Quasi-one-dimensional disordered bipolaronic superconductor

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We investigate superconductivity in a quasi-one-dimensional bipolaron system with random potentials, treating the interchain Josephson couplings in the mean-field approximation. The model is transformed into the $S=\frac{1}{2}$ XXZ-spin chain with random fields along the z axis and the ordering field along the x axis. Using the quantum transfer-matrix method by the Suzuki-Trotter formula, we calculate the order parameter and the rigidity as functions of the temperature, the mean strength of the random fields, and the ordering field. We obtain the two-parameter scaling laws for these quantities. We also discuss these scaling relations by using the cumulant expansion in the phase Hamiltonian. This analytical method is extended to the more general models of one-dimensional interacting electrons, and the generalized susceptibilities for various long-range orderings are discussed in the light of the scaling laws.

I. INTRODUCTION

The interplay between randomness and superconductivity (SC) has recently gained renewed interest in the light of interest in Anderson localization and metal-insulator transition. It is recognized that Anderson's theorem[1,2] for nonmagnetic impurities is valid to zeroth order in $\hbar/\varepsilon_F\tau$, where $\hbar, \varepsilon_F$ and τ are the Planck's constant, the Fermi energy, and the elastic mean free time, respectively. Quantum effects, i.e., higher-order effects in $\hbar/\varepsilon_F\tau$, are roughly considered in two steps. One effect is on the pairing of two electrons. The randomness enhances the effective Coulomb repulsion and decreases the transition temperature, as has been discussed for the weakly localized regime using perturbation theory[3-5] and in the critical region near the metal-insulator transition.[6] The other effect is on the coherency. Even if all the electrons form tightly bound pairs (we shall call this a bipolaron in this paper), superconductivity as the result of their Bose condensation is disturbed by randomness. The most fundamental quantity which characterizes the superconductivity and superfluidity is the rigidity of the macroscopic quantum phase, which is nothing but the superfluidity density. Incorporating the scaling theory of Anderson localization with BCS theory, Ma and Lee[7] discussed the rigidity and the order parameter as functions of the mean strength of the random potentials. They claimed that the order parameter was homogeneous and little affected by the random potentials even into the insulating phase if the condition $N_0\Delta(0)\xi^3>1$ is satisfied, where $N_0, \Delta(0)$ and ξ are the density of states at the Fermi energy, half of the zero-temperature gap in the absence of the random potentials and the localization length, respectively. The rigidity, on the other hand, is a rapidly decreasing function of the randomness and is of the order of $[\Delta(0)/\varepsilon_F]^{1/3}$ times its pure value near the metal-insulator transition. This indicates that the rigidity is more sensitive to the extent of the wave functions because the overlap between the wave functions transmits the coherency and rigidity. Ma, Lee, and Halperin[8] further discussed the strongly localized regime where $N_0\Delta(0)\xi^3>1$ is not satisfied. The relevant model in this case is the XY-spin model with the transverse random fields along the z direction. They concluded that the destruction of SC is due to the interplay between the quantum fluctuations and the randomness.

(Quasi)-one-dimensional (1D) systems have some particular features in this problem. Interactions between electrons and the random potentials both have the essential influence on the electronic states in 1D because the electron motion is strictly restricted and the effects of the interactions are essential. Mathematically speaking, there are many kinds of infrared-divergent diagrams, which leads to the following conclusions: (1) In the absence of the electron-electron interactions, all the electronic states are localized exponentially, however weak the random potentials are. (2) The asymptotic behavior of the various correlation functions of the interacting electron gas in 1D is governed by the fixed line (not a point) with the continuously varying critical exponents. (3) The disordered interacting electron system in 1D shows a metal-insulator transition when the strength of the interactions changes.

The correlations of the SC and the charge-density wave (CDW) in the 1D disordered interacting electron system has been investigated by the Monte Carlo simulation[9] and by the quantum transfer matrix method.[10] The former work stressed the conjugate nature of the SC and CDW fluctuations. The latter work found the one-parameter scaling law, and the correlation function of the SC is also cutoff at the Fukuyama-Lee length, which characterizes the size of the coherent domain of the CDW.

Taking the 3D interactions into account in the mean-field approximation, we can discuss the long-range ordering of the SC in quasi-1D systems. Fukuyama *et al.*[11] discussed the effect of a single impurity in the superconducting phase. They pointed out the possibility of the soliton formation around the impurity. Suzumura[12] ex-

tended this work to the many-impurities case and discussed the locally pinned states in the superconductor. By using the Monte Carlo method, Imada[13] investigated the same problem. The information concerning the phase of the order parameter could not be obtained because the Metropolis method cannot be applied to the problem which includes complex quantities.

In this paper, we investigate the quasi-1D bipolaron system with random potentials treating the interchain Josephson couplings in the mean-field approximation. The model is transformed into the $S=\frac{1}{2}$ XXZ chain with random fields along the z axis and the ordering field along the x axis. This model itself is of interest from the viewpoint of the statistical mechanics of the quantum spin chains, and is closely related to the more general models of 1D interacting electron systems as will be discussed in Sec. V. Using the quantum transfer matrix method[14–16] using the Suzuki-Trotter formula,[17] we calculate the average of the order parameter $\langle\langle S^x\rangle\rangle$ and the rigidity R as functions of the inverse of the temperature β, the mean strength of the random fields σ and the ordering fields H^x.

Our main conclusions are summarized as follows:

(1) Around the single impurity, the amplitude of the order parameter $\langle S_l^x\rangle$ at the site l is little affected though the charge density $\langle S_l^z\rangle$ changes drastically. The phase soliton is not introduced.

(2) The average of the order parameter $\langle\langle S^x\rangle\rangle$ and the rigidity R are described by the following two-parameter scaling laws:

$$\langle\langle S^x\rangle\rangle \sim (H^x)^{1/(4\eta-1)} f((H^x)^2\beta^{4-1/\eta},\sigma^2\beta^{3-\eta}) , \tag{1a}$$

$$R \sim g((H^x)^2\beta^{4-1/\eta},\sigma^2\beta^{3-\eta}) , \tag{1b}$$

where η is a measure of the quantum fluctuation and is given in Eq. (16c). The scaling functions $f(x,y)$ and $g(x,y)$ are obtained numerically (Figs. 9). It is found that R decreases more rapidly than $\langle\langle S^x\rangle\rangle$ as a function of σ, showing that R is more sensitive to the localization effect.

(3) Combining the self-consistent equation and Eq. (1a), we obtain the following scaling relations for the transition temperature T_c, the order parameter $\langle\langle S^x\rangle\rangle$, and the rigidity R as functions of σ:

$$T_c(\sigma)=T_{c0}h\left[\frac{\sigma}{T_{c0}^{(3-\eta)/2}}\right] . \tag{2a}$$

At zero temperature,

$$\langle\langle S^x\rangle\rangle(\sigma)=T_{c0}^{1/2\eta}\psi\left[\frac{\sigma}{T_{c0}^{(3-\eta)/2}}\right] , \tag{2b}$$

$$R(\sigma)=\varphi\left[\frac{\sigma}{T_{c0}^{(3-\eta)/2}}\right] , \tag{2c}$$

where T_{c0} is the transition temperature in the pure system.

(4) The generalized susceptibilities $\chi(\beta)$' s for various long-range orderings in the more general 1D disordered interacting electron system are described by the following one-parameter scaling law:

$$\chi(\beta)=\chi_0(\beta)f\left[\frac{\xi_\sigma}{\beta}\right] , \tag{3}$$

where $\chi_0(\beta)$ is the generalized susceptibility in the pure system ($\sigma=0$). The physical meaning of this scaling law is to prevent the system to go to the "low temperature region" expressed as follows:

$$\chi(\beta)\sim\chi_0[\min(\beta,\xi_\sigma)] . \tag{3'}$$

In Sec. II, the model is introduced. The approach using the phase Hamiltonian, which is believed to describe the low-energy properties of our lattice model, and the cumulant expansion is discussed in Sec. III. The results of the numerical simulation are given in Sec. IV. Discussion and conclusions are given in Sec. V.

