Effective electron-electron interaction in metals and superconductors

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The effective electron-electron interaction (EEEI) arising in metals and superconductors due to exchange of virtual quasiparticles is investigated. The quasiparticles such as phonons, scalar and vector photons, and magnons in ferromagnetic metals are taken into consideration. The energy relaxation time and the conductivity induced by EEEI are calculated.

I. INTRODUCTION

Investigating the electron-phonon interaction in metals,[1] Migdal noticed that if the phonon lifetime was taken into account, the electron damping at low-frequency ε was $\mathrm{Im}\Sigma(\varepsilon)\sim\varepsilon^2/\varepsilon_F$, where Σ is the electron self-energy, ε_F is the Fermi energy. The electron damping arising from the screened electron-electron Coulomb interaction has the same form. The result obtained was interpreted as the effective electron-electron interaction (EEEI) associated with the exchange of virtual phonons.

This effect has been investigated in many papers.[2–5] The general description of EEEI including both the direct electron-electron interaction and the interaction due to virtual phonons was developed in Refs. 4 and 5, where additional exchange diagrams were also considered. However, in Refs. 4 and 5 only $\mathrm{Im}\Sigma$ was calculated.

Now the electron-electron interaction is studied intensively in both pure[6] and impure[7,8] metals, so it seems timely to analyze the influence of EEEI on kinetic properties of metals. The main purpose of the present paper is to derive the collision integral for EEEI and with its help to calculate such measurable values as the electron energy relaxation time and the conductivity. The electron-phonon interaction is not the only one which contributes to EEEI. As an example we will consider the electron-magnon interaction in ferromagnetic metals.

The paper is organized as follows. In Sec. II we introduce the electron-electron interaction in a gauge invariant form which allows us to consider a new electron relaxation mechanism, the interaction of electrons with vector photons. The screening effects for both the electron-electron interaction and the electron-phonon interaction are treated in the random-phase approximation (RPA). In Sec. III the EEEI via exchange of virtual magnons in ferromagnetic metals is considered. The EEEI in superconductors is investigated in Sec. IV.

II. NORMAL METAL

We use the Keldysh diagram technique for inequilibrium processes[9] in which the electron and phonon Green's functions, along with the electron and phonon self-energies, are represented by matrices

$$\hat{G}=\begin{bmatrix}0 & G^A\\ G^R & G^C\end{bmatrix},\quad \hat{D}=\begin{bmatrix}0 & D^A\\ D^R & D^C\end{bmatrix},$$
$$\hat{\Sigma}=\begin{bmatrix}\Sigma^C & \Sigma^R\\ \Sigma^A & 0\end{bmatrix},\quad \hat{\Pi}=\begin{bmatrix}\Pi^C & \Pi^R\\ \Pi^A & 0\end{bmatrix}. \tag{1}$$

At low temperatures the electron-impurity interaction is the main electron momentum relaxation process and the electron Green's functions averaged over impurity positions equals

$$G^R(\mathbf{p},\varepsilon)=\{G^A(\mathbf{p},\varepsilon)\}^*=(\varepsilon-\xi_{\mathbf{p}}+i/2\tau)^{-1},$$
$$\xi_{\mathbf{p}}=(p^2-p_F^2)/2m, \tag{2}$$

where τ is the electron momentum relaxation time due to electron-impurity scattering, p_F is the Fermi momentum, m is the electron mass, and $\{\}^*$ means complex conjugate.

To the lowest-order accuracy in a spatially inhomogeneous system, G^C is connected with G^R and G^A by the equation

$$G^C(\mathbf{p},\varepsilon)=S(\mathbf{p},\varepsilon)[G^A(\mathbf{p},\varepsilon)-G^R(\mathbf{p},\varepsilon)]. \tag{3}$$

The collision integral equals

$$I(\mathbf{p},\varepsilon)=-i\{\Sigma^C(\mathbf{p},\varepsilon)-S(\mathbf{p},\varepsilon)[\Sigma^A(\mathbf{p},\varepsilon)-\Sigma^R(\mathbf{p},\varepsilon)]\}. \tag{4}$$

When solving the problem of energy relaxation in a spatially homogeneous system, $S(\mathbf{p},\varepsilon)=S_0(\varepsilon)=-\tanh(\varepsilon/2T)=2n_\varepsilon-1$, where n_ε can be interpreted as the electron energy distribution function. According to Refs. 10 and 11 the electron energy relaxation time τ_ε is defined by the following kinetic equation:

$$\frac{1}{\tau_\varepsilon}=-\frac{\delta}{\delta n_\varepsilon}\frac{dn_\varepsilon}{dt},$$
$$\frac{dn_\varepsilon}{dt}=\frac{1}{\pi\nu}\int d\mathbf{p}\frac{1}{(2\pi)^3}\mathrm{Im}[G^A(\mathbf{p},\varepsilon)]I(\mathbf{p},\varepsilon), \tag{5}$$

where $\nu=mp_F/\pi^2$ is the electron two-spin density of states.

We treat the electron-electron interaction associated with the electromagnetic field as an effective interaction corresponding to the exchange of virtual photons. A

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similar treatment was used in Refs. 12 and 13.

