Destruction by fluctuations of superconducting long-range order in the Abrikosov flux lattice

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Previous studies have demonstrated that thermal fluctuations destroy superconducting long-range order within the mixed state of a type-II superconductor. These fluctuations are shown to be incompressible shear motions of the flux lines forming the flux lattice. Despite the absence of superconducting order, the existence of a flux lattice phase is indicated by a renormalization-group analysis. The Lindemann melting criterion suggests that the lattice might melt when $(1-T/T_c)$ is of order $H^{2/3}$.

As long ago as 1971, Maki and Takayama[1] had extended the earlier analysis by Eilenberger[2] of the thermally excited fluctuations of the Abrikosov flux lattice to show that these fluctuations would destroy superconducting off-diagonal long-range order (ODLRO). However, the destruction of ODLRO in conventional superconductors is significant only for fields H very close to H_{c2}, such that $(H_{c2}-H)/H_{c2}<R$, where R increases logarithmically with the size of the system, and for a sample of size 1 cm lying in the range $10^{-4}-10^{-5}$. These fluctuations turn out to be much more important for the high-temperature superconductors due to their shorter correlation lengths.[3]

In this paper it is shown that the fluctuations which destroy ODLRO correspond to an incompressible shear motion of the flux lines forming the flux lattice and an effective Hamiltonian which greatly aids in their analysis is derived. A renormalization-group treatment of this Hamiltonian indicates that the system is at its lower critical dimension, but that there should be a phase transition at finite temperature. There exist useful analogies with the Kosterlitz-Thouless[4] (KT) theory of the two-dimensional superfluid which also has no ODLRO, but which has a nonvanishing superfluid density ρ_s. Rather surprisingly the root-mean-square displacement $d(T)$ of a flux line due to the thermal fluctuations remains finite in the thermodynamic limit. Thus the fluctuations which destroy ODLRO leave the flux lattice intact. The possibility of a flux lattice, but no ODLRO was anticipated by Fisher and Lee[5] and described as "exotic." While $d(T)$ is finite it becomes comparable to the flux lattice spacing l, at temperatures high enough such that $(1-T/T_c)\sim H^{2/3}$, which according to the Lindemann criterion would imply that the lattice should melt.[6] It is perhaps significant that the line which marks the onset of glassy behavior has the same functional form.[7]

The starting point of the calculation is the conventional Ginzburg-Landau phenomenological free-energy density functional

$$F=\tau|\psi|^2+\tfrac{1}{2}u|\psi|^4+(2m)^{-1}|(-i\hbar\nabla-2e\,\mathbf{A}/c)\psi|^2+(\mathbf{B}-\mathbf{H})^2/8\pi \quad (1)$$

which is valid for $H\approx H_{c2}(T)$. The vector potential $\mathbf{A}$ is related to $\mathbf{B}$ via $\mathbf{B}=\operatorname{curl}\mathbf{A}$, and following Eilenberger,[2] it will be approximated by $\bar{B}(-y,0,0)$ where $\bar{B}$ denotes the spatial average of the magnetic induction, whose direction is that of the applied field $\mathbf{H}$, i.e., the z axis. The complex order parameter ψ can be expanded in the eigenvectors of the operator $(-i\hbar\nabla-2e\,\mathbf{A}/c)^2$. As usual, we shall retain throughout only the lowest Landau level in this expansion which again should be a valid approximation for $H\approx H_{c2}(T)$. Denote by $\phi(r|0)$ (with amplitude α) the function which is obtained by minimizing the functional F with respect to ψ and $\mathbf{A}$ under the above approximations. $|\phi(r|0)|$ has the periodicity of a triangular lattice of lattice constant l, with periods

$$r_{\rm I}=(x_{\rm I},y_{\rm I})=(l,0)\ ,$$
$$r_{\rm II}=(x_{\rm II},y_{\rm II})=(\tfrac{1}{2},\sqrt{3}/2)l\ ,$$

