Surface corrugation and surface-polariton binding in the infrared frequency range

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(Received 12 September 1988)

We consider the influence of a small-amplitude grating on the binding of surface polaritons to metal surfaces, with emphasis on the infrared frequency range. The grating can compress the field in the vacuum above the structure very substantially in this regime. Indeed, a grating can bind the surface polariton in the limit of zero skin depth (or infinite conductivity). The results of recent experiments are discussed in light of these results. We also comment on the influence of small-amplitude random roughness on the spatial profile of surface-polariton fields.

I. INTRODUCTION

Surface polaritons are electromagnetic waves bound to the smooth surface of a dielectric material.[1] On simple metals, these waves can propagate over a very wide range of frequencies. If $\epsilon(\omega)$ is the frequency-dependent dielectric constant of the substrate (assumed real here), then the surface will support surface-polariton propagation at any frequency for which $\epsilon(\omega) < -1$. If we model $\epsilon(\omega)$ by writing $\epsilon(\omega)=\epsilon-\omega_p^2/\omega^2$, with ω_p the conduction electron plasma frequency and ϵ the contribution from interband transitions, then surface polaritons propagate at all frequencies below the cutoff $\omega_s=\omega_p/(1+\epsilon)^{1/2}$. Typically, ω_s lies in the visible or near uv frequency range.

In the infrared, or far infrared, the waves are very weakly bound to a perfectly flat surface. One may see this in a variety of ways. If the wave vector and frequency of the wave are $k_\parallel$ and ω, respectively, the dispersion relation is

$$\frac{c^2k_\parallel^2}{\omega^2}=\frac{\epsilon(\omega)}{1+\epsilon(\omega)}\cong 1+\frac{1}{|\epsilon(\omega)|} , \tag{1.1}$$

where the last form applies in the infrared, $\epsilon(\omega)$ is negative, and $|\epsilon(\omega)| \gg 1$. For a given value of $k_\parallel$, the difference in frequency between a plane wave photon propagating parallel to the surface and a surface polariton is controlled by the term $1/|\epsilon(\omega)|$, which is very small when $\omega \ll \omega_p$. The quantity $1/|\epsilon(\omega)|$ is a measure of the binding energy of the wave.

In the vacuum above the surface, the electromagnetic fields associated with the surface polariton fall off exponentially as $\exp[-\alpha_0(\omega)z]$, with the z axis normal to the surface. One has $\alpha_0(\omega)=[k_\parallel^2-(\omega^2/c^2)]^{1/2}$, and in the infrared, $\alpha_0(\omega)\cong(\omega/c)|\epsilon(\omega)|^{-1/2}$. The skin depth $\delta(\omega)=(c/\omega)|\epsilon(\omega)|^{-1/2}$ here, so we may also write $\alpha_0(\omega)=(\omega/c)^2\delta(\omega)$. At a frequency of 100 μm, for a metal with a skin depth $\delta(\omega)=500$ Å, the decay constant $\alpha_0(\omega)=7.2$ cm^{-1}, so that the wave field decays sensibly to zero within a distance of about 5 mm from the surface. In the infrared frequency range when studying the generation of surface polaritons by various coupling schemes, it is often difficult to discriminate between the surface polariton and true electromagnetic radiation propagating parallel to the surface,[2] because the surface mode is so weakly bound.

In the infrared, a very large fraction of the energy stored in the surface-polariton fields resides in the vacuum, where no dissipation of energy occurs. A consequence is that the propagation length of the surface polariton is expected to be very long. This is indeed found to be the case[3] but, so far as the present authors know, virtually every experimental study finds the propagation length to be quite a bit shorter than expected from the theory of a perfectly planar metal surface, described by dielectric theory. The discrepancies are typically a factor of 2,[3] or perhaps more. The difference is often ascribed, quite reasonably, to surface roughness, which can scatter the surface polaritons in a variety of ways.[4]

Recent experiments by Stegeman and his colleagues[5] prove more disturbing. These authors excite surface polaritons in the far infrared (116 μm) by use of a grating coupler, to find excitation efficiencies very much larger than expected on the basis of perturbation theory.[6] The results can be understood qualitatively if, for some reason, the waves are bound to the surface more tightly than suggested by the dielectric theory of a flat surface.