The interaction between the electrons and the electromagnetic field is described by the Hamiltonian

$$H_{e\text{-}\gamma}=-\sum_{\mathbf{p}}\sum_{\mathbf{q}\neq 0}\varphi_{\mathbf{q}}C^{\dagger}_{\mathbf{p}+\mathbf{q},s}C_{\mathbf{p},s}+\sum_{\mathbf{p}}\sum_{\mathbf{q}\neq 0}\mathbf{A}_{\mathbf{q}}\frac{2\mathbf{p}+\mathbf{q}}{2mc}C^{\dagger}_{\mathbf{p}+\mathbf{q},s}C_{\mathbf{p},s}+\frac{1}{2mc^2}\sum_{\mathbf{p}}\sum_{\mathbf{q},\mathbf{q}'\neq 0}\mathbf{A}_{\mathbf{q}}\mathbf{A}_{\mathbf{q}'}C^{\dagger}_{\mathbf{p}+\mathbf{q}+\mathbf{q}',s}C_{\mathbf{p},s}\ , \tag{6}$$

where $\varphi_{\mathbf{q}}$ and $\mathbf{A}_{\mathbf{q}}$ are the Fourier representations of the scalar and vector potentials, $C^{\dagger}_{\mathbf{p},s}$ is the creation operator for an electron with momentum $\mathbf{p}$ and spin s, c is the velocity of light, and the absolute value of the electron charge is absorbed in the definition of electromagnetic potentials.

The electron-photon vertices corresponding to the Hamiltonian (6) are

$$a^0=-1,\quad \mathbf{a}^1=\frac{1}{mc}\left[\mathbf{p}+\frac{\mathbf{q}}{2}\right],\quad a^{11}=\frac{1}{2mc^2}\ . \tag{7}$$

From here on, the index 0 will refer to vertices describing interactions with scalar potentials, while 1 will denote interactions with vector potentials.

The electromagnetic field Green's function $V_{\mu\nu}$ in the Coulomb gauge (div $\mathbf{A}=0$) is diagonal. If the screening is taken into consideration, $V_{\mu\mu}$ equals[14]

$$V^R_{00}(\mathbf{q},\omega)=\frac{4\pi e^2}{q^2-4\pi e^2P^R_{00}(\mathbf{q},\omega)}\ , \tag{8}$$

$$(V^R_{11}(\mathbf{q},\omega))_{mn}=V^R_{11}(\mathbf{q},\omega)T_{mn}\ ,\qquad V^R_{11}(\mathbf{q},\omega)=\frac{4\pi e^2c^2}{\omega^2-c^2q^2-4\pi e^2c^2P^R_{11}(\mathbf{q},\omega)}\ , \tag{9}$$

where e is the electron charge and m and n stand for the Cartesian coordinates x,y,z. If the vector $\mathbf{q}$ is directed along z axis

$$T_{mn}=\delta_{mn}-\frac{q_mq_n}{q^2}\ . \tag{10}$$

$P_{\mu\mu}$ is the polarization operator (Fig. 1) with the vertices a^{μ}_x, where $a^0_x=-1$ and $a^1_x=(2p_x+q_x)/2mc$.

In the Keldysh diagram technique, functions $V_{\mu\mu}$ have the same matrix structure as the phonon Green's function D, and

$$V^C_{\mu\mu}(\mathbf{q},\omega)=[2N_T(\omega)+1][V^R_{\mu\mu}(\mathbf{q},\omega)-V^A_{\mu\mu}(\mathbf{q},\omega)]\ ,\qquad N_T(\omega)=[\exp(\omega/T)-1]^{-1}\ . \tag{11}$$

Vertices corresponding to the electron-boson interaction have tensor structure Q^k_{ij}, where upper index is for bosons, the lower for electrons;

$$(a^{\mu})^k_{ij}=a^{\mu}K^k_{ij},\quad K^1_{ij}=\frac{1}{\sqrt{2}}\delta_{ij},\quad K^2_{ij}=\frac{1}{\sqrt{2}}(\sigma_x)_{ij}\ , \tag{12}$$

where σ_x is the Pauli matrix. So for $P^R_{\mu\mu}$ we have

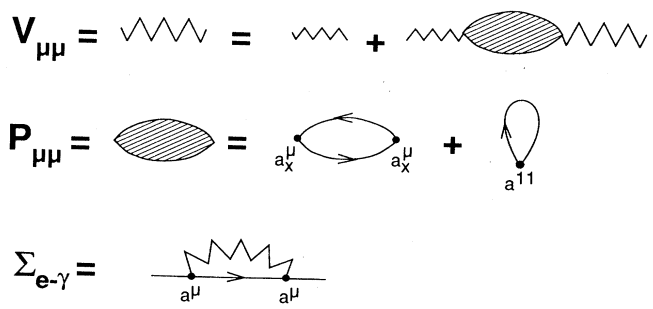

FIG. 1. Equation for the Green's function of electromagnetic field $V_{\mu\mu}$. $P_{\mu\mu}$ is the photon polarization operator. $\Sigma_{e\text{-}\gamma}$ is the electron self-energy corresponding to the electron-photon interaction.

$$P^R_{\mu\mu}(\mathbf{q},\omega)=-2i\int\int d\bar{p}\,d\varepsilon\frac{1}{(2\pi)^4}(a^{\mu}_x)^2\times K^1_{ij}[G(\mathbf{p}+\mathbf{q},\varepsilon+\omega)]_{jl}\times K^2_{lk}[G(\mathbf{p},\varepsilon)]_{ki}+\frac{n_e}{mc^2}\delta_{\mu 1}\ , \tag{13}$$

where n_e is the electron density. Summation is implied by the pairs of repeated indices.

