Brief Reports

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Fractional quantum Hall effect in two-layered systems

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(Received 6 June 1988)

The ground-state wave function is investigated at strong magnetic field for a system that consists of two parallel two-dimensional electron systems, which are coupled by the Coulomb interaction. Ground-state wave functions for finite-size systems on a sphere are obtained numerically and compared with Jastrow-type wave functions at Landau-level filling factors $\nu=1$, $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{1}{3}$. It is shown that the layer separations at which Jastrow-type wave functions give a good approximation to the ground state are filling-factor dependent.

The fractional quantum Hall effect is observed in a two-dimensional electron system in a strong magnetic field.[1,2] This effect comes from the realization of incompressible ground states around certain Landau-level filling factors ν. ($\nu=2\pi l^2 n$, where l is the Larmor radius, n is the electron density.) The strongest anomalies are associated with incompressible states occurring at $\nu=1/q$ (q odd) where wave functions are extremely well approximated by the Jastrow functions,[3]

$$\Psi_q(r_1,r_2,\ldots,r_N) = \prod_{1\le i<j\le N}(z_i-z_j)^q \exp\left[-\sum_i |z_i|^2/4l^2 2\right], \quad (1)$$

where $r_j=(x_j,y_j)$ is the coordinate of the jth electron and $z_j=x_j-iy_j$ is its complex representation. In Eq. (1) the spin part of the wave functions, which is left implicit, has each spin aligned with the magnetic field. The restriction of attention to this class of wave functions is usually accepted because of the strong magnetic field. On the other hand, Halperin[4] suggested the possibility of reversed spins at some filling factors and proposed Jastrow-type wave functions for two-component i.e., spin-up and spin-down fermion systems. Although such states are unlikely to be realized under the usual experimental conditions,[5] the possibility that some of the electrons have reversed spin becomes realistic at lower magnetic fields and in particular at $\nu=\frac{5}{2}=2+\frac{1}{2}$, where the fractional quantum Hall effect was observed recently.[6] In fact, Haldane and Rezayi[7] have suggested a ground-state wave function to explain this experiment, in which half of the electrons have reversed spin. Their wave function, however, is not a simple Jastrow type, and cannot be because of the following facts. The Coulomb interaction between electrons is spin independent. Thus the total spin S of the system is conserved. As a result, the orbital part of the wave function must satisfy a condition known as the Fock condition.[8] It can be shown that there is no Jastrow-type wave function at $\nu=\frac{1}{2}$ which satisfies this condition. Haldane and Rezayi[7] noticed that the Jastrow wave function, when multiplied by the permanent of a certain matrix,[9] does satisfy the Fock condition for an $S=0$ state. This modification drastically alters the correlations in the state, however, and in our judgement is unlikely to lead to a strong anomaly. (The observed anomaly at $\nu=\frac{5}{2}$ is weak.)

These developments have motivated us to investigate the conditions under which Jastrow ground states, expected to produce strong anomalies, can be realized in two-component systems. Fortunately, it has become possible to fabricate multilayer semiconductor structures, where parallel sheets of two-dimensional electrons are realized with quite high mobility.[10,11] When there are only two sheets, we can consider the system as a two-dimensional electron system with pseudospin one-half, where the up and down of the z component of the pseudospin corresponds to the sheet index. In this sense this system is equivalent to a single-layer two-dimensional electron system with negligible Zeeman energy. However, due to the separation between the two layers, the Coulomb interaction depends on the pseudospin of the electrons, and the Hamiltonian does not conserve the total pseudospin S. This fact profoundly influences the set of filling factors at which Jastrow-type wave functions become good approximations to the ground state for finite layer separation.