II. MODEL

We consider the following 1D Hamiltonian.

$$\mathcal{H}=-\sum_{l,\alpha}\left[t\exp\left[-i\frac{eAa}{c\hbar}\right]C_{l\alpha}^\dagger C_{l+1\alpha}+t\exp\left[i\frac{eAa}{c\hbar}\right]C_{l+1\alpha}^\dagger C_{l\alpha}\right] , \tag{4}$$

where $C_{l\alpha}(C_{l\alpha}^\dagger)$ is the annihilation (creation) operator of the electron at site l with spin α. $n_{l\alpha}$ is $C_{l\alpha}^\dagger C_{l\alpha}$ and $n_l=\sum_\alpha n_{l\alpha}$. t is the hopping integral and the vector potential A induces the phase change

$$(e/c\hbar)\int_C \mathbf{A}\cdot d\mathbf{s}=\pm(eAa/c\hbar) ,$$

where the line integral is performed along the electron path C. (e is the minus of the electron charge, c is the velocity of light, and a is the lattice constant. Below in this paper, a is taken to be the unit of the length.) It should be noted that this vector potential does not represent the physical magnetic field because the rotation of it vanishes. It is the test field to measure the rigidity as will be discussed herein. ε_l is the random potential at site l with the average 0 because we deal with the half-filled case. $-U$ and V are the on-site attraction and the nearest-neighbor interaction, respectively. In the limit of large U, all the electrons form singlet electron pairs (bipolarons) with the binding energy U. As a result, the spin degrees of freedom are quenched, but there remain large degeneracies with respect to the distribution of the bipolarons. All the other interactions (t,ε_l,V) are treated perturbatively and the effective Hamiltonian is derived for the charge degrees of freedom. This has already been discussed by Emery[18] for the pure ($\varepsilon_l=0$) system, and including ε_l is an easy task to reach the following effective Hamiltonian:

$$\mathcal{H}_{\text{eff}}=-\sum_l\left\{\frac{J_x}{2}\left[\exp\left[-i\frac{2eA}{c\hbar}\right]S_l^+S_{l+1}^- + \exp\left[i\frac{2eA}{c\hbar}\right]S_{l+1}^+S_l^-\right]+J_zS_l^zS_{l+1}^z\right\} -\sum_l H_l^zS_l^z , \tag{5}$$

where

$$S_l^+ = C_{l\uparrow}^\dagger C_{l\downarrow}^\dagger \ , \tag{6a}$$

$$S_l^- = C_{l\downarrow} C_{l\uparrow} \ , \tag{6b}$$

$$S_l^z = (C_{l\uparrow}^\dagger C_{l\uparrow} + C_{l\downarrow}^\dagger C_{l\downarrow} - 1)/2 \ . \tag{6c}$$

The phase change doubles because a bipolaron (two electrons) moves. The exchange integrals J_x and J_z, and the random field H_l^z are given by

$$J_x = 2t^2/U \ , \tag{7a}$$

$$J_z = -2t^2/U - V \ , \tag{7b}$$

$$H_l^z = 2\varepsilon_l \ , \tag{7c}$$

respectively. The random field H_l^z at each site is generated obeying the Gaussian distribution with the average 0 (half-filled) and the correlation given by

$$\langle H_l^z H_{l'}^z \rangle = \sigma^2 \delta_{l,l'} \ , \tag{8}$$

where $\delta_{l,l'}$ is the Kronecker delta. In Eq. (5), we have neglected the randomness in J_x and J_z because they are of higher order in $1/U$. In addition to the Hamiltonian for one chain Eq. (5), we introduce the Josephson couplings between nearest-neighbor chains given by

$$\mathcal{H}_{\rm JC} = -J_\perp \sum_l \sum_{\langle n,m \rangle} (S_{ln}^x S_{lm}^x + S_{ln}^y S_{lm}^y) \ , \tag{9}$$

where n and m are the indices of chain and $\langle n,m \rangle$ indicates nearest-neighbor pairs. The exchange $J_\perp$ is $2t_\perp^2/U$ where $t_\perp$ is the hopping integral between nearest-neighbor chains.

When the interchain coupling $J_\perp$ is small compared with J^x or J^z, it is only after the correlation of superconductivity develops enough along the chain that the interchain coupling $J_\perp$ plays roles. If the correlation length of superconductivity along the chain is large enough, the fluctuations perpendicular to the chain are suppressed. Below in this paper, we treat the interchain coupling $J_\perp$ in the mean-field approximation assuming that $J_\perp$ is small enough compared with J^x or J^z, and the random potential is weak enough to allow the correlation length to be large at low temperatures. It should be noticed that "mean field" in this paper refers to the configurational averaging as well as the quantum mechanical and thermal averagings. The mean-field approximation is equivalent to replace $\mathcal{H}_{\rm JC}$ in Eq. (9) by the infinitely long-ranged interaction $\mathcal{H}_{\rm JC}^{\rm MF}$ given by

$$\mathcal{H}_{\rm JC}^{\rm MF} = -\frac{zJ_\perp}{N} \sum_l \sum_n \sum_m \left[S_{ln}^x S_{lm}^x + S_{ln}^y S_{lm}^y \right] \ , \tag{9'}$$

where z is the number of the nearest-neighbor chains, N is the total number of chains, and the summations, with respect to n and m, run over all the chains. The Hamiltonian Eq. (4) together with the interchain Hamiltonian Eq. (9') gives the following Hamiltonian for one chain:

$$\mathcal{H} = -\sum_l \left\{ \frac{J_x}{2} \left[\exp\left[-i\frac{2eA}{c\hbar} \right] S_l^+ S_{l+1}^- + \exp\left[i\frac{2eA}{c\hbar} \right] S_{l+1}^+ S_l^- \right] + J_z S_l^z S_{l+1}^z \right\} - \sum_l H_l^z S_l^z - \sum_l H_l^x S_l^x \ , \tag{10}$$

where

$$H_l^x = 2zJ_\perp \frac{1}{N} \sum_n S_{ln}^x \ , \tag{11}$$

and using the hypothesis of self-averaging in the thermodynamic limit ($N \to \infty$), we can replace Eq. (11) by

$$H_l^x = 2zJ_\perp \langle\langle S_{ln}^x \rangle\rangle \ , \tag{12}$$

where the double bracket means that the ensemble average with respect to the configuration of random potentials as well as the quantum statistical average is taken. As a result, the right-hand side of Eq. (12) is independent of the site index l and we denote it $\langle\langle S^x \rangle\rangle$. Therefore, Eq. (12) is rewritten as

$$H_l^x = H^x = 2zJ_\perp \langle\langle S^x \rangle\rangle \ \text{(independent of } l\text{)} \ . \tag{13}$$

Using the similar argument, $\langle\langle S^x \rangle\rangle$ has another expression,

$$\langle\langle S^x \rangle\rangle = \frac{1}{L} \sum_l \langle S_l^x \rangle \ , \tag{14}$$

where L is the number of sites along the chain.

In summary, we calculate $\langle\langle S^x \rangle\rangle$ through Eq. (14) in the 1D Hamiltonian Eq. (10) as a function of the temperature $T(=1/\beta)$, the mean strength of the random potentials σ and the ordering field H^x, and solve the self-consistent equation (13) to determine H^x (and $\langle\langle S^x \rangle\rangle$) as a function of β and σ.

It should be noticed that there are some essential differences between Eq. (9) and Eq. (9'). The domain argument is crucially dependent on the dimensionality of the system,[13] and the frustration, which exists in higher dimensional systems, does not occur in our 1D Hamiltonian Eq. (10).

In the following part of this paper, J_x is taken to be the unit of energy and Δ is J_z/J_x.