The purpose of this paper is to point out that the presence of a diffraction grating, even with an amplitude quite small compared to the wavelength, can increase the binding energy of surface polaritons on metal surfaces very substantially over that expected from the dielectric theory of the flat surface. The grating also compresses the fields more tightly against the surface, i.e., the decay constant $\alpha_0(\omega)$ discussed above can be much larger than expected from dielectric theory. These effects operate to some degree in any frequency range where surface polaritons propagate, but they can dominate for waves on metal surfaces at far infrared frequencies, where the "binding" provided by the dielectric response of the substrate is so very weak.

On the flat surface, $1/|\epsilon(\omega)|$ must be finite for the surface polariton to bind, as we have seen. In the limit $|\epsilon(\omega)|\to\infty$ (also the limit of infinite conductivity), the fields do not penetrate the substrate (the skin depth vanishes), and we have no surface polaritons. The binding of the waves to the surface is thus a consequence of dielectric response characteristics of the substrate. We show here that in the limit of infinite conductivity, the waves do bind to a grating in the absence of field penetration into the substrate. This results follows directly from the literature[7-9] on a mathematically isomorphic problem: The surface of an isotropic elastic solid does not support surface acoustic waves of shear horizontal character. But shear horizontal waves do "bind" if a periodic grating is present, as first noted by Auld and co-workers.[7] If one formulates the surface-polartion problem in terms of the magnetic field in the wave, which is everywhere parallel to the surface for propagation normal to the grooves of the grating, then in the limit of infinite conductivity the wave equation and boundary condition at the surface become identical to those encountered in the discussion of shear horiztonal surface acoustic waves in the isotropic elastic medium. The velocity of sound is replaced by that of light in vacuum.

Our view, then, is that numerous experimental studies of surface-polariton propagation on metal gratings in the far infrared range have been interpreted within the framework of the incorrect zero-order picture, which assumes that the dominant contribution to the binding energy comes from the dielectric response of the substrate. The proper zero-order picture in numerous instances is that of a wave on a surface whose conductivity is infinite; the dominant contribution to the binding energy is provided by the interaction of the fields with the grating structure, and the dielectric response of the substrate plays a minor role.

It has been shown recently that the presence of random roughness[10] of one-dimensional character can also "bind" shear horizontal acoustic waves to the surface of a semi-infinite isotropic elastic solid. We show here that such roughness also may compress the fields of a surface polariton more tightly to the surface of a metal in the far infrared range than expected from dielectric theory [$\alpha_0(\omega)$ is increased by roughness]. This effect may be an important ingredient in understanding attenuation-length studies in the infrared frequency range. In its presence, a larger fraction of the energy stored in the wave resides within the substrate, where energy dissipation occurs. The attenuation length is thus shortened by an effect distinct from roughness-induced scattering. It is difficult to explore this issure in practice since, in the usual circumstances, the nature of the roughness on a sample surface is poorly understood.

In Sec. II we present the calculation that provides the basis for the above remarks. Our aim is to present a simple discussion, with approximations introduced that are directed toward the frequency regime of interest here. We obtain rather simple final results as a consequence.

II. THEORETICAL ANALYSIS

The extinction theorem provides a useful approach to formulating a description of the interaction of light with diffraction gratings, and the propagation of surface polaritons on such structures.[11] Here we shall use the extinction theorem, in combination with certain approximations, to obtain a description of the influence of a periodic grating on the properties of surface polaritons, with attention to the infrared frequency range. The notation used here follows that used earlier by Weber and Mills.[12]

The surface of our grating is defined by the equation $z=\zeta(x)$; its grooves thus are oriented perpendicular to the x axis, and the z axis is perpendicular to the surface of the semi-infinite sample upon which the grating is ruled. The period of the grating is a, so that $\zeta(x+na)=\zeta(x)$ for any integer n. The substrate lies in the region $z\leq\zeta(x)$.