The model we consider here is as follows. There are two ideal two-dimensional electron systems separated by distance d. By ideal we mean that the spread of the wave

function in the direction perpendicular to the plane is negligible. The dimensionless parameters which characterize the system are $\nu=2\pi l^2 n$, and $\delta=d/l$. Here n is the total areal electron density of the two layers. For present purposes we limit our attention to the case where the electron density is the same $(n/2)$ in each layer and neglect the possibility of tunneling between the layers so that the z component of total pseudospin is a good quantum number. We emphasize that neither assumption is necessary and that interesting phenomena can occur when either assumption is relaxed. In particular, the difference between subband energies in the two layers acts like a Zeeman term which influences the z component of the isospin for which the ground state occurs and which can be controlled with a gate. It is our opinion that the fabrication of two-layer systems of sufficiently high quality to allow the observation of the phenomena discussed here will open up a rich and fruitful area for fractionally quantized Hall effect (FQHE) studies.[11] Chakraborty and Pietiläinen[12] have previously investigated this model by diagonalization of finite-size systems. They considered only the excitation spectrum of the system with $\nu=1$ and $\delta=2$. In this paper we investigate the ground-state wave function as a function of δ for various values of ν. For $\nu=1$ and $\delta=2$ our results are consistent with those of Ref. 12. We remark that as d approaches zero our model becomes equivalent to a single-layer model with vanishing Zeeman energy. In this limit the Hamiltonian is invariant under all rotations in pseudospin and the ground state must be a pseudospin eigenstate.

The general wave function for such a system with N electrons is written as

$$\Phi(r_1,\ldots,r_N)=A[\,\Psi(z_1,\ldots,z_{N/2},z_{[1]},\ldots,z_{[N/2]}) \times u_1 u_2\cdots u_{N/2} d_{[1]}\cdots d_{[N/2]}]\,, \quad (2)$$

where A is the antisymmetrization operator, $[i]=i+N/2$, $z_i=x_i-iy_i$, and u_i and d_i are eigenstates for the pseudospin component which measures the layer to which an electron belongs. We consider the following Jastrow wave function for the ("Roman-Greek") wave function $\Psi(z_1,\ldots,z_{N/2},z_{[1]},\ldots z_{[N/2]})$:

$$\Psi_{m,m,n}(z_1,\ldots,z_{N/2},z_{[1]},\ldots z_{[N/2]}) = \prod_{1\le i<j\le N/2}(z_i-z_j)^m \prod_{1\le i'<j'\le N/2}(z_{[i']}-z_{[j']})^m \prod_{\substack{1\le i''\le N/2\\ 1\le j''\le N/2}}(z_{i''}-z_{[j'']})^n \exp\left[-\sum_{i=1}^{N/2}|z_i|^2/4l^2-\sum_{i'=1}^{N/2}|z_{[i']}|^2/4l^2\right]. \quad (3)$$

The filling factor ν for this function is $2/(m+n)$ and m must be odd. This wave function generally does not satisfy the Fock condition. However, for special choices of m and n it does. For example, if $m-n=0$, this wave function gives the total pseudospin $S=N/2$, and $S_z=0$ state. This is the same state as Laughlin's Jastrow wave function at $\nu=1/m$ rotated in the pseudospin space. Therefore it gives a very good approximation to the ground state for $\nu=1$, $\frac{1}{3}$, etc., at least as d approaches zero, since there the interaction is nearly isotropic in the pseudospin space. For $m-n=1$, the wave function can be shown to give a total pseudospin $S=0$ state which closely approximate the ground states as d goes to zero. For other values of $m-n$ the Jastrow wave function is not a pseudospin eigenstate and cannot be the $d=0$ ground state. For larger layer separations the Jastrow-like ground states may occur for larger values of $m-n$ until as d approaches infinity they occur only for $n=0$, corresponding to isolated layers.