If $ql\gg 1$, where the electron mean free path $l=v_F\tau$, then

$$P^R_{00}(\mathbf{q},\omega)=-\nu\left[1+i\frac{\pi\omega}{2qv_F}\right],\qquad P^R_{11}(\mathbf{q},\omega)=-i\frac{\pi\nu\omega v_F}{4qc^2}\ . \tag{14}$$

In RPA the collision integral $I_{e\text{-}\gamma}$ corresponding to the electron-photon interaction is defined by the electron self-energy shown in Fig. 1,

$$I_{e\text{-}\gamma}(\mathbf{p},\varepsilon)=-2\int\int d\mathbf{q}\,d\omega\frac{1}{(2\pi)^4}\mathrm{Im}[G^A(\mathbf{p}+\mathbf{q},\varepsilon+\omega)]\times(a^{\mu}_m)\mathrm{Im}[(V^R_{\mu\mu})_{mn}]\times a^{\mu}_nR_T\mathbf{p},\mathbf{q},\varepsilon,\omega)\ , \tag{15}$$

where

$$R_T(\mathbf{p},\mathbf{q},\varepsilon,\omega)=[2N_T(\omega)+1][S(\mathbf{p}+\mathbf{q},\varepsilon+\omega)-S(\mathbf{p},\varepsilon)]-S(\mathbf{p}+\mathbf{q},\varepsilon+\omega)S(\mathbf{p},\varepsilon)+1\ . \tag{16}$$

Calculating the energy relaxation times due to the interaction between electron and scalar photons $\tau_{e\text{-}s\gamma}$ and between electron and vector photons $\tau_{e\text{-}v\gamma}$ we should take into account the inequalities $ql\gg 1$ and $qv_F\gg\omega$ to be true for the characteristic values q and ω, which are determined by the Eqs. (5) and (15). As a result for $\varepsilon=0$ we have

$$\frac{1}{\tau_{e\text{-}s\gamma}}=\frac{\pi^4}{64}\frac{\kappa}{p_F}\frac{T^2}{\varepsilon_F},\quad \kappa=4\pi e^2\nu,\quad \kappa<2p_F \tag{17}$$

$$\frac{1}{\tau_{e\text{-}v\gamma}}=\frac{\pi}{6}\left[\frac{\kappa}{mc}\right]^2T\ln\frac{T}{T_1},\quad T_1<T<T_2 \tag{18}$$

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where

$$T_1=\frac{1}{\tau}\left[\frac{c}{v_F^2\kappa\tau}\right]^2,\quad T_2=\frac{\kappa v_F^2}{c}\ . \tag{20}$$

For pure metals $T_2\sim 10^2-10^3$ K, $v_F\kappa\tau\sim 10^3$, and $T_1\sim 1$ K. It is clear from Eqs. (17) and (18) that for 1 $K<T<10$ K the contribution to the energy relaxation time from the electron-vector photon interaction is dominant, $\tau_{e\text{-}v\gamma}<\tau_{e\text{-}s\gamma}$. At the helium temperatures and above, the scattering of electrons on the thermal phonons is more important and the corresponding energy relaxation time is $\tau_{e\text{-ph}}\sim T^{-3}(p_F u_l)^2$, where u_l is the velocity of longitudinal sound.

For $ql\ll 1$ and $\omega\tau\ll 1$ in P_{00}, the renormalization of the scalar vertices a^0 due to the electron-impurity interaction should be taken into account, which leads to well known results.[7,8] The vector vertices a^1 in P_{11} are not renormalized and hence

$$P_{11}^R(\mathbf{q},\omega)=-\frac{i\omega\nu v_F^2\tau}{3c^2}\ . \tag{21}$$

As a result,

$$\frac{1}{\tau_{e\text{-}v\gamma}}=\frac{2\sqrt{2}-1}{\sqrt{3}}\Gamma(\tfrac{3}{2})\zeta(\tfrac{3}{2})(\kappa/mc)^3(\varepsilon_F\tau)(T\tau)^{1/2}T\ . \tag{22}$$

Here $\Gamma(x)$ and $\zeta(x)$ are Γ and ζ functions. We note that the condition $ql\ll 1$ leads to inequality

$$\frac{1}{c}v_F^2\kappa\tau(T\tau)^{1/2}\ll 1\ , \tag{23}$$

and hence the contribution to the energy relaxation from the region $ql\ll 1$ for vector photons is less then from the region $ql\gg 1$.

In the two-dimensional case the ω integral for $\tau_{e\text{-}v\gamma}$ diverges logarithmically for $ql\ll 1$. This situation was completely solved for the phase relaxation time in Ref. 12.

Now consider the electron-phonon interaction. The bare phonon Green's function is

$$D_0^R(\mathbf{q},\omega)=\{D_0^A(\mathbf{q},\omega)\}^* =(\omega-\omega_{q\lambda}+i0)^{-1}-(\omega+\omega_{q\lambda}+i0)^{-1}\ , \tag{24}$$

where $\omega_{q\lambda}$ is the phonon frequency and λ stands for the polarization of the phonon branch.
For equilibrium phonons we have

$$D_0^C(\mathbf{q},\omega)=[2N_\Theta(\omega)+1][D_0^R(\mathbf{q},\omega)-D_0^A(\mathbf{q},\omega)]\ , \tag{25}$$

where Θ is the temperature of the heat bath.