Let $H^{>}(x,z)$ be the magnetic field associated with the surface polariton, in the vacuum above the sample, $z>\zeta(x)$. The extinction theorem[11,13] provides the following statement about $H^{>}(x,z)$:

$$\int ds' \left[H^{>}(x',z')\frac{\partial G_0(\mathbf{x}-\mathbf{x}')}{\partial n'_{>}} - G_0(\mathbf{x}-\mathbf{x}')\frac{\partial H^{>}(x'z')}{\partial n'_{>}} \right]_{z'=\zeta(x')} =0 \ . \tag{2.1}$$

In this expression, the integration is over the surface of the grating, $\partial/\partial n_{>}$ denotes differentiation along the direction normal to the grating (with $\hat{n}_{>}$ directed toward the vacuum), and $G_0(\mathbf{x}-\mathbf{x}')$ is the free-space Green's function

$$G_0(\mathbf{x}-\mathbf{x}')=\frac{e^{i(\omega/c)|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|} \ . \tag{2.2}$$

Equation (2.1) is valid for any choice of $\mathbf{x}$, provided that $\mathbf{x}$ lies below the grating in the substrate, i.e., $z<\zeta(x)$.

A full treatment of the dispersion relation of surface polaritons requires use of Eq. (2.1) in combination with a second statement provided by the extinction theorem.[14] However, in the far infrared frequency range, the problem may be simplified through introduction of an approximation. There are several lengths in our problem: the skin depth δ, the height and period of the grating, d and a, respectively, and the wavelength λ and penetration length $l_0=\alpha_0^{-1}$ of the surface polariton into the vacuum. The three lengths a, d, and λ are all of the same order of magnitude, and as we saw in Sec. I, $l_0\gg\lambda$. The skin depth of a good metal will be smaller than the four other lengths by several orders of magnitude in the infrared range. Over lengths of the order of the skin depth, the grating is quite flat, and the fields above the grating exhibit very little spatial variation. Under these conditions, we may relate $\partial H^{>}/\partial n_{>}$ and $H^{>}$ just above the surface by the relation valid for a perfectly flat surface:[15]

$$\frac{\partial H^{>}}{\partial n_{>}}(x,z)|_{z=\zeta(x)}=-\frac{\omega}{c|\epsilon(\omega)|^{1/2}}H^{>}(x,z)|_{z=\zeta(x)} \ . \tag{2.3}$$

This boundary condition is also referred to as the surface-impedance boundary condition. Quite recently, a careful discussion has established its range of validity and given the explicit form of the leading corrections to it.[16]

With use of Eq. (2.3), Eq. (2.1) becomes

$$\int ds' \left[\frac{\partial G_0(\mathbf{x}-\mathbf{x}')}{\partial n'_{>}} + \frac{\omega}{c|\epsilon(\omega)|^{1/2}}G_0(\mathbf{x}-\mathbf{x}') \right] h(x')=0 \ , \tag{2.4}$$

where $h(x')=H^{>}(x',\zeta(x'))$.

Surface polaritons have the form of Bloch waves in the presence of the grating. It follows that we may write

$$h(x)=e^{ikx}u(x) , \tag{2.5}$$

where $u(x)$ is a periodic function of x, with period a, and the wave vector k lies within the first Brillouin zone, $-\pi/a \le k \le +\pi/a$. Thus, we also have

$$h(x)=\sum_{s=-\infty}^{\infty} e^{ik_s x} h_s , \tag{2.6}$$

where $k_s=k+(2\pi/a)s$.