To determine where Jastrow ground states will occur, we diagonalize the Hamiltonian of a finite-size system and calculate the overlap between the ground-state wave function and $\Psi_{m,m,n}$. The actual calculations are done for spherical systems.[13] As a convenience we consider two concentric spheres with the same diameter for each layer. The surfaces of these spheres are separated by d in the fourth spatial dimension. The interaction between the electrons is the ordinary Coulomb interaction using geometrical distances. Thus

$$v^{\sigma\sigma'}(\hat{\Omega}_1,\hat{\Omega}_2)=\frac{e^2}{\epsilon(R^2|\hat{\Omega}_1-\hat{\Omega}_2|^2+d^2\delta_{\sigma,-\sigma'})^{1/2}}\,, \quad (4)$$

where R is the radius of the spheres, $\hat{\Omega}_1$ and $\hat{\Omega}_2$ are two unit vectors on the sphere that give the position of two electrons, $\sigma=\pm1$ indicates the two layers, and ϵ is the dielectric constant.

In the calculation we use pseudopotential parameters $V_l^{\sigma\sigma'}$ calculated from Eq. (4). This $V_l^{\sigma\sigma'}$ has the meaning that it is the energy of pairs of particles with relative angular momentum l. The expression for $V_l^{\sigma\sigma}$ is given by Haldane[14] for an infinite-size system and that for finite-size system is given by Fano *et al.*[15] $V_l^{\sigma-\sigma}$ is defined with obvious replacement. It is important to realize that for an infinite-size system the filling factor is given by $\nu=n/n_\phi$, where n_ϕ is the magnetic flux density measured in units of h/e. However, for a finite-size system on a sphere, the Jastrow-type wave function for ν is not realized at $\nu=N/N_\phi$, where N is the total number of electrons and N_ϕ is the total number of flux quanta. For example, Laughlin's wave function, Eq. (1), is realized at $N_\phi=q(N-1)$.[14] Similarly, $\Psi_{5,5,0}$ and $\Psi_{3,3,2}$ give filling factor $\nu=\frac{2}{5}$ for the infinite-size system, but they are realized at $N_\phi=\frac{5}{2}N-5$ and $\frac{5}{2}N-3$, respectively, for finite-size systems.

The calculation is done in the second quantized form. We obtain the Jastrow function $\Psi_{m,m,n}$ in this form numerically with appropriate choice of N_ϕ and by use of the

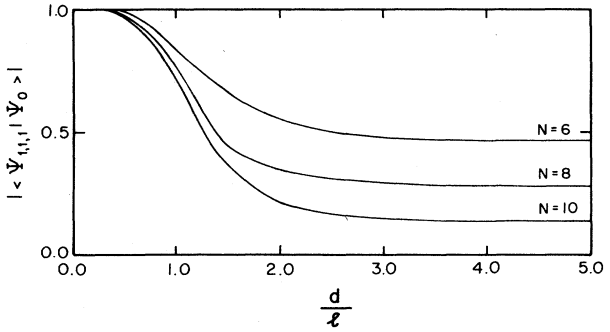

FIG. 1. Overlap between the ground-state wave function Ψ_0 and the Jastrow function $\Psi_{1,1,1}$ as a function of $d/l=\delta$. The filling factor ν is 1. Three choices of the total number of electrons, $N=6$, 8, and 10, are shown.

model pseudopotential parameters:

$$V_l^{\sigma\sigma}=\begin{cases}1, & l<m\\ 0, & l\geq m\end{cases}, \tag{5}$$

and

$$V_l^{\sigma-\sigma}=\begin{cases}1, & l<m\\ 0, & l\geq n\end{cases}. \tag{6}$$

At the appropriate value of N_ϕ the ground state with these potentials is nondegenerate, has zero energy, and is the $\Psi_{m,m,n}$ state. We have considered four different filling factors to illustrate the different possibilities. The results are presented in Figs. 1–4.

At $\nu=1$ we have only one Jastrow wave function, $\Psi_{1,1,1}$. As noticed above, this is the pseudospin polarized filled Landau-level state, which is rotated in the pseudospin space. Therefore the overlap is unity at $d=0$, and decays as $\delta=d/l$ becomes larger. We believe that this state is responsible for the excitation gap in a $\nu=1$ two-layer system noticed by Chakraborty and Pietiläinen.[12] (Notice that their definition of ν differs from ours by a factor of 2.)