The equation for the electron-phonon vertex g_q, which takes into account screening effects is shown in Fig. 2(a). For longitudinal phonons we have[15]

$$g_q=\frac{2}{3}\frac{\varepsilon_F q}{(2MN\omega_{ql})^{1/2}}\frac{\kappa^2}{q^2+\kappa^2}\ , \tag{26}$$

where M is the ion mass and N is number of unit cells. We represent g_q in the form

$$g_q^2=\beta\frac{\omega_{ql}}{\nu}\left[\frac{\kappa^2}{q^2+\kappa^2}\right]^2,\quad \beta=(\tfrac{2}{3}\varepsilon_F)\frac{\nu}{2MNu_l^2}\ , \tag{27}$$

where β is the dimensionless constant.

It is convenient to consider the electron-electron interaction via exchange of scalar photons, virtual longitudinal phonons, and their interference on the basis of a single approach. In RPA for $\kappa\ll 2p_F$ the essential diagrams for the electron self-energy are shown in Fig. 2, where the first diagram represents the direct electron-electron interaction due to the screened Coulomb potential $V_0^S=4\pi e^2/(q^2+\kappa^2)$. The second diagram corresponding to exchange of virtual phonons is considered by introducing the renormalized phonon Green's function

$$D^R=(D_0^R)^2\Pi^R,\quad \Pi^R=g_q^2P_{00}^R\ , \tag{28}$$

which means that the phonon lifetime is taken into consideration.

Virtual phonons have the following characteristic values: $\omega\sim T$, $q\sim q_D$, where q_D is the zone boundary wave vector. Hence we may assume $(D_0^R)^2=(D_0^A)^2=4/\omega_{q\lambda}^2$.

We note the phonon polarization operator is defined by an equation, similar to (11),

$$\Pi^C(\mathbf{q},\omega)=[2N_T(\omega)+1][\Pi^R(\mathbf{q},\omega)-\Pi^A(\mathbf{q},\omega)]\ , \tag{29}$$

which contains the dependence on the electron temperature.

The collision integral for EEEI containing all contributions mentioned above is

$$I_{\text{eff}}(\mathbf{p},\varepsilon)=-\frac{2}{\nu^2}\int\int d\mathbf{q}\,d\omega\frac{1}{(2\pi)^4}\mathrm{Im}[G^A(\mathbf{p}+\mathbf{q},\varepsilon+\omega)]\mathrm{Im}[P^R(\mathbf{q},\omega)]R_T(\mathbf{p},\mathbf{q},\varepsilon,\omega)\left[\frac{\kappa^2}{q^2+\kappa^2}\right]^2 \times\left[1+4\beta^2\left[\frac{\kappa^2}{q^2+\kappa^2}\right]^2+4\beta\frac{\kappa^2}{q^2+\kappa^2}\right]\ . \tag{30}$$

The fact that the collision integral I_{eff} depends only on the electron temperature means that EEEI cannot transfer energy from electrons to the lattice and thus cannot bring the electrons and the phonons into thermal equilibrium.

The upper limit of q integration is different for each term in the bracket in (30). For the first term it is $2p_F$ and for the last two terms it is $\min\{q_D,2p_F\}$. For simpli-

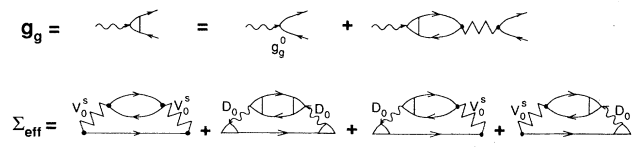

FIG. 2. Equation for the screened electron-phonon vertex g_q, g_q^0 is the vertex without screening. Σ_{eff} is the electron self-energy corresponding to the electron-scalar photon, electron-longitudinal phonon interactions and their interference.

city we consider only the case $q_D \ll \kappa \ll 2p_F$. The electron energy relaxation time for $\varepsilon=0$ is

$$\frac{1}{\tau_{\text{eff}}}=\Lambda\frac{\pi^4}{64}\frac{T^2}{\varepsilon_F}\ ,$$
$$\Lambda=1+\frac{16p_F}{\pi\kappa}\left[\frac{2}{Z}\right]^{1/3}(\beta^2+\beta)\ , \tag{31}$$
$$\frac{Z}{2}=\left[\frac{p_F}{q_D}\right]^3\ ,$$

where Z is the valence of an atom of the lattice, $ZN=n_e$.

The whole approach is easily adapted for the case of virtual optical longitudinal phonons. For transverse phonons one must take into account transverse electromagnetic fields in the same way as we described the electron–vector-photon interaction. It was shown in Ref. 13 that for large q and ω the contribution of transverse phonons is negligible.

For EEEI virtual phonons with large $q \sim q_D$ are essential, hence τ_{eff} is not sensitive to impurities. On the contrary, the energy relaxation time due to the electron–thermal-phonon interaction in impure metals under the condition $Tl \ll u$ becomes[16,15] $\tau_{e\text{-ph}}^{-1} \sim T^4 p_F l/(p_F u_l)^3$. In impure metals and especially in low-dimensional systems at low temperatures the electron-electron Coulomb interaction gives the main contribution to the electron energy relaxation time.[7,8]

In considering the conductivity due to EEEI we use the method developed in Refs. 10 and 13. Assuming the electron-impurity scattering to be the main electron momentum relaxation process we calculate the temperature-dependent correction to the dc conductivity σ_0 due to EEEI.