When Eq. (2.6) is inserted into Eq. (2.4), algebra along the lines given in Refs. 11 or 12 allows one to generate a hierarchy of equations for h_s. We proceed here by ignoring the influence of roughness on the term proportional to $|\epsilon(\omega)|^{-1/2}$ in Eq. (2.4). Such corrections are quite small, when the depth d of the grating grooves is small compared to the period a.[17]

After some algebra, we find the following hierarchy of equations, which must be satisfied for each integer m:

$$\sum_{s=-\infty}^{\infty} \left[\frac{\omega^2}{c^2} - k_m k_s + \alpha_m \frac{\omega}{c|\epsilon|^{1/2}} \right] I^{(-)}(s-m;m)h_s = 0 . \tag{2.7}$$

In this expression, $k_m=k+(2\pi/a)m$, and

$$\alpha_m = \left[k_m^2 - \frac{1}{c^2}(\omega+i\eta)^2 \right]^{1/2} ,$$

with η a positive infinitesimal and the convention $\mathrm{Re}(\alpha_m)>0$ employed. We also have

$$I^{(-)}(p;m)=\int_{-a/2}^{+a/2} dx \frac{1}{a} e^{2\pi i p x/a} e^{-\alpha_m \zeta(x)} . \tag{2.8}$$

In the limit $|\epsilon|\to\infty$, Eq. (2.7) yields for surface polaritons precisely the same dispersion relation as obtained for shear horizontal acoustic waves bound to the surface of an isotropic, elastic continuum upon which a diffraction grating has been ruled,[9] provided the shear wave sound velocity is replaced by the velocity of light in vacuum. We see, at this point, that the grating "binds" the surface polariton, even in the limit $|\epsilon(\omega)|\to\infty$.

It is also the case that for the perfectly flat surface Eq. (2.7) yields a proper description of the surface polaritons. If we let $\zeta(x)\to 0$ to obtain a flat surface, we have $I^{(-)}(p;m)\to\delta_{p,0}$. The hierarchy in Eq. (2.7) then reduces to

$$\left[\frac{\omega^2}{c^2} - k_s^2 + \alpha_s \frac{\omega}{c|\epsilon|^{1/2}} \right] h_s = 0 , \tag{2.9}$$

which requires

$$\alpha_s = \frac{\omega}{c|\epsilon|^{1/2}} \tag{2.10a}$$

or

$$k_s^2 = \frac{\omega^2}{c^2} + \frac{\omega^2}{c^2}\frac{1}{|\epsilon|} . \tag{2.10b}$$

The expression in Eq. (2.10b) is the dispersion relation of the surface polariton of frequency ω, expanded in powers of $1/|\epsilon|$ with the first correction term retained. Equation (2.10a) gives the attenuation constant which describes the field profile in the vacuum above the substrate (this was denoted by α_0 in Sec. I). The formulas here have all wave vectors k within the first Brillouin zone of the grating.

We can develop an approximate dispersion relation from Eq. (2.6) that proves accurate for gratings with small amplitude. We can write Eq. (2.6) in the form

$$\left[\alpha_m - \frac{\omega}{c|\epsilon|^{1/2}} \right] h_m = \frac{1}{\alpha_m I^{(-)}(0;m)} \sum_{s(\neq m)} \left[\frac{\omega^2}{c^2} - k_m k_s + \alpha_m \frac{\omega}{c|\epsilon|^{1/2}} \right] I^{(-)}(s-m;m) h_s , \tag{2.11a}$$

where h_s satisfies

$$\left[\alpha_s - \frac{\omega}{c|\epsilon|^{1/2}} \right] h_s = \frac{I^{(-)}(m-s;s)}{\alpha_s I^{(-)}(0;s)} \left[\frac{\omega^2}{c^2} - k_m k_s + \alpha_s \frac{\omega}{c|\epsilon|^{1/2}} \right] h_m + \frac{1}{\alpha_s I^{(-)}(0;s)} \sum_{p\,(\neq s,m)} \left[\frac{\omega^2}{c^2} - k_p k_s + \alpha_s \frac{\omega}{c|\epsilon|^{1/2}} \right] I^{(-)}(p-s;s) h_p . \tag{2.11b}$$

A form of self-consistent second-order perturbation theory follows if we retain only the term proportional to h_m on the right-hand side of Eq. (2.11b). This is equivalent, for our problem, to the method employed by Glass, Weber, and Mills,[18] who found the procedure to be quite accurate, when results obtained with it are compared with exact results.

