikosov flux lattice first order below six dimensions. We expect that a similar effect takes place in our case and the superconducting transition in very high magnetic field may be discontinuous.

To summarize, we propose here that a new type of superconducting state may exist at very high magnetic field. This is no upper critical field in such a state, and the superconductivity may be enhanced by increasing the magnetic field. The stable configuration of the superconducting order parameter resembles the Abrikosov vortex lattice for type-II superconductors, but the scale of variation of the order parameter is set by magnetic length. The density of zeros of the order parameter is nearly independent of temperature and is fixed by magnetic field; there is always a unit flux per elementary plaquette. Experimentally, multivalley semiconductors and semimetals, with low electronic density and strong electron-phonon coupling, appear to be good candidates to observe this new superconducting state. In these systems we do not expect the magnetic field to significantly affect the band structure and change the nature of interactions from that in zero field. The mixing of the valleys due to orbital coupling to the field which changes the effective Coulomb repulsion is proportional to $\exp(-Q^2l^2)$, where Q is the wave vector separating the valleys in reciprocal space. Q^{-1} is of the order of *interatomic* spacing while the superconducting state proposed here develops for l comparable to *interelectronic* separation. In degenerately doped semiconductors and semimetals the latter is much larger than the former and the intervalley mixing due to strong magnetic field will be small. One would particularly like to utilize those materials which are superconductors in zero field and are known to have a strong intervalley component of electron-phonon coupling.

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