The kinetic equation for the electron distribution function $S(\mathbf{p},\varepsilon)$ has the form

$$e\mathbf{v}\cdot\mathbf{E}\frac{\delta S_0(\varepsilon)}{\delta\varepsilon}=I_{e\text{-imp}}(S)+I_{\text{eff}}(S)+I_{e\text{-}v\gamma}(S)\ , \tag{32}$$

where $\mathbf{E}$ is the electric field, $I_{e\text{-imp}}$ is the electron-impurity collision integral $I_{e\text{-imp}}=(S_0-S)/\tau$, and I_{eff} and $I_{e-v\gamma}$ are determined by Eqs. (30) and (15).

We solve Eq. (32) by iterations: $S=S_0+\varphi_0+\varphi_1$. Without electron-photon and electron-phonon interactions we have

$$\varphi_0(\mathbf{p},\varepsilon)=-e\tau\mathbf{v}\cdot\mathbf{E}\frac{\delta S_0(\varepsilon)}{\delta\varepsilon}\ . \tag{33}$$

The electric current is defined by the equation

$$\mathbf{j}=\sigma\mathbf{E}=2e\int\int d\mathbf{p}\,d\varepsilon\frac{1}{(2\pi)^4}\mathbf{v}S(\mathbf{p},\varepsilon)\text{Im}[G^A(\mathbf{p},\varepsilon)]\ . \tag{34}$$

Substituting (33) in (34) we obtain

$$\sigma_0=\tfrac{1}{3}e^2\nu v_F^2\tau\ . \tag{35}$$

Nonequilibrium correction φ_1 is defined by the equation

$$\varphi_1=\tau[I_{\text{eff}}(\varphi_0)+I_{e\text{-}v\gamma}(\varphi_0)]\ . \tag{36}$$

After substituting φ_1 into (34) we get the following corrections to σ_0:

$$\frac{\Delta\sigma_{\text{eff}}}{\sigma_0}=-\left[\frac{\beta^2+\beta}{Z}+\frac{3\pi}{16}\left[\frac{\kappa}{p_F}\right]^2\right]\frac{\pi^3}{18}\frac{\tau T^2}{\varepsilon_F}\ , \tag{37}$$

$$\frac{\Delta\sigma_{e\text{-}v\gamma}}{\sigma_0}\sim\left[\frac{v_F}{c}\right]^{10/3}\left[\frac{\varepsilon_F}{T}\right]^{1/3}\frac{\Delta\sigma_{\text{eff}}}{\sigma_0}\ . \tag{38}$$

It is clear from (38) that for reasonable temperatures $\Delta\sigma_{e\text{-}v\gamma}$ may be neglected.

III. FERROMAGNETIC METAL

The method described in Sec. II may be easily used to treat the exchange of other virtual bosons. Let us consider magnons in ferromagnetic metals. The electron-magnon interaction is defined by the s-d exchange Hamiltonian.[17] Taking into account only one-magnon processes we have

$$H_{s\text{-}d}=-J\left[\frac{2S}{N}\right]^{1/2}\sum_{\mathbf{q},\mathbf{p}}(b_{\mathbf{q}}c^{\dagger}_{\mathbf{p}+\mathbf{q},\downarrow}c_{\mathbf{p},\uparrow}+b^{\dagger}_{\mathbf{q}}c^{\dagger}_{\mathbf{p}\uparrow}c_{\mathbf{p}+\mathbf{q},\downarrow})\ , \tag{39}$$

where $b^{\dagger}_{\mathbf{q}}$ is the magnon creation operator, arrows in the electron operators stand for the electron spin, J is the exchange integral, and N is the number of magnetic atoms with spin S.

The magnon Green's function and the magnon spectrum are

$$D^R_{0m}(\mathbf{q},\omega)=(\omega-\Omega_q+i0)^{-1},\quad \Omega_q=Bq^2 \tag{40}$$

where $B=\Theta_C p_F^{-2}$ and the Curie temperature $\Theta_C=dJ^2/\varepsilon_F$, $d\sim 1$. The electron spectrum for each of the spin-split subbands is $\varepsilon_{\uparrow,\downarrow}=p^2/2m \mp JS$. Due to the splitting in the electron spectrum, one-magnon scattering processes take place only if $q>q_0$, where q_0 is the threshold value of the magnon wave vector, which is defined from the equation $q_0 v_F=2JS$. For low temperatures $T<T_0$, where $T_0=Bq_0^2$, one-magnon processes for thermal magnons are forbidden. For the usual value $J\sim 0.1\varepsilon_F$ we have $T_0\sim 10$ K. For $T<T_0$ the electron–two-magnon processes were taken into account in Ref. 18, where the energy relaxation time and the conductivity were calculated and as a result the following temperature dependences were obtained: $\tau_{e\text{-}2m}^{-1}\sim T^{7/2}$, $\sigma_{e\text{-}2m}\sim T^{-9/2}$.

We now show that EEEI due to exchange of virtual magnons is more important for $T<T_0$ than the

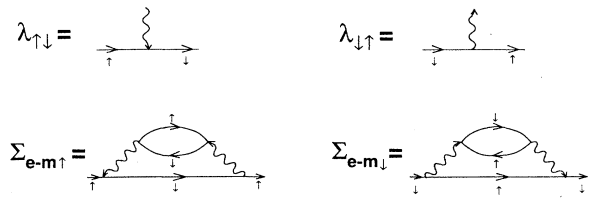

FIG. 3. Vertices of electron-magnon interactions and the electron-magnon self-energy.

electron–two-magnon interaction. The vertices $\lambda_{\downarrow\uparrow}$ and $\lambda_{\uparrow\downarrow}$ related to absorption and emission of magnons according to (39) equal $\lambda_{\downarrow\uparrow}=\lambda_{\uparrow\downarrow}=-J(2S/N)^{1/2}$. Diagrams of the electron self-energy describing EEEI via virtual magnons are shown in Fig. 3. Note that the Hamiltonian (39) forbids the electron self-energy exchange diagrams with two-magnon Green's functions.