III. CUMULANT EXPANSION

It is well known that the ground state and the low-energy excitations can be described in terms of the bosonized Hamiltonians or the phase Hamiltonian derived from it.[19,20] We skip the derivation, and only write down the phase Hamiltonian,

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}' \ , \tag{15a}$$

$$\mathcal{H}_0 = \int dx \left[\hat{A} \left[\frac{d\theta_+(x)}{dx} \right]^2 + \hat{C} \left[p_+(x) - \frac{eA(x)}{2\pi\hbar c} \right]^2 \right] \ , \tag{15b}$$

$$\mathcal{H}' = -\int dx \left[\frac{H^x}{\pi\alpha}\cos\theta_-(x) + \frac{H^z(x)}{\pi\alpha}\cos[\theta_+(x)+2k_F x] \right] , \tag{15c}$$

where α and k_F are the momentum cutoff and the Fermi momentum, respectively. The coefficients $\hat{A}, \hat{C}$ and the measure of the quantum fluctuation η are given by

$$\hat{A} = \frac{(1-\Delta^2)^{1/2}(1-\frac{2}{\pi}\arcsin\Delta)}{16\arccos\Delta} , \tag{16a}$$

$$\hat{C} = \frac{\pi^2(1-\Delta^2)^{1/2}}{1-\frac{2}{\pi}\arccos\Delta} , \tag{16b}$$

$$\eta = \frac{1}{2\pi}\sqrt{C/A} = \frac{2}{1-\frac{2}{\pi}\arcsin\Delta} . \tag{16c}$$

$H^z(x)$ is the continuum version of the random field H_l^z, and its average and correlation are given by

$$\langle H^z(x)\rangle = 0 , \tag{17a}$$

$$\langle H^z(x)H^z(x')\rangle = \sigma^2\delta(x-x') . \tag{17b}$$

The phases $\theta_+(x)$ and $\theta_-(x)$ are conjugate to each other quantum mechanically. $p_+(x)$ is the momentum operator conjugate to $\theta_+(x)$.

$$[\theta_+(x), p_+(x')] = i\delta(x-x') , \tag{18}$$

and $\theta_-(x)$ is related to $p_+(x)$ by the following relation:

$$p_+(x) = -\frac{1}{4\pi}\frac{d\theta_-(x)}{dx} . \tag{19}$$

In the original spin language, $\theta_+(x)$ is related to the z component of the spin:

$$S_l^z = \left[\frac{1}{2\pi}\frac{d\theta_+(x)}{dx} + \frac{1}{\alpha}\cos[\theta_+(x)+2k_F x] \right]_{x=la} , \tag{20}$$

$\theta_-(x)$, on the other hand, is related to the x and y components of the spin.

$$S_l^x \sim [\cos\theta_-(x)]_{x=la} , \tag{21a}$$

$$S_l^y \sim [\sin\theta_-(x)]_{x=la} , \tag{21b}$$

From the preceding equation, it is clear that θ_- is the angle within the xy plane, and is nothing but the Josephson phase. Superconductivity is the breaking of the rotational symmetry in this xy plane, and the ordering field H^x prefers the phase to be the multiple of 2π. The effect of the vector potential $A(x)$ is to replace $\theta_-(x)$ $[p_+(x)]$ by

$$\theta_-(x) + \frac{2e}{\hbar c}\int^x A(x')dx' \; [p_+(x) - (e/2\pi\hbar c)A(x)]$$

as is evident from Eqs. (10) and (21).

We discuss in this paper the two physical quantities, i.e., the order parameter $\langle\langle S^x\rangle\rangle$ and the rigidity R. From Eq. (21a), $\langle\langle S^x\rangle\rangle$ is roughly given by $\langle\langle\cos\theta_-(x)\rangle\rangle$ where the double bracket has the same meaning as in Eq. (12). The rigidity R is related to the increase in the free energy F due to the small vector potential A.

$$F(A) - F(0) \propto R A^2 , \tag{22}$$

and this leads to the London equation for the supercurrent J:

$$J = -\frac{\partial F}{\partial A} \propto -R A . \tag{23}$$

The above discussion clarifies that the rigidity R is essentially the density of superconducting electrons. In the spin language, the rigidity is the stiffness constant against the distortion of the phase in the xy plane.

Now we derive the explicit expression of the rigidity R. We rewrite Eqs. (15) as follows:

$$\mathcal{H}_A = \mathcal{H}_{A=0} - \frac{Ce}{\pi\hbar c}\int dx \; A(x)p_+(x) + \frac{Ce^2}{(2\pi\hbar c)^2}\int dx \; A(x)^2 . \tag{24}$$

The free energy up to the second order in $A(x)$ is

$$F(A) - F(0) = \frac{Ce^2}{(2\pi\hbar c)^2}\int dx \; A(x)^2 - \frac{C^2e^2}{2(\pi\hbar c)^2\beta}\int_0^\beta d\tau_1\int_0^\beta d\tau_2\int dx_1\int dx_2 \, A(x_1)A(x_2)\langle\langle T_\tau p_+(x_1,\tau_1)p_+(x_2,\tau_2)\rangle\rangle , \tag{25}$$

where T_τ is the time ordering operator. In the absence of the H^x term in Eqs. (15), it can be shown that the first term cancels the second term in the right-hand side of Eq. (25) because $A(x)$ does not represent the physical magnetic field (see Appendix).

At finite temperatures, there are no phase transitions or sharp change in 1D. All the physical quantities are continuous as functions of temperature $T(=1/\beta)$ and the parameters in the Hamiltonian. Considering this fact, we expand the physical quantities in terms of the perturbative Hamiltonian $\mathcal{H}'$ in Eqs. (15). This expansion has already been discussed in the case where $H^z(x)$ exists.[10,21] Our discussion below is its extension to the case of two kinds of fields H^x and $H^z(x)$.

The explicit forms are as follows:

$$\langle\langle \cos\theta_-(x)\rangle\rangle = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{n!}\frac{1}{m!}\int_0^\beta d\tau_1\cdots\int_0^\beta d\tau_n\int_0^\beta d\tau_1'\cdots\int_0^\beta d\tau_m'\int dx_1\cdots\int dx_n\int dx_1'\cdots\int dx_m'$$
$$\times\left[\frac{H^x}{\pi\alpha}\right]^n\frac{1}{(\pi\alpha)^m}\langle H^z(x_1')\cdots H^z(x_m')\rangle_{\rm ens}$$
$$\times\langle T_\tau\cos\theta_-(x)\cos\theta_-(x_1,\tau_1)\cdots\cos\theta_-(x_n,\tau_n)$$
$$\times\cos[\theta_+(x_1',\tau_1')+2k_Fx_1']\cdots\cos[\theta_+(x_m',\tau_m')+2k_Fx_m']\rangle_{0C}\ , \tag{26a}$$

$$\langle\langle p_+(x,\tau)p_+(0,0)\rangle\rangle = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{n!}\frac{1}{m!}$$
$$\times\int_0^\beta d\tau_1\cdots\int_0^\beta d\tau_n\int_0^\beta d\tau_1'\cdots\int_0^\beta d\tau_m'\int dx_1\cdots\int dx_n\int dx_1'\cdots\int dx_m'$$
$$\times\left[\frac{H^x}{\pi\alpha}\right]^n\frac{1}{(\pi\alpha)^m}\langle H^z(x_1')\cdots H^z(x_m')\rangle_{\rm ens}$$
$$\times\langle T_\tau p_+(x,\tau)p_+(0,0)\cos\theta_-(x_1,\tau_1)\cdots\cos\theta_-(x_n,\tau_n)$$
$$\times\cos[\theta_+(x_1',\tau_1')+2k_Fx_1']\cdots\cos[\theta_+(x_m',\tau_m')+2k_Fx_m']\rangle_{0C}\ , \tag{26b}$$

where $\langle\ \rangle_{0C}$ and $\langle\ \rangle_{\rm ens}$ means the cumulant averaged over $\mathcal{H}_0$ and the ensemble average, respectively.

In Eqs. (26), not all the terms contribute. Only the even terms with respect to $H^z(x)$ survive the ensemble average. And in Eq. (26a) only odd order terms with respect to H^x contribute because all the $\cos\theta_-$ should pair. In Eq. (26b), only even terms with respect to H^x contribute due to the same reason, and the contribution from the terms with $n=0$ cancel the first term in Eq. (25). The resultant expressions are as follows:

$$\langle\langle \cos\theta_-(x)\rangle\rangle = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{(2n+1)!}\frac{1}{(2m)!}$$
$$\times\int_0^\beta d\tau_1\cdots\int_0^\beta d\tau_{2n+1}\int_0^\beta d\tau_1'\cdots\int_0^\beta d\tau_{2m}'\int dx_1\cdots\int dx_{2n+1}\int dx_1'\cdots\int dx_{2m}'$$
$$\times\left[\frac{H^x}{\pi\alpha}\right]^{2n+1}\frac{1}{(\pi\alpha)^{2m}}\langle H^z(x_1')\cdots H^z(x_{2m}')\rangle_{\rm ens}$$
$$\times\langle T_\tau\cos\theta_-(x)\cos\theta_-(x_1,\tau_1)\cdots\cos\theta_-(x_{2n+1},\tau_{2n+1})$$
$$\times\cos[\theta_+(x_1',\tau_1')+2k_Fx_1']\cdots\cos[\theta_+(x_{2m}',\tau_{2m}')+2k_Fx_{2m}']\rangle_{0C}\ , \tag{27a}$$