At $\nu=\frac{1}{2}$, $\Psi_{3,3,1}$ is realized. This state is not an eigenstate of S. Thus it cannot be the eigenstate of the Hamiltonian at $d=0$. However, the overlap approaches unity, when δ is around 1.5. (There is a possibility that for some pseudopotential parameters $\Psi_{3,3,1}$ has much larger overlap with the ground state at $d=0$.) Since this function has better short-range correlation than the hollow core model ground state of Haldane and Rezayi,[7] this state may be related to the fractional quantum Hall effect at $\nu=\frac{5}{2}$.[6,16]

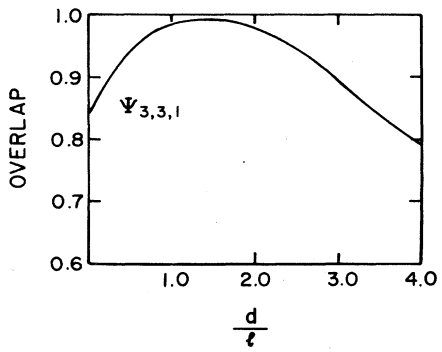

FIG. 2. Overlap between the ground-state wave function and the Jastrow function $\Psi_{3,3,1}$. The filling factor ν is $\frac{1}{2}$. The total number of electrons is $N=6$.

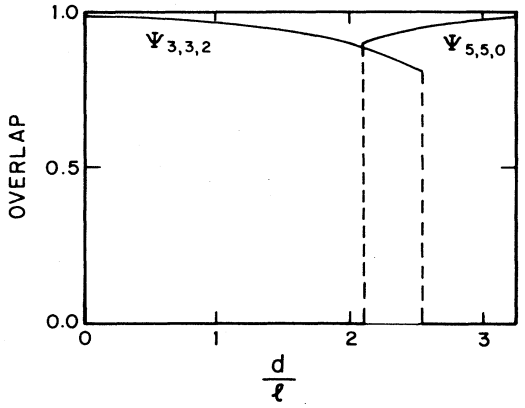

FIG. 3. Overlap between the ground-state wave function and the Jastrow functions $\Psi_{3,3,2}$ and $\Psi_{5,5,0}$. The filling factor ν is $\frac{3}{5}$. The total number of electrons is $N=6$.

At $\nu=\frac{2}{5}$ we can consider two Jastrow functions, $\Psi_{3,3,2}$ and $\Psi_{5,5,0}$. As mentioned above, $\Psi_{3,3,2}$ is the state suggested by Halperin, and it is known that the ground state at $d=0$ is well approximated by it.[5,17,18] On the other hand, as $\delta\rightarrow\infty$, the system separates into two independent systems with filling factor $\frac{1}{5}$ each. In this limit it is evident that $\Psi_{5,5,0}$ gives a good approximation. The ground state crosses over between these two states as δ changes. The numerical calculation suggests that this is a first-order transition. Since $\Psi_{3,3,2}$ and $\Psi_{5,5,0}$ are realized at different N_ϕ, both can be approximate ground states at the same value of δ. In the thermodynamic limit there should be a first-order phase transition at $\delta\simeq 2$, accom-

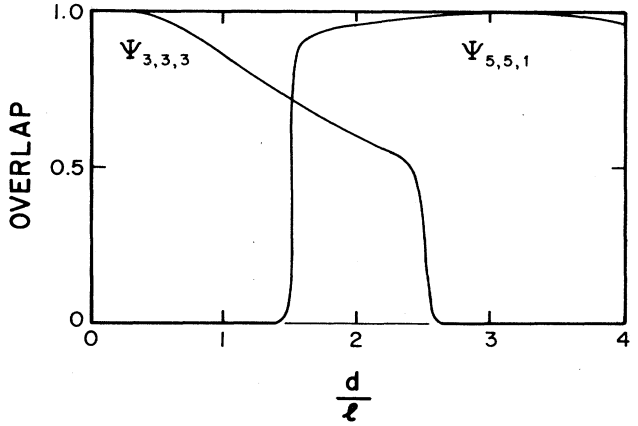

FIG. 4. Overlap between the ground-state wave function and the Jastrow functions $\Psi_{3,3,3}$ and $\Psi_{5,5,1}$. The filling factor ν is $\frac{1}{3}$. The total number of electrons is $N=6$.