This procedure then provides the following expression for the decay constant α_m, which in fact is to be calculated self-consistently from this expression:

$$\alpha_m = \frac{\omega}{c|\epsilon|^{1/2}} + \sum_{s\,(\neq m)} \frac{I^{(-)}(s-m;m) I^{(-)}(m-s;s)}{\alpha_m \alpha_s I^{(-)}(0;s) I^{(-)}(0;m)} \frac{\left[\dfrac{\omega^2}{c^2} - k_m k_s + \dfrac{\alpha_s \omega}{c|\epsilon|^{1/2}} \right] \left[\dfrac{\omega^2}{c^2} - k_m k_s + \dfrac{\alpha_m \omega}{c|\epsilon|^{1/2}} \right]}{\left[\alpha_s - \dfrac{\omega}{c|\epsilon|^{1/2}} \right]} . \tag{2.12}$$

We shall be content to substitute the unperturbed frequency into the right-hand side of Eq. (2.12) to obtain the description provided by second-order perturbation theory. In the numerators, the terms involving $|\epsilon|^{-1/2}$ are small, as is the case also in the denominator, unless k_m lies very near the zone boundary, and k_s does also. Thus, we have

$$\alpha_m \cong \frac{\omega}{c|\epsilon|^{1/2}} + k_m^2 \sum_{s\,(\neq m)} \frac{I^{(-)}(s-m;m)I^{(-)}(m-s;s)(k_m-k_s)^2}{\alpha_s^2\alpha_m I^{(-)}(0;s)I^{(-)}(0;m)} . \tag{2.13}$$

For small-amplitude gratings, $I^{(-)}(0;p)\cong 1$ and $I^{(-)}(p;m)\cong -\zeta_p\alpha_m$, where

$$\zeta_p = \int_{-a/2}^{+a/2} dx \frac{1}{a}\zeta(x)e^{-2\pi ipx/a} . \tag{2.14}$$

The skin depth of the substrate, $\delta = c/\omega|\epsilon|^{1/2}$, so that we may arrange Eq. (2.14) to read

$$\alpha_m \cong \frac{\omega^2}{c^2}\left[\delta + \sum_{s\,(\neq m)} \frac{|\zeta_{s-m}|^2(k_m-k_s)^2}{\alpha_s}\right] . \tag{2.15}$$

The expression in Eq. (2.15) is quite simple, and may be used to decide in any particular case whether the surface polariton "binds" primarily because of the presence of the grating structure or because of the dielectric response characteristics of the substrate. In the former case, the second term in Eq. (2.15) will dominate the first, and in the latter the first will dominate the second. If the second term dominates the first, which we believe to be the case for the experimental configuration used in Ref. 5, then the correct zero-order picture of the surface polariton is that its binding to the substrate is a consequence of the presence of the grating; the mode will be much more tightly bound than expected if α_m is approximated by the value appropriate to the flat surface, which is given by the first term in Eq. (2.15).

The attenuation constants α_s which appear in Eq. (2.15) need not all be real. If $k_s < \omega/c$, then α_s is pure imaginary in the limit that the infinitesimal η in its definition approaches zero. Then α_m has an imaginary as well as a real part. The grating can couple the surface polariton to radiative waves, whose wave-vector component projected onto the plane of the surface is smaller than ω/c. In the presence of such couplings, α_m acquires an imaginary part. In the interest of simplicity, we assume all α_s are real in what follows.