$$\langle\langle p_+(x,\tau)p_+(0,0)\rangle\rangle - \langle\langle p_+(x,\tau)p_+(0,0)\rangle\rangle_{H^x=0}$$
$$= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{(2n+2)!}\frac{1}{(2m)!}\int_0^\beta d\tau_1\cdots\int_0^\beta d\tau_{2n+2}\int_0^\beta d\tau_1'\cdots\int_0^\beta d\tau_{2m}'\int dx_1\cdots\int dx_{2n+2}\int dx_1'\cdots\int dx_{2m}'$$
$$\times\left[\frac{H^x}{\pi\alpha}\right]^{2n+2}\frac{1}{(\pi\alpha)^{2m}}\langle H^z(x_1')\cdots H^z(x_{2m}')\rangle_{\rm ens}$$
$$\times\langle T_\tau p_+(x,\tau)p_+(0,0)\cos\theta_-(x_1,\tau_1)\cdots\cos\theta_-(x_{2n+2},\tau_{2n+2})$$
$$\times\cos[\theta_+(x_1',\tau_1')+2k_Fx_1']\cdots\cos[\theta_+(x_{2m}',\tau_{2m}')+2k_Fx_{2m}']\rangle_{0C}\ . \tag{27b}$$

Now we carry out the power counting of the integrand of each term. When $x, x_i (1\le i\le 2n+1$ [Eq. (27a)] or $2n+2$ [Eq. (27b)]), $x_j' (1\le j\le 2m)$ and $\tau,\tau_i (1\le i\le 2n+1$ [Eq. (27a)] or $2n+2$ [Eq. (27b)]), $\tau_j' (1\le j\le 2m)$ are of the order $\xi(\gg a)$, the asymptotic form of the (n,m) term is

$$\sigma^m\xi^{-m\eta}(H^x)^{2n+1}\xi^{-(n+1)/\eta}$$

in Eq. (27a) and

$$\sigma^m\xi^{-m\eta}(H^x)^{2n+2}\xi^{-(n+1)/\eta}\xi^{-2}$$

in Eq. (27b). The contribution to $\langle\langle S^x\rangle\rangle$ contains $(4n+3m)$ integrations up to $\xi\sim\beta$ while that to R contains $(4n+3m+2)$ integrations. Now we discuss the asymptotic behavior in the limit of large β (low temperature). When $\frac{1}{4}<\eta<3$, both $4-1/\eta$ and $3-\eta$ are positive, and each term is diverging as $\beta\to\infty$. In this case, both H^x and $H^z(x)$ are so-called relevant, and the following scaling relations are obtained for large enough β and small enough H^x and σ with the products $(H^x)^2\beta^{4-1/\eta}$ and $\sigma^2\beta^{3-\eta}$ being finite.

$$\langle\langle S^x\rangle\rangle = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} a_{n,m}(H^x)^{2n+1}\sigma^{2m} \times\beta^{2-1/\eta+n(4-1/\eta)+m(3-\eta)}$$
$$= H^x\beta^{2-1/\eta}\tilde{f}((H^x)^2\beta^{4-1/\eta},\sigma^2\beta^{3-\eta})$$
$$= (H^x)^{1/(4\eta-1)}f((H^x)^2\beta^{4-1/\eta},\sigma^2\beta^{3-\eta}) , \quad (28a)$$

$$R = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} b_{n,m}(H^x)^{2n+2}\sigma^{2m}\beta^{(n+1)(4-1/\eta)+m(3-\eta)}$$
$$= g((H^x)^2\beta^{4-1/\eta},\sigma^2\beta^{3-\eta}) , \quad (28b)$$

where $a_{n,m}$ and $b_{n,m}$ are the expansion coefficients. When $3-\eta$ is negative and $H^z(x)$ (or σ) is irrelevant, the asymptotic form of the (n,m) term is not so simple as $\sigma^{2m}\beta^{3-\eta}$. In this case, the system is metallic when σ is smaller than some critical value σ^* with $H^x=0$. The metal-insulator transition at σ^* is beyond the scope of this paper. We restrict our discussion to the case where $H^z(x)$ is relevant, that is, the insulating phase in the absence of H^x.

IV. SIMULATION

A. Method

We used the quantum transfer matrix method proceeding along the spatial direction[14–16] based on the Suzuki-Trotter formula.[17] The details of the method are found in the literatures.[10,14–16] We make some comments on it here. We believe this method is advantageous in the following respects. (1) There are no statistical errors, which enables the numerical differentiation. (2) The theoretical basis of the extrapolation with respect to $1/N_T$ (N_T is the number of the breakups along the Trotter axis) is established.[22] (3) The lattice size L can be easily extended with the CPU time proportional to L. This is advantageous for the study of random systems where the average in large system is necessary. (Self-averaging property is assumed.) (4) All the metastable configurations are taken into account exactly.

The lattice size L is 100 and the extrapolation has been performed using the values with $N_T=5$, 6, 7, and 8. We checked that the extrapolation works even at the lowest temperature ($\beta=20.0$) by comparing with the result obtained using the data with $N_T=4$, 5, 6, and 7.

All the simulations in this paper concern the case of XY model ($\Delta=0.0$ and $\eta=2.0$).

The order parameter $\langle\langle S^x\rangle\rangle$ and the rigidity R are calculated by the numerical differentiations of the free energy F with respect to the ordering field along the x axis and the phase angle $\chi\equiv 2eA/c\hbar$, respectively:

$$\langle\langle S^x\rangle\rangle = -\frac{1}{L}\frac{\partial F}{\partial H^x} , \quad (29a)$$

$$R = \left.\frac{\partial^2 F}{\partial\chi^2}\right|_{\chi=0} . \quad (29b)$$

B. Pure system ($\sigma=0$)

In the absence of the random fields, we expect the one-parameter scaling laws from Eqs. (28). Figures 1 and 2

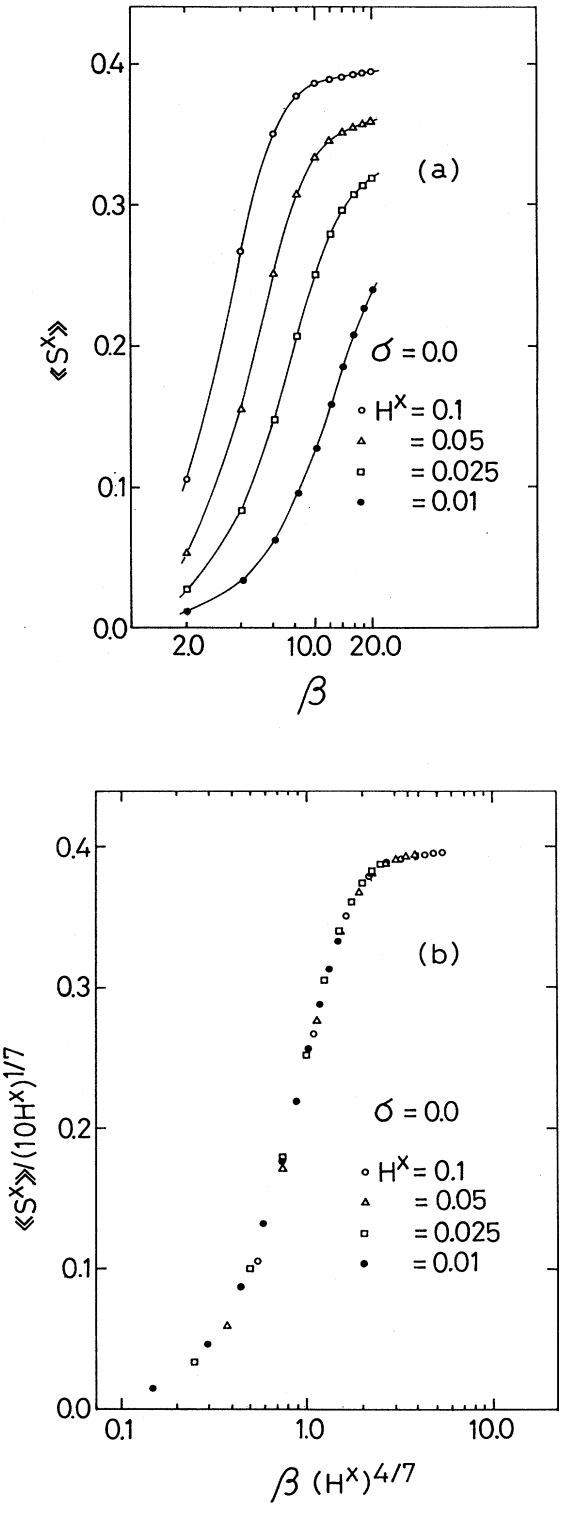


FIG. 1. (a) The order parameter $\langle\langle S^x\rangle\rangle$ as a function of the inverse temperature β for several values of H^x and $\sigma=0$ in the log-log plot. The solid curves are only the guide to the eyes. (b) The scaling analysis of the above (a). $\langle\langle S^x\rangle\rangle/(10H^x)^{1/7}$ is plotted against $\beta(H^x)^{4/7}$. We obtain a single smooth curve from the data in (a). It should be noted that we take the unit of energy as $J^x=1.0$.