Using the method of Sec. II it is easy to see that for $T<T_0$ there the characteristic values $\omega\sim T$ and $q_0<q<q_D$. Hence $(D^R_{0m})^2=(D^A_{0m})^2=\Omega_q^{-2}$. For the electrons of each of the spin-split subbands the energy relaxation time is

$$\frac{1}{\tau_{e\text{-}m\uparrow}}=\frac{1}{\tau_{e\text{-}m\downarrow}}=\frac{27\pi^3Z^2}{16Sd}\frac{\varepsilon_F}{J}\frac{T^2}{\Theta_C}\ . \tag{41}$$

For the conductivity we take into account the contributions of the electrons of both subbands,

$$\frac{\Delta\sigma_{e\text{-}m}}{\sigma_0}=-\frac{3\pi^3SZ^2}{16d}\frac{\tau JT^2}{\varepsilon_F\Theta_C}\ . \tag{42}$$

EEEI in ferromagnetic metals via virtual magnons may explain the fact that the quadratic temperature dependence of the resistivity was often observed in many experiments[17] even for low temperatures $T<T_0$. The quadratic temperature dependence of resistivity due to the electron-thermal magnon scattering for $T>T_0$ is well known. In impure ferromagnetic metals the corrections to σ_0 due to the electron–magnon-impurity interference are also important.[13] An antiferromagnetic metal may be considered in the same manner.

IV. SUPERCONDUCTOR

The electron Green's function in a superconductor in the Numbu representation has the form

$$\hat{G}^R(\mathbf{p},\varepsilon)=\frac{-\xi_{\mathbf{p}}\hat{\sigma}_z-\varepsilon\hat{1}+\Delta\hat{\sigma}_x}{(\xi_{\mathbf{p}}-\xi_\varepsilon-i0)(\xi_{\mathbf{p}}+\xi_\varepsilon+i0)}\ , \tag{43}$$

where

$$\xi_\varepsilon=(\varepsilon^2-\Delta^2)^{1/2}\mathrm{sgn}(\varepsilon),\quad |\varepsilon>\Delta \tag{44}$$

and Δ is the energy gap. Vertices a^0 and g_q carry factor $\hat{\sigma}_z$ and vertex a^1 carries $\hat{1}$.

The kinetic equation, which describes the energy relaxation in a superconductor, is

$$\frac{dn_\varepsilon}{dt}=-\frac{i}{\pi\nu}\frac{\xi_\varepsilon}{\varepsilon}\tfrac{1}{2}\mathrm{Tr}\int d\mathbf{p}\frac{1}{(2\pi)^3}\mathrm{Im}[\hat{G}^A(\mathbf{p},\varepsilon)]\times\{\hat{\Sigma}^C(\mathbf{p},\varepsilon)-S(\varepsilon)[\hat{\Sigma}^A(\mathbf{p},\varepsilon)-\hat{\Sigma}^R(\mathbf{p},\varepsilon)]\}\ . \tag{45}$$

In RPA for EEEI due to virtual scalar photons and longitudinal phonons we have

$$\frac{dn_\varepsilon}{dt}=-\frac{1}{\pi\nu^3}\frac{\xi_\varepsilon}{\varepsilon}\mathrm{Tr}\int\int d\mathbf{p}\,d\mathbf{q}\,d\omega\frac{1}{(2\pi)^7}\left[\frac{\kappa^2}{q^2+\kappa^2}\right]^2\left[1+4\beta^2\left[\frac{\kappa^2}{q^2+\kappa^2}\right]^2+4\beta\frac{\kappa^2}{q^2+\kappa^2}\right]R_T(\varepsilon,\omega)$$
$$\times\mathrm{Im}[\hat{G}^A(\mathbf{p},\varepsilon)]\hat{\sigma}_z\mathrm{Im}[\hat{G}^A(\mathbf{p}+\mathbf{q},\varepsilon+\omega)]\hat{\sigma}_z\mathrm{Im}[P^R_{00}(\mathbf{q},\omega)]\ . \tag{46}$$

After some algebra equation (46) takes the form

$$\frac{dn_\varepsilon}{dt}=\frac{\pi^2\Lambda\kappa}{256\varepsilon_F p_F}\int d\omega\frac{\xi_\varepsilon}{\varepsilon}\left[1+\frac{\varepsilon(\varepsilon+\omega)-\Delta^2}{\xi_\varepsilon\xi_{\varepsilon+\omega}}\right]F(\omega,T,\Delta)R_T(\varepsilon,\omega) \tag{47}$$

and the energy relaxation time

$$\frac{1}{\tau_{\mathrm{eff}}}=\frac{\pi^2\Lambda\kappa}{64\varepsilon_F p_F}\int d\omega\frac{\varepsilon+\omega}{\xi_{\varepsilon+\omega}}\left[1-\frac{\Delta^2}{\varepsilon(\varepsilon+\omega)}\right]F(\omega,T,\Delta)[N_T(\omega)+n_{\varepsilon+\omega}]\ , \tag{48}$$

where function F is defined by the equations

$$\mathrm{Im}[P^R_{00}(\mathbf{q},\omega)]=-\frac{\pi\nu}{4qv_F}F(\omega,T,\Delta)\ , \tag{49}$$

$$F(\omega,T,\Delta)=\int d\varepsilon'\frac{\varepsilon'(\varepsilon'+\omega)-\Delta^2}{\xi_{\varepsilon'}\xi_{\varepsilon'+\omega}}[S(\varepsilon')-S(\varepsilon'+\omega)]\ . \tag{50}$$