panied by a large change in ρ_{xx} associated with the change in activation energies.[19]

Similarly, at $\nu=\frac{1}{3}$ we have two Jastrow functions $\Psi_{3,3,3}$ and $\Psi_{5,5,1}$. The former is the $S=N/2$ Laughlin state, and the overlap at $d=0$ is almost unity. For $\delta\rightarrow\infty$ we have two $\nu=\frac{1}{6}$ systems. The ground state there is not known but obviously differs from $\Psi_{5,5,1}$. Thus $\Psi_{5,5,1}$ is realized at an intermediate value of δ.

We have seen that simple Jastrow-type wave functions for two-component systems give a good approximation to the ground state of the two-layered system for certain choices of the parameters. Although quantitative conclusions will require investigations of the excitation spectrum,[20] it is clear that large excitation gaps and strong anomalies in the transport properties will be associated with these Jastrow-like ground states.

Crossovers in the ground states occur for d of the order of l, $\simeq 100$ Å. This value may change slightly when we take into account the finite spread of the electron wave function in the direction perpendicular to the two-dimensional plane, and if we take into account the hopping between the two layers. However, if the value does not change drastically, it should be possible to perform interesting experiments.

The authors acknowledge useful conversations with F. D. M. Haldane and E. H. Rezayi. This work was supported in part by the National Science Foundation under Grant No. DMR-88-02383 and by an allocation of computer time from the NCSA.

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[1]D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

[2]*The Quantum Hall Effect,* edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987).

[3]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[4]B. I. Halperin, Helv. Phys. Acta **56**, 75 (1983).

[5]D. Yoshioka, J. Phys. Soc. Jpn. **55**, 3960 (1986).

[6]R. Willett, J. P. Eisenstein, H. L. Störmer, D. C. Tsui, A. C. Gossard, and J. H. English, Phys. Rev. Lett. **59**, 1776 (1987).

[7]F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **60**, 956 (1988).

[8]M. Hamermesh, *Group Theory and its Application to Physical Problems* (Addison-Wesley, Reading, Mass., 1962), p. 249.

[9]Wave functions satisfying the Fock condition may be constructed at many different filling factors by combining Jastrow wave functions with the permanents of certain matrices. D. Yoshioka, A. H. MacDonald, and S. M. Girvin (Ref. 19 and unpublished).

[10]See, for example, A. Pinczuk, M. G. Lamont, and A. C. Gossard, Phys. Rev. Lett. **56**, 2092 (1986); G. Fasol, N. Mestres, H. P. Hughes, A. Fischer, and K. Ploog, *ibid.* **56**, 2517 (1986).

[11]The interesting possibilities for the fractional quantum Hall effect in two-layer systems were first discussed by E. H. Rezayi and F.D.M. Haldane, Bull. Am. Phys. Soc. **32**, 892 (1987).

[12]Numerical studies for a model in the same class as those used here were recently reported by T. Chakraborty and P. Pietiläinen, Phys. Rev. Lett. **59**, 2784 (1987).

[13]F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **54**, 237 (1985).

[14]F. D. M. Haldane, Ref. 2, Chap. 8.

[15]G. Fano, F. Ortolani, and E. Colombo, Phys. Rev. B **34**, 2670 (1986).

[16]A. H. MacDonald, D. Yoshioka, and S. M. Girvin (unpublished).

[17]E. H. Rezayi, Phys. Rev. B **36**, 5454 (1987).

[18]M. Rasolt and A. H. MacDonald, Phys. Rev. B **34**, 5530 (1986).

[19]D. Yoshioka, A. H. MacDonald, and S. M. Girvin, Phys. Rev. B **38**, 3636 (1988).

[20]A. H. MacDonald, M. Rasolt, and R. Perrot (unpublished).