show the results of our simulations. Figures 1(a) and 2(a) show $\langle\langle S^x\rangle\rangle$ and R, respectively, as functions of the inverse temperature β for several values of H^x in the log-log plot. These several curves are again plotted with the abscissa $\beta(H^x)^{4/7}$, and $\langle\langle S^x\rangle\rangle$ is divided by $(10H^x)^{1/7}$. The results are shown in Figs. 1(b) and 2(b), and all the data points form a single smooth curve in each figure. This means that the following scaling relations hold:

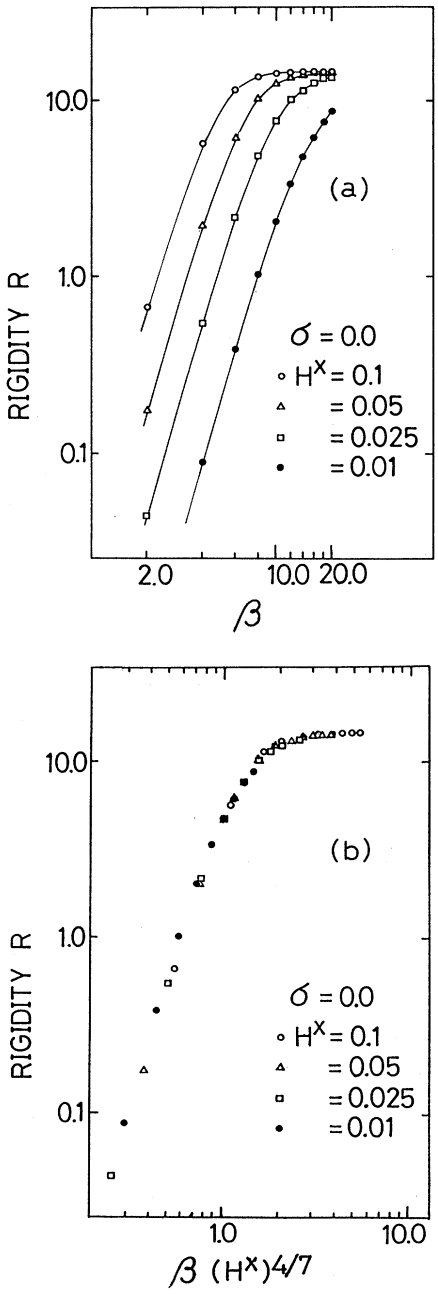


FIG. 2. (a) The rigidity R as a function of the inverse temperature β for several values of H^x and $\sigma=0$ in the log-log plot. The solid curves are only the guide to the eyes. (b) The scaling analysis of the above (a). R is plotted against $\beta(H^x)^{4/7}$. A single smooth curve is obtained from the data in (a).

$$\langle\langle S^x\rangle\rangle=(H^x)^{1/7}f_1((H^x)^{4/7}\beta)\ , \tag{30a}$$

$$R=g_1((H^x)^{4/7}\beta)\ , \tag{30b}$$

and the scaling functions $f_1(x)$ and $g_1(x)$ are given numerically in Figs. 1(b) and 2(b), respectively. Equations (30) are equivalent to Eqs. (28) with $\sigma=0$ and $\eta=2$. The Hamiltonian Eqs. (15) is reduced to the quantum sine-Gordon model in the absence of σ. The H^x term is relevant of $\eta>\frac{1}{4}$, and it leads to the gap in the energy spectrum which is proportional to $(H^x)^{2\eta/(4\eta-1)}$. Equation (30b) means that the rigidity is determined by the dimensionless ratio of the gap to the temperature with η being 2. It should be noted that if $H^x=0$, i.e., $(H^x)^{4/7}\beta=0$, the rigidity is zero, because of the gauge invariance of the system (Sec. III and Appendix). And the rigidity reaches the value $g_1(\infty)$ independent of H^x at zero temperature ($\beta\rightarrow\infty$), i.e., R has a step from zero to $g_1(\infty)$ at $H^x=0$. This is the case also in the BCS theory.

C. Configurations in the presence of the random potentials

Now we study the effects of the random potentials. As the first step, we show the expectation values $\langle S_l^x\rangle$ and $\langle S_l^z\rangle$ around a single impurity. Figure 3 shows the results. H_l^z at the impurity site is 240 times larger than the uniform field H^x along the x axis. The expectation value of the bipolaron occupation number $\langle S_l^z\rangle+\frac{1}{2}$ is almost saturated (~ 0.9) at the impurity site. $\langle S_l^z\rangle$'s show oscillatory behavior around the impurity site. This is due to the tendency toward the CDW ordering. Compared to the drastic change of $\langle S_l^z\rangle, \langle S_l^x\rangle$ is little affected by the potential.

The formation of the soliton, if it occurs, should be detected by the negative value of $\langle S_l^x\rangle$ at the impurity site. Because the 2π rotation of the phase θ_- should result in the change in the sign of $\langle S_l^x\rangle$. $\langle S_l^y\rangle$ is always zero because of the symmetry. We investigate the case of extremely large H_l^z and extremely small H^x, but we could not observe the negative value of $\langle S_l^x\rangle$. This means that

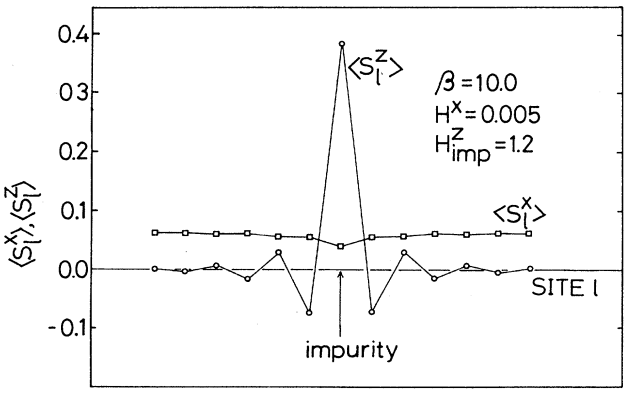


FIG. 3. Configurations around the single impurity. The charge density $\langle S_l^z\rangle$ and the amplitude of the order parameter $\langle S_l^x\rangle$ at each site are shown. $\langle S_l^z\rangle$ changes drastically compared to $\langle S_l^x\rangle$. $\langle S_l^y\rangle$ is zero due to the symmetry.

the expectation value of the phase $\langle \theta_-(x) \rangle$ is zero (or the integral multiple of 2π).

Figure 4 shows the expectation values of $\langle S_l^x \rangle$ and $\langle S_l^z \rangle$ with the random potential H_l^z at every site. Comparing with the single impurity case Fig. 3, the collaboration of the impurity potentials enhances the inhomogeneity of the order parameter.

D. $\langle\langle S^x \rangle\rangle$ and R in the presence of the random potentials

In the presence of the random potentials, we expect the two-parameter scaling laws. Then, we fix one of the scaling arguments in Eqs. (28), and examine the scaling relation with respect to the other argument. In Fig. 5(a), we plot $\langle\langle S^x \rangle\rangle$ as a function of β for several values of σ with $(H^x)^2\beta^{7/2}$ being fixed. In Fig. 5(b), these several curves are again plotted with the ordinate $\langle\langle S^x \rangle\rangle / (10H^x)^{1/7}$ and the abscissa $\sigma^2\beta$. We perform the similar scaling analysis for $\langle\langle S^x \rangle\rangle$ with $\sigma^2\beta$ being fixed. A single universal curve is again obtained in Fig. 6(b). These results confirm the two-parameter scaling law Eq. (28a) with $\eta = 2$.