For a simple sinusoidal grating with

$$\zeta(x) = d\cos(2\pi x/a) , \tag{2.16}$$

we have

$$\zeta_{s-m} = \frac{d}{2}(\delta_{s,m+1}+\delta_{s,m-1}) . \tag{2.17}$$

Then $(k_m-k_{m\pm1})^2=(2\pi/a)^2$. Thus, for α_m we find

$$\alpha_m = \frac{\omega^2}{c^2}\left[\delta + \frac{\pi^2 d^2}{a^2}\left[\frac{1}{a_{m+1}}+\frac{1}{\alpha_{m-1}}\right]\right] . \tag{2.18}$$

If we have the experiments of Ref. 5 in mind, the wavelengths of the surface polaritons generated are of the order of the grating spacing. Thus, both α_{m+1} and α_{m-1} will be of the order of π/a; the precise values depend on details of the geometry. Thus, if β is a dimensionless constant of order unity, we have

$$\alpha_m = \frac{\omega^2}{c^2}\left[\delta + \beta\frac{d^2}{a}\right] , \tag{2.19}$$

and the dimensionless ratio

$$R = \frac{d^2}{a\delta} \tag{2.20}$$

serves to tell which limit we are in. When $R \gg 1$, the binding is dominated by the interaction of the wave with the grating, while when $R \ll 1$ the profile of the wave is well approximated by that appropriate to the flat surface. For the experiments of Ref. 5, $d \cong 6$ μm, $a \cong 100$ μm, and $\delta \cong 500$ Å. This gives $R \cong 7$ well into the regime where the waves are much more tightly bound to the grating structure than they would be on a perfectly flat surface.

In Sec. I we commented that the increased binding provided by the grating (or roughness, as discussed next) will shorten the mean free path of the surface polariton, because a larger fraction of its energy density resides in the substrate, where energy dissipation may occur. This grating-induced damping mechanism is distinct from the scattering effects discussed elsewhere,[4] as noted earlier. A description of this additional damping follows from Eq. (2.15), modified to allow the substrate dielectric constant to have a nonzero imaginary part. When $\epsilon(\omega)$ is complex, and we use the definitions of k_m and k_s, Eq. (2.15) may be arranged to read

$$\alpha_m = \frac{\omega}{c}\frac{i}{\epsilon^{1/2}} + \frac{4\pi^2\omega^2}{a^2c^2}\sum_{s\,(\neq m)}\frac{|\zeta_{s-m}|^2(m-s)^2}{\alpha_s} . \tag{2.21}$$

We may obtain an expression for the complex wave vector k_m of the surface polariton by noting $\alpha_m=(k_m^2-\omega^2/c^2)^{1/2}$, and squaring Eq. (2.21). For simplicity, we assume the second term on the right-hand side of Eq. (2.21) is real, and we let $\epsilon=\epsilon_1+i\epsilon_2$, where $\epsilon_2 \ll \epsilon_1$. To first order in ϵ_2, we have

$$k_m^2 = \frac{\omega^2}{c^2} + \alpha_m^{(r)2} + i\frac{\omega}{c}\frac{\epsilon_2}{|\epsilon_1|^{3/2}}\alpha_m^{(r)} , \tag{2.22a}$$

where

$$\alpha_m^{(r)} = \frac{\omega}{c}\frac{1}{|\epsilon_1|^{1/2}} + \frac{4\pi^2\omega}{a^2c^2}\sum_{s\,(\neq m)}\frac{|\zeta_{s-m}|^2(m-s)^2}{\alpha_s} \tag{2.22b}$$

is the decay constant which controls the spatial profile of the wave in the vacuum above the grating. We write $k_m=k_m^{(1)}+ik_m^{(2)}$ where in the far infrared frequency range, $k_m^{(1)} \cong \omega/c$. Then to first order in ϵ_2, we find

$$k_m^{(2)} = \frac{1}{2}\frac{\omega}{c}\frac{\epsilon_2}{|\epsilon_1|} f \ , \tag{2.23}$$

where $f = \delta\alpha_m^{(r)} \equiv \delta/l$; $l = (\alpha_m^{(r)})^{-1}$ is the penetration length of the wave into the vacuum above the grating.