such that the area Q of the fundamental cell is given by the flux quantization condition $Q=x_{\rm I}y_{\rm II}=l^2\sqrt{3}/2=\Phi_0/\bar{B}$. Explicitly

$$\phi(r|0)=(2y_{\rm II}/x_{\rm I})^{1/4}\exp(-y^2/2P)\times\theta_3(\pi r/x_{\rm I}|(x_{\rm II}+iy_{\rm II})/x_{\rm I})\ , \quad (2)$$

where $Q=2\pi P$, $r=(x+iy)$ and θ_3 is a Jacobi theta function.[8] Eilenberger[2] observed that the functions

$$\phi(r|r_0)=\exp(ik_x x)\phi(r+r_0|0)\ , \quad (3)$$

where $r_0=(x_0+iy_0)\equiv P(-k_y+ik_x)$ form a complete orthonormal set which span the space of the lowest Landau level for $\mathbf{k}=(k_x,k_y)$ within the first Brillouin zone (BZ) associated with the triangular lattice. The reader requiring further details of these functions and the approximation scheme used here should consult Ref. 2.

Writing $\psi=\alpha\phi(r|0)+\delta\psi$ and expanding F only to second order in $\delta\psi$ leaves a quadratic form which when diagonalized leads to two branches of excitations—a "hard" mode whose contributions are small[2] and which henceforth will be neglected and a "soft" mode whose eigenmodes in the long-wavelength limit are such that the contribution from the mode labeled by $q,\mathbf{k}$ to $\delta\psi$ is

$$\delta\psi = ia[\exp(iqz)\phi(r|r_0)+\exp(-iqz)\phi(r|-r_0)] \ . \tag{4}$$

The amplitude a is real. Then correct to $O(a^2)$

$$F = F_{\rm MF} + \tfrac{1}{2}\sum_{q,k}(\rho_s q^2 + c_{66}P^2k^4)a^2/\alpha^2 \ , \tag{5}$$

where $F_{\rm MF}$ is the mean-field free energy, i.e., the minimum of $F\{\psi\}$. The superfluid density ρ_s at the level of the mean-field theory is given by

$$\rho_s = \Phi_0 H_{c2}(1-\bar{B}/H_{c2})/\{4\pi^2[1+(2\kappa^2-1)\beta]\} \ , \tag{6}$$

the shear modulus of the flux lattice[9]

$$c_{66} = \frac{0.48H_{c2}^2(2\kappa^2-1)(1-\bar{B}/H_{c2})^2}{\{8\pi[1+(2\kappa^2-1)\beta]^2\}} \ , \tag{7}$$

where β is a numerical constant $\simeq 1.16$, κ is the Ginsburg parameter, and $\rho_s = \hbar^2\alpha^2/m$. Maki and Takayama[1] had previously demonstrated the existence of the k^4 in the fluctuations but had not realized its coefficient was just $c_{66}P^2$. This connection provides an important clue as to the nature of the fluctuations which destroy ODLRO.

To estimate the importance of these fluctuations it is convenient to compute[1,2]

$$D = \langle |\psi - \alpha\phi(r|0)|^2\rangle_{\rm sp}/\alpha^2 \ , \tag{8}$$

sp denotes a spatial average and the angular brackets a thermal average calculated with a relative probability $\exp(-F/k_BT)$. Fluctuation effects are small if $D \ll 1$ where

$$D = \int \frac{dq}{2\pi}\int_{\rm BZ}\frac{d^2k}{(2\pi)^2}\frac{k_BT}{\rho_s q^2 + c_{66}P^2k^4} \approx \frac{1}{4\pi P}\frac{k_BT}{(\rho_s c_{66})^{1/2}}\ln[L(2/P)^{1/2}] \ , \tag{9}$$

and the k integral has been approximated by taking it over a circular Brillouin zone of radius Λ where $\Lambda^2 = 2/P$, and cutoff at $|\mathbf{k}| \approx 1/L$, where L is a linear dimension of the sample in a direction perpendicular to the applied field. Writing $H_{c2}(T) = H_{c2}^0(1-T/T_c)$ one obtains for large κ

$$D \approx 2.6\epsilon^{1/2}\ln[L(2/P)^{1/2}](\bar{B}/H_{c2}^0) \times (T/T_c)[(1-\bar{B}/H_{c2})(1-T/T_c)]^{-3/2} \ , \tag{10}$$

where the Ginsburg criterion parameter[10]

$$\epsilon = 16\pi^3\kappa^4(k_BT_c)^2/\Phi_0^3H_{c2}^0 \ .$$

For $T > T_c$, $H = 0$, fluctuations in ψ are only important when $|(1-T/T_c)| < \epsilon$.