We have carried out the same procedure also for the rigidity R. Figure 7 shows one of the results. But we could not obtain a single smooth curve with the abscissa $\sigma^2\beta$ from the data in Fig. 7. To investigate further the origin of this deviation from the scaling law, we studied the case of smaller σ. Figure 8(a) shows the rigidity R measured from its pure value, because $R(\beta, H^x, \sigma = 0)$ with $(H^x)^2\beta^{7/2}$ being fixed has small β dependence which mask the even smaller contribution from the random fields. With the abscissa $\sigma^2\beta$, we obtain a single straight line up to $\sigma^2\beta \sim 0.2$ whose slope is 1 in the log-log plot in Fig. 8(b). This means that the terms with $m = 1$ in Eq. (28b) dominate for this smaller values of the scaling variable $\sigma^2\beta$. From these results, it is concluded that for the rigidity, the values of σ greater than 0.2 with the inverse temperature β less than 20.0 do not reach the asymptotic region where our discussion in Sec. III is valid.

In Figs. 9, $\langle\langle S^x \rangle\rangle / (10H^x)^{1/7}$ and R are shown as functions of $(10H^x)^2$ and σ^2 with the inverse temperature β being fixed at 10.0. From the preceding discussion, the function in Fig. 9(a) can be regarded as the scaling function f in Eq. (28a). Strictly speaking, the function in Fig. 9(b) does not represent the scaling function g in Eq. (28b) in the region $\sigma > 0.2$, but the global feature of g is the

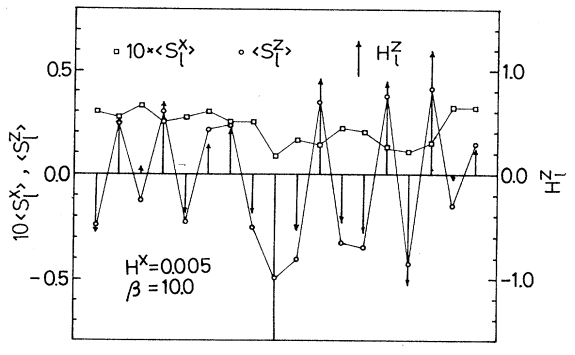


FIG. 4. The charge density $\langle S_l^z \rangle$ and the amplitude of the order parameter $\langle S_l^x \rangle$ when the random potential H_l^z (represented by the arrow) is applied at every site. $\langle S_l^y \rangle$ is zero due to the symmetry.

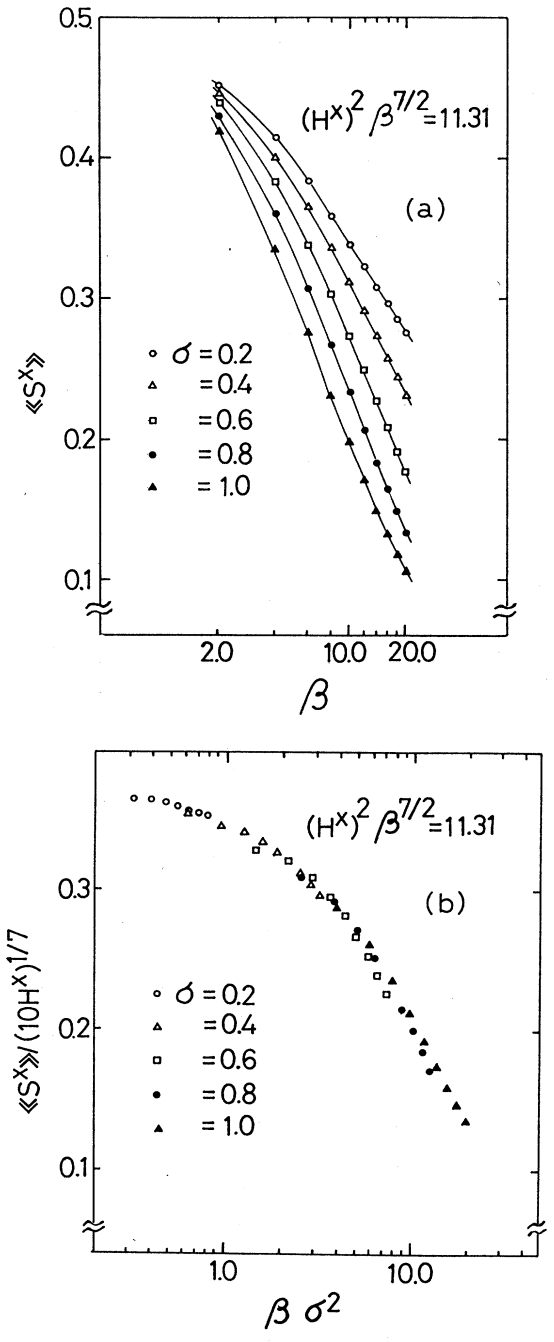
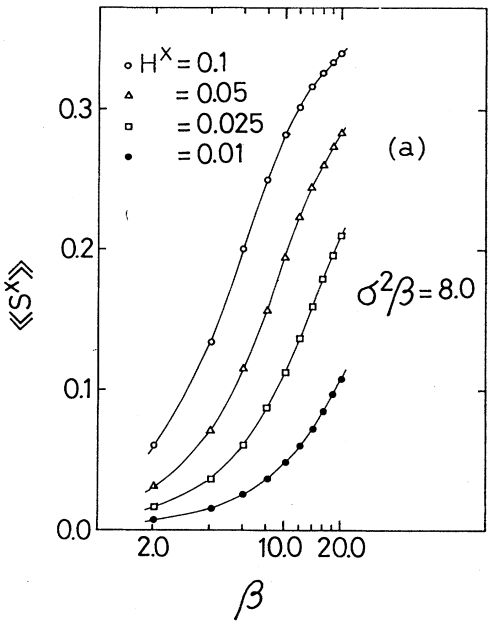


FIG. 5. (a) The order parameter $\langle\langle S^x \rangle\rangle$ as a function of the inverse temperature β for several values of σ with $(H^x)^2\beta^{7/2}$ being fixed. The solid curves are only the guide to the eyes. (b) The scaling analysis of the above (a). $\langle\langle S^x \rangle\rangle / (10H^x)^{1/7}$ is plotted against $\beta\sigma^2$. We obtain a single smooth curve from the data in (a).

same as it. Comparing Figs. 9(a) and 9(b), we conclude that the rigidity R decreases more rapidly than the order parameter $\langle\langle S^x \rangle\rangle$ as a function of σ. This is because the rigidity is more sensitive to the extent of the wave function than the order parameter, though the scaling variables are the same.



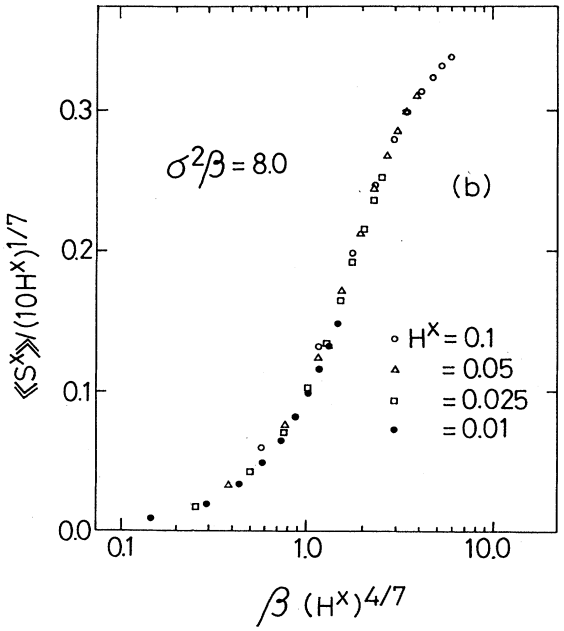


FIG. 6. (a) The order parameter $\langle\langle S^x \rangle\rangle$ as a function of the inverse temperature β for several values of H^x with $\sigma^2\beta$ being fixed. The solid curves are only the guide to the eyes. (b) The scaling analysis of the above (a). $\langle\langle S^x \rangle\rangle/(10H^x)^{1/7}$ is plotted against $\beta(H^x)^{4/7}$. We obtain a single smooth curve from the data in (a).