When $\delta \ll l$ as is true for the limit of interest here, the dimensionless number f is a measure of the fraction of the energy density of the wave which is stored within the substrate, where dissipation may occur when $\epsilon_2 \neq 0$. Interaction of the wave with the grating compacts it onto the grating, f thus increases, and the mean free path of the surface polariton is thus shortened by the increased dissipation.

It is also straightforward to derive the influence of small-amplitude random roughness of one-dimensional character on the attenuation constant from Eq. (2.15). Formally, one lets the period a of the grating approach infinity, then assumes that for a very large and finite, $\zeta(x)$ is a random function. If angular brackets denote an average over an ensemble of surface profiles, $\langle \zeta(x)\zeta(x') \rangle$ becomes a function of only $x - x'$, as $a \rightarrow \infty$.

One may then establish the relation

$$\langle |\zeta_p|^2 \rangle = \frac{1}{a}\int_{-a/2}^{+a/2} dx \, e^{-2\pi i p x/a} \langle \zeta(x)\zeta(0) \rangle \ , \tag{2.24}$$

and it is conventional to let $\langle \zeta(x)\zeta(x') \rangle = \sigma^2 g(x-x')$, where σ is the root-mean-square roughness height, and $g(x-x')$ is normalized so that $g(0)=1$. A common choice for $g(x-x')$ is the Gaussian $g(x-x') = \exp[-(x-x')^2/r_0^2]$, where r_0 is called the transverse correlation length.

After some algebra, we find

$$\alpha(k,\omega) = \frac{\omega}{c|\epsilon|^{1/2}} + \frac{a\sigma^2}{2\sqrt{\pi}}\frac{\omega^4}{c^4}\left\{ \left[\left(\int_1^{\infty} dy + \int_{-\infty}^{-1} dy \right) \exp\left[-\frac{1}{4} r_0^2 \frac{\omega^2}{c^2}(1-y)^2 \right] \frac{(y-1)^{3/2}}{(y+1)^{1/2}} \right] + i \left[\int_{-1}^{+1} dy \, \exp\left[-\frac{1}{4} r_0^2 \frac{\omega^2}{c^2}(1-y)^2 \right] \frac{(1-y)^{3/2}}{(1+y)^{1/2}} \right] \right\} . \tag{2.25}$$

The term on the right-hand side of Eq. (2.25) contributed by the one-dimensional roughness is identical to the expression which applies to shear horizontal waves, discussed earlier in Ref. 9. Since the quantity in curly brackets has a positive definite real part, we see that in the limit of infinite conductivity, roughness of the assumed character will bind surface polaritons to the surface. When $|\epsilon|$ is finite, the presence of such roughness will increase the value of $\alpha(k,\omega)$, and thus compress the surface polariton fields more tightly to the surface. It would be of considerable interest to extend the discussion to the case of true random roughness of two-dimensional character. The extension is not straightforward, since in this case one may no longer assume the surface polariton is a true TM wave.

From Eq. (2.25) extended to allow ϵ to be complex, it is easy to see that roughness-induced compaction of the wave onto the surface shortens its mean free path. One may see this by proceeding along lines similar to the analysis which follows Eq. (2.20).

In conclusion, the analysis of experimental data on surface-polariton propagation on metal surfaces in the far infrared frequency range must take into account the possibility that the spatial profile of the wave may differ substantially from that appropriate to the perfectly flat ideal surface. A small-amplitude grating, or the presence of roughness, may give rise to a wave more tightly bound to the surface than expected from dielectric theory applied to the perfectly flat surface.

ACKNOWLEDGMENTS

We are grateful to Professor G. I. Stegeman for stimulating our interest in this problem, and for many useful discussions. The research of D.L.M. was supported by the U.S. Department of Energy, through Grant No. DE-FG03-84ER45083, while that of A.A.M. was supported by National Science Foundation (NSF) Grant DMR-85-17634.

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