In order to separate out the processes of scattering and recombination of quasiparticles in (50) it is necessary to go from the electronic representation to the quasiparticle representation,

$$F_{\text{scatt}}(\omega,T,\Delta)=2\int_{\Delta}^{\infty}d\varepsilon'\frac{\varepsilon'(\varepsilon'+\omega)-\Delta^2}{[\varepsilon'^2-\Delta^2]^{1/2}[(\varepsilon'+\omega)^2-\Delta^2]^{1/2}}[S(\varepsilon')-S(\varepsilon'+\omega)]\ , \tag{51}$$

$$F_{\text{rec}}(\omega,T,\Delta)=\Theta(\omega-2\Delta)\int_{\Delta}^{\omega-\Delta}d\varepsilon'\frac{\varepsilon'(\omega-\varepsilon')+\Delta^2}{[\varepsilon'^2-\Delta^2]^{1/2}[(\omega-\varepsilon')^2-\Delta^2]^{1/2}}[S(\varepsilon'-\omega)-S(\varepsilon')]\ , \tag{52}$$

where $\Theta(x)=1$, $x>0$, and $\Theta(x)=0$, $x<0$.
In some limited cases we have

$$F_{\text{scatt}}=\left[\frac{\pi T\Delta}{2}\right]^{1/2}\exp(-\Delta/T),\quad T\ll\Delta\ll\omega \tag{53}$$

$$F_{\text{scatt}}=\omega\exp(-\Delta/T),\quad \omega\ll T\ll\Delta$$

$$F_{\text{rec}}=\omega,\quad 2\Delta\ll\omega \tag{54}$$

$$F_{\text{rec}}=\pi\omega/2,\quad \omega-2\Delta\ll\Delta\ .$$

In order to get the quasiparticle recombination time τ^r_{eff} and the scattering time τ^s_{eff} we should go to the quasiparticle representation also in Eq. (48). For the low-temperature region $T\ll\Delta$ we have for $\varepsilon=\Delta$

$$\frac{1}{\tau^r_{\text{eff}}}=\frac{\pi^3\kappa\Lambda T\Delta}{64p_F\varepsilon_F}\exp(-2\Delta/T)\ , \tag{55}$$

$$\frac{1}{\tau^s_{\text{eff}}}=\frac{\pi^3\kappa\Lambda T}{128p_F\varepsilon_F}(2\pi\Delta T)^{1/2}\exp(-\Delta/T)\ . \tag{56}$$

The kinetic equation which determines the quasiparticle energy relaxation due to the processes of interaction between electron quasiparticles and vector photons has the form

$$\frac{dn_\varepsilon}{dt}=-\frac{v_F}{8\pi^2c^2}\int dq\,q\int d\omega\frac{\xi_\varepsilon}{\varepsilon}\left[1+\frac{\varepsilon(\varepsilon+\omega)+\Delta^2}{\xi_\varepsilon\xi_{\varepsilon+\omega}}\right]\times\text{Im}[V^R_{11}(\mathbf{q},\omega)]R_T(\omega,\varepsilon)\ , \tag{57}$$

where $V^R_{11}(\mathbf{q},\omega)$ is defined by Eq. (9), where $P^R_{11}(\mathbf{q},\omega)$ for $ql\gg1$ equals

$$P^R_{11}(\mathbf{q},\omega)=-i\frac{\pi\nu v_F}{8qc^2}\mathcal{J}(\omega,T,\Delta)\ , \tag{58}$$

$$\mathcal{J}(\omega,T,\Delta)=\int d\varepsilon'\frac{\varepsilon'(\varepsilon'+\omega)+\Delta^2}{\xi_{\varepsilon'}\xi_{\varepsilon'+\omega}}[S(\varepsilon')-S(\varepsilon'+\omega)]\ . \tag{59}$$

We will not give detailed calculation of function $\mathcal{J}$, because after $\mathbf{q}$ integration in (57) the dependence on the function $\mathcal{J}$ for $T_1<T<T_c$ (T_c is the transition temperature) disappears and as a result the energy relaxation time

$$\frac{1}{\tau_{e\text{-}v\gamma}}=\frac{\pi}{12}\left[\frac{\kappa}{mc}\right]^2\int d\omega\frac{\varepsilon+\omega}{\xi_{\varepsilon+\omega}}\left[1+\frac{\Delta^2}{\varepsilon(\varepsilon+\omega)}\right]\times[N_T(\omega)+n_{\varepsilon+\omega}]\ . \tag{60}$$

For $T\ll\Delta$ and $\varepsilon=\Delta$ the recombination and scattering relaxation times equal

$$\frac{1}{\tau^r_{e\text{-}v\gamma}}=\frac{\pi}{24}\left[\frac{\kappa}{mc}\right]^2\left[\frac{\pi T}{2\Delta}\right]^{1/2}T\exp(-\Delta/T)\ , \tag{61}$$

$$\frac{1}{\tau^s_{e\text{-}v\gamma}}=\frac{\pi}{12}\left[\frac{\kappa}{mc}\right]^2(2\pi\Delta T)^{1/2}\ . \tag{62}$$

Note that the expressions (61) and (62) in comparison with (55) and (56) do not contain the additional small factor $\exp(-\Delta/T)$, and hence the electron–vector-photon interaction is more important then EEEI at low temperatures. The comparison of $\tau_{e\text{-}v\gamma}$ with the relaxation time $\tau_{e\text{-ph}}$ associated with the scattering of quasiparticles on thermal phonons in pure[19] and impure[15] superconductors shows that they may be of the same order.