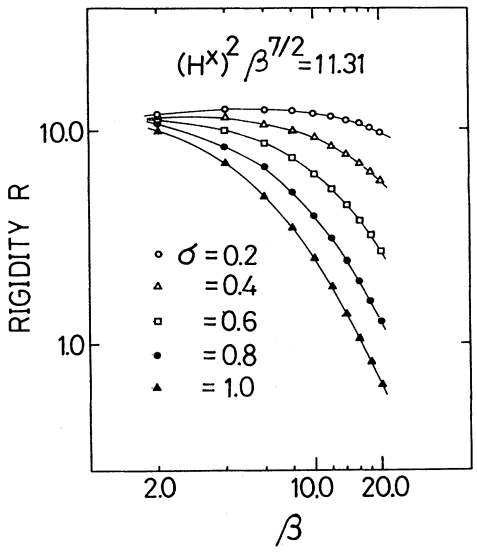


FIG. 7. The rigidity R as a function of the inverse temperature β for several values of σ with $(H^x)^2\beta^{7/2}$ being fixed. The solid curves are only the guide to the eyes. We can not obtain a scaling relation from these data. See the text.

V. DISCUSSION AND CONCLUSIONS

A. Physical meaning of the scaling laws

The scaling laws discussed thus far have simple physical meaning as follows. We have three characteristic lengths in the present problem. The first is the thermal cutoff length $\xi_\beta \sim \beta$. The second is the Fukuyama-Lee length ξ_σ including the quantum fluctuations.[10] This is the length over which the correlation of the charge density phase θ_+ and, hence, the S_l^z decays exponentially, and is proportional to $\sigma^{2/(\eta-3)}$. The last one is the width of the Josephson soliton ξ_S which is proportional to $(H^x)^{2\eta/(1-4\eta)}$. The physics is determined by the dimensionless ratios of these three length scales, and the scaling relations Eqs. (28) are rewritten as follows:

$$\langle\langle S^x \rangle\rangle = (H^x)^{1/(4\eta-1)} \bar{f}\left[\frac{\xi_S}{\xi_\beta}, \frac{\xi_\sigma}{\xi_\beta}\right], \tag{31a}$$

$$R = \bar{g}\left[\frac{\xi_S}{\xi_\beta}, \frac{\xi_\sigma}{\xi_\beta}\right]. \tag{31b}$$

The radius of the Cooper pair is zero and disappears from the problem. In the limit of zero temperature ($\beta \to \infty$), ξ_β should cancel in Eqs. (31), and we have the following relations:

$$\langle\langle S^x \rangle\rangle = (H^x)^{1/(4\eta-1)} \hat{f}\left[\frac{\xi_S}{\xi_\sigma}\right], \tag{32a}$$

$$R = \hat{g}\left[\frac{\xi_S}{\xi_\sigma}\right]. \tag{32b}$$

B. Self-consistent equation

We have established the scaling relations Eqs. (28), and the next step is to solve the self-consistent equation (13).

$$H^x = 2zJ_\perp (H^x)^{1/(4\eta-1)} f((H^x)^2\beta^{4-1/\eta}, \sigma^2\beta^{3-\eta}) \ . \tag{13'}$$

We discuss here the following two cases.

1. *Determination of T_c*

The transition temperature T_c as a function of the randomness σ is determined as a point where the nontrivial solution ($H^x \neq 0$) of Eq. (13') disappears. The result is

$$T_c(\sigma) = T_{c0} h\left[\frac{\sigma}{T_{c0}^{(3-\eta)/2}}\right], \tag{33}$$

where T_{c0} is the transition temperature in the pure system. h is the scaling function which depends only on η, and $h(0)=1$. Equation (33) implies that the critical randomness $\sigma = \sigma^*$, at which $T_c(\sigma)$ vanishes and the ground

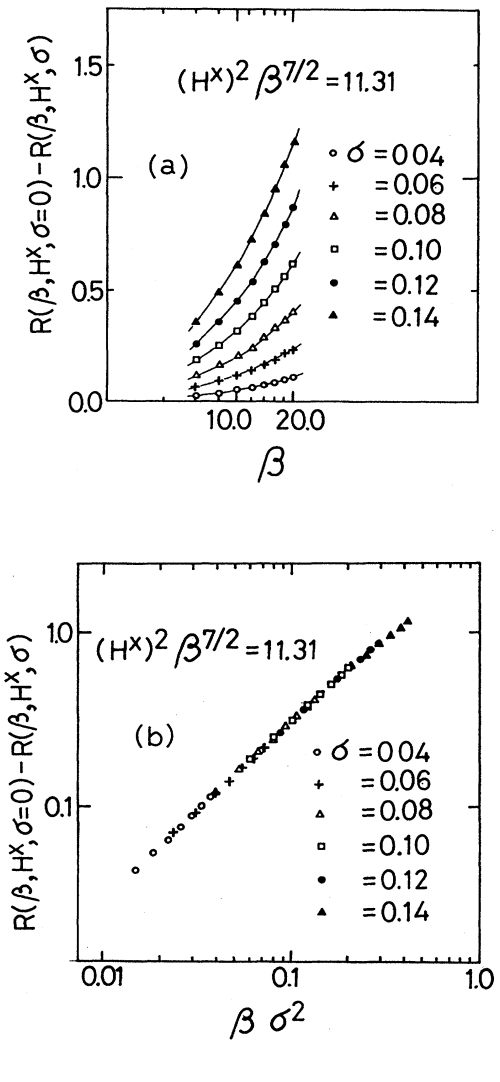


FIG. 8. (a) The rigidity R measured from its pure value ($\sigma=0$) as a function of the inverse temperature β for several values of σ with $(H^x)^2\beta^{7/2}$ being fixed. The solid curves are only the guide to the eyes. (b) The scaling analysis of the above (a). The data are again plotted with the abscissa $\beta\sigma^2$ in the log-log plot. We obtain a single straight line with the slope 1.0 for $\beta\sigma^2 < 0.2$.

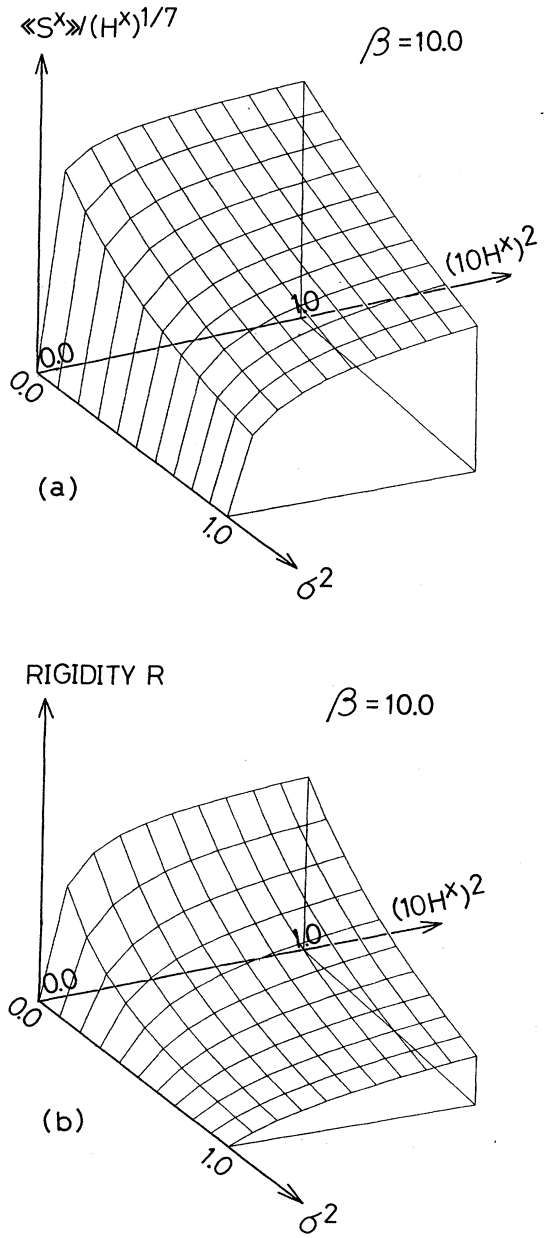


FIG. 9. (a) $\langle\langle S^x \rangle\rangle / (10H^x)^{1/7}$ as a function of $(10H^x)^2$ and σ^2 with the inverse temperature β being fixed at 10.0. This can be regarded as the scaling function f in Eq. (28a). (b) The rigidity R as a function of $(10H^x)^2$ and σ^2 with the inverse temperature β being fixed at 10.0. This can be approximately regarded as the scaling function g in Eq. (28b). See the text.

state ceases to be superconducting, is proportional to $T_{c0}^{(3-\eta)/2}$.