The electron energy relaxation time determines a number of parameters which characterize the superconducting state, e.g., the relaxation times for the amplitude and phase of the order parameter,[20,21] and general contributions of τ_{eff} and $\tau_{e\text{-}v\gamma}$ together with $\tau_{e\text{-ph}}$ are important. Besides it there are some effects in nonequilibrium superconductors[22] which depend on the comparison between the electron-electron and the electron-phonon relaxation times. For this phenomena the contribution of $\tau_{e\text{-}v\gamma}$ is especially important at low temperatures.

V. SUMMARY

The main results of the paper can be summarized as follows. The contribution to the energy relaxation time and the conductivity from EEEI associated with virtual longitudinal phonons, scalar photons, and their interference is calculated in RPA.

It has been shown that the processes of the electron–vector-photon interaction are dominant in the electron energy relaxation in pure metals below the helium temperature. Unfortunately, the most reliable method of measuring the inelastic electron relaxation time is studying the localization effects on the conductivity of disordered systems,[7,8] where the contribution of the electron–vector-photon interaction is negligible. However, the electron–vector-phonon interaction may be the most important mechanism of the electron-electron interaction in superconductors at low temperatures $T\ll\Delta$, where the relaxation time associated with EEEI has the additional small factor $\exp(-\Delta/T)$. It was also shown that EEEI due to the exchange of virtual magnons in ferromagnetic metals is essential for both the conductivity and the energy relaxation time at low temperatures, where the process of interaction of electron with one thermal magnon is forbidden.

The recent study of the electron-electron interaction in inpure and low-dimensional metals[7,8,12] shows nontrivial modification of the inelastic electron relaxation time (more exactly, the dephasing time of the electron wave function). The results of the present paper show that

even in pure metals the electron-electron energy relaxation time is not universal and strongly depends on the type of boson, which carries the interaction.

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[1]A. B. Migdal, Zh. Eksp. Teor. Fiz. **34**, 1438 (1958) [Sov. Phys.—JETP **7**, 996 (1958)].
[2]T. Holstein, Ann. Phys. (N.Y.) **29**, 410 (1964).
[3]P. B. Allen and R. Silberglitt, Phys. Rev. B **9**, 4733 (1974).
[4]J. M. B. Lopes dos Santos and D. Sherington, J. Phys. F **13**, 1233 (1983); **14**, 2039 (1984).
[5]K. Schwartzman and W. E. Lawrence, Phys. Rev. B **37**, 1136 (1988).
[6]M. Kaveh and N. Wiser, Adv. Phys. **33**, 257 (1984).
[7]B. L. Altshuler and A. G. Aronov, *Electron-Electron Interaction in Disordered Systems,* edited by A. L. Efros and M. Polak (North-Holland, Amsterdam, 1985).
[8]P. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[9]L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1514 (1964) [Sov. Phys.—JETP **20**, 1018 (1965)].
[10]B. L. Altshuler, Zh. Eksp. Teor. Fiz. **75**, 1330 (1978) [Sov. Phys.—JETP **48**, 670 (1978)].
[11]B. L. Altshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **75**, 1610 (1978) [Sov. Phys. —JETP **48**, 812 (1978)].
[12]B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitzkii, J. Phys. C **15**, 7367 (1982).
[13]M. Yu. Reizer and A. V. Sergeev, Zh. Eksp. Fiz. **92**, 2291 (1987) [Sov. Phys.—JETP **65**, 1291 (1987)].
[14]A. A. Abrikosov, L. P. Gorkov, and I. Ye. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics* (Prentice-Hall, Englewood Cliffs, 1963).
[15]M. Yu. Reizer and A. V. Sergeev, Zh. Eksp. Teor. Fiz. **90**, 1056 (1986)[Sov. Phys.—JETP **63**, 616 (1986)].
[16]A. Schmid, Z. Phys. **259**, 421 (1973).
[17]S. V. Vonsovskii, *Magnetism* (Halsted, New York, 1975).
[18]V. S. Lutovinov and M. Yu. Reizer, Zh. Eksp. Teor. Fiz. **77**, 707 (1979) [Sov. Phys.—JETP **50**, 355 (1979)].
[19]S. B. Kaplan, C. C. Chi, D. N. Langenberg, J. J. Chang, S. Jaffreg, and D. J. Scalapino, Phys. Rev. B **14**, 4854 (1976).
[20]A. Schmid, in *Nonequilibrium Superconductivity, Phonons, and Kapitza Boundaries,* Vol. 65 of *Proceedings of the NATO Institute of Advanced Study,* edited by K. E. Gray (Plenum, New York, 1980), Chap. 14.
[21]Yu. N. Ovchinnikov, A. Schmid, and G. Schon, Phys. Rev. Lett. **46**, 1013 (1981).
[22]A. G. Aronov and B. Z. Spivak, Zh. Eksp. Teor. Fiz. **77**, 652 (1979) [Sov. Phys.—JETP **50**, 328 (1979)].