2. Zero-temperature properties

At zero temperature, we use the one-parameter scaling laws, Eqs. (32). For the pure system, the order parameter at zero temperature is related to the transition temperature as follows:

$$\langle\langle S^x\rangle\rangle_0 = \text{const} T_{c0}^{1/2\eta} . \tag{34}$$

The order parameter $\langle\langle S^x\rangle\rangle$ and the rigidity R as functions of σ are given by

$$\langle\langle S^x\rangle\rangle(\sigma) = \langle\langle S^x\rangle\rangle_0 \psi\left[\frac{\sigma}{T_{c0}^{(3-\eta)/2}}\right] , \tag{35a}$$

$$R(\sigma) = \varphi\left[\frac{\sigma}{T_{c0}^{(3-\eta)/2}}\right] , \tag{35b}$$

where ψ and φ are the universal scaling functions which depend only on η. $\psi(0)=1$ and $\varphi(0)\cong 14.5$ (for $\eta=2$).

C. More general 1D electron models

We thus far discussed the limiting case where the electrons form tightly bound singlet pairs. The effective Hamiltonian in this case is that of $S=\frac{1}{2}$ XXZ-spin chain and the quantum sine-Gordon model is equivalent to it in the continuum limit. It is well known that the most general models of interacting electron systems are described by the two kinds of phases, θ and ϕ, which correspond to the charge and spin degrees of freedom, respectively.[23,24] The Hamiltonian for each phase is that of the quantum sine-Gordon model, and the random potentials couple the charge and spin degrees of freedom.[23,24] The cumulant expansion method can be extended to this model, and the one-parameter scaling laws are obtained for the generalized susceptibilities (GS's) corresponding to various long-range orderings. The details will be published elsewhere, and we describe here only the physical picture. In the absence of the random potentials, the GS $\chi_0(\beta)$ is described by the product of the powers of the inverse temperature β, ξ_{charge} and ξ_{spin}, where ξ_{charge} and ξ_{spin} are the characteristic lengths of the spatial variation of the phases θ and ϕ, respectively. The effect of the random potentials can be described by the following scaling law:

$$\chi(\beta) = \chi_0(\beta) f\left[\frac{\xi_\sigma}{\beta}\right] , \tag{36}$$

where ξ_σ is the localization length which is proportional to some negative powers of σ (the mean strength of the random potentials). The physical meaning of the scaling function f is the saturation of the correlation at the localization length, i.e., to replace the inverse temperature β in χ_0 by min (β,ξ_σ). Then, the random potentials only prevent the system from going to the "low-temperature region," and it would be difficult to obtain another ordering suppressing the ordering in the pure system by introducing the randomness in quasi-1D systems.

In conclusion, we have studied the quasi-1D disordered bipoloranic superconductor by the quantum transfer matrix method. The order parameter and the rigidity are discussed analytically and numerically as functions of the temperature, the strength of the random potentials and the ordering field, and we obtain the two-parameter scaling relations.

The following problems are left for future investigations:

(1) We have restricted ourselves to the insulating region when the superconductivity is absent. The effect of the metal-insulator transition on the superconductivity in this quasi-1D system is an open problem.

(2) What is the effect of the random potentials on the competition among various long-range orderings, i.e., CDW, spin-density wave (SDW), and superconductivity.[25]

(3) The extension to higher-dimensional system[26] is desirable. In particular, the effect of the transverse random fields on the Kosterlitz-Thouless transition in 2D system is unknown.

ACKNOWLEDGMENTS

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APPENDIX

We show that when $H^x=0$, the first term cancels the second term in the right-hand side of Eq. (25). It is rather obvious from the viewpoint of the gauge invariance of the Hamiltonian Eq. (4) or the rotational symmetry within the xy plane in the spin Hamiltonian Eq. (10), but we show it explicitly to see the details of the cancellation.

When the Hamiltonian $\mathcal{H}$ in Eq. (24) does not contain the $\cos\theta_-$ term, the imaginary-time derivative of $\theta_+(x)$, i.e., the commutator of $\mathcal{H}$ and $\theta_+(x)$ is given by

$$\frac{\partial\theta_+(x,\tau)}{\partial\tau} = [\mathcal{H},\theta_+(x)] = -2Cip_+(x) . \tag{A1}$$

Then

$$\begin{aligned}\int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \langle p_+(x_1,\tau_1)p_+(x_2,\tau_2)\rangle &= \frac{i}{2C}\int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \left\langle p_+(x_1,\tau_1)\frac{\partial\theta_+(x_2,\tau_2)}{\partial\tau_2}\right\rangle \\ &= \frac{i}{2C}\int_0^\beta d\tau_1 \langle p_+(x_1,\tau_1)[\theta_+(x_2,\tau_1)-\theta_+(x_2)]\rangle \\ &= \frac{i}{2C}\left[\beta\langle p_+(x_1)\theta_+(x_2)\rangle - \int_0^\beta d\tau_1\langle p_+(x_1,\tau_1)\theta_+(x_2)\rangle\right] .\end{aligned} \tag{A2}$$

We use again Eq. (A1) in the second term of the last part of Eq. (A2).

$$\int_0^\beta d\tau_1 \langle p_+(x_1,\tau_1)\theta_+(x_2)\rangle = \frac{i}{2C}\int_0^\beta d\tau_1 \left\langle \frac{\partial\theta_+(x_1,\tau_1)}{\partial\tau_1}\theta_+(x_2)\right\rangle$$
$$= \frac{i}{2C}\langle [\theta_+(x_1,\beta)-\theta_+(x_1)]\theta_+(x_2)\rangle = \frac{i}{2C}\langle [\theta_+(x_2),\theta_+(x_1)]\rangle = 0 \ . \tag{A3}$$

Then Eq. (A2) becomes

$$\int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \langle p_+(x_1,\tau_1)p_+(x_2,\tau_2)\rangle = \frac{i}{2C}\beta\langle p_+(x_1)\theta_+(x_2)\rangle \ . \tag{A4}$$

We rewrite the left-hand side of Eq. (A2) as follows:

$$\int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \langle p_+(x_1,\tau_1)p_+(x_2,\tau_2)\rangle = \int_0^\beta d\tau_2 \int_{\tau_2}^\beta d\tau_1 \langle p_+(x_1,\tau_1)p_+(x_2,\tau_2)\rangle \ . \tag{A5}$$

Repeating the similar discussion from Eq. (A2) to Eq. (A4), we obtain another expression:

$$\int_0^\beta d\tau_1 \int_{\tau_2}^\beta \langle p_+(x_1,\tau_1)p_+(x_2,\tau_2) = -\frac{i}{2C}\beta\langle \theta_+(x_1)p_+(x_2)\rangle \ . \tag{A6}$$

Therefore, our final result is

$$\frac{C^2e^2}{2(\pi\hbar c)^2\beta}\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int dx_1 \int dx_2 \langle T_\tau p_+(x_1,\tau_1)p_+(x_2,\tau_2)\rangle A(x_1)A(x_2)$$
$$= \frac{C^2e^2}{(\pi\hbar c)^2\beta}\int dx_1 \int dx_2 \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \langle p_+(x_1,\tau_1)p_+(x_2,\tau_2)\rangle A(x_1)A(x_2)$$
$$= \frac{iCe^2}{2(\pi\hbar c)^2\beta}\int dx_1 \int dx_2 \frac{\beta}{2}\langle p_+(x_1)\theta_+(x_2)-\theta_+(x_1)p_+(x_2)\rangle A(x_1)A(x_2)$$
$$= \frac{iCe^2}{4(\pi\hbar c)^2}\int dx_1 \int dx_2 \langle [p_+(x_1),\theta_+(x_2)]\rangle A(x_1)A(x_2) = \frac{Ce^2}{(2\pi\hbar c)^2}\int dx\, A(x)^2 \ . \tag{A7}$$

Equation (A7) is nothing but the first term in the right-hand side of Eq. (25).

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