The magnitude of ϵ is very sensitive to the only poorly known value of κ and to effects of crystal lattice anisotropy which have been neglected in this treatment but which are readily incorporated. From experiments[11] on the specific heat near T_c we estimate that in YBCO $\epsilon \approx 0.002$. For a sample of 1 cm dimension $\ln[L(2/P)^{1/2}] > 10$ for $\bar{B} > 10$ G. Note that D becomes logarithmically infinite in the thermodynamic limit, which implies the absence of ODLRO, i.e., $\langle\psi\rangle = 0$, and it becomes large whenever $\bar{B} \to H_{c2}$. Experimentally perhaps the easiest region in which to explore the effects of fluctuations is for fields H larger than prescribed by $(1-T/T_c) \approx 2\times 10^{-4}H^{2/3}$. Along this line for temperatures close to T_c D is unity. [$\bar{B}$ is always well approximated by H in the region of validity of Eq. (1). We have set $H_{c2}^0 \approx 400$ kG in making these estimates[12]—a value appropriate to a field along the crystal's c axis.]

We shall next proceed to identify the flux line motion which accompanies the soft mode. The special case when $r_0 = (x_0, 0)$ will be studied first. Then using Eqs. (2), (3), and (4) and expanding to second order in x_0 gives

$$\begin{aligned}\psi = \alpha 3^{1/8}\exp(-y^2/2P) \times [&\theta_3(\pi r/l|\tau)(1+i\theta) \\ &-2a\sin(qz)(\pi x_0/\alpha l)\theta_3'(\pi r/l|\tau) \\ &+i\theta(\pi^2x_0^2/l^2)\theta_3''(\pi r/l|\tau)/2+\cdots] \ ,\end{aligned} \tag{11}$$

where $\tau = (1+i\sqrt{3})/2$ and $\theta = 2a\cos(qz)/\alpha$. Using the identity[8]

$$\theta_3''(z|\tau) = -(4/\pi i)\partial\theta_3(z|\tau)/\partial\tau \ ,$$

Eq. (11) gives, correct again to second order in $x_0(=-Pk_y)$,

$$\psi \approx \alpha 3^{1/8}\exp(i\theta - y^2/2P) \times \theta_3(\pi\{r+[2a\sin(qz)/\alpha]Pk_y\}/l)|\tau - \sqrt{3}Pk_y^2\theta/2) \ .$$

This is of the same form as Eq. (2), except for the phase factor $\exp(i\theta)$, a changed flux lattice periodicity in which

$$x_{\rm I} = l, \quad y_{\rm I} = 0 \ ,$$

$$x_{\rm II} = (\tfrac{1}{2} - \sqrt{3}Pk_y^2\theta/2)l, \quad y_{\rm II} = \sqrt{3}l/2 \ ,$$

and a displacement u_x of the flux lines along the x axis by $-[2a\sin(qz)/\alpha]Pk_y$. Notice that the area $x_{\rm I}y_{\rm II}$ of the new unit cell is unchanged by the fluctuation. The flux lines move in the soft mode as if the flux lattice were incompressible. It is easily deduced from the changed dimensions of the unit cell that the only nonvanishing strain is $\partial_y u_x = -Pk_y^2\theta$.

The case of a general wave vector $\mathbf{k} = (k_x, k_y)$, i.e., a value of $(x_0, y_0) = P(-k_y, k_x)$ can be similarly analyzed provided one first rotates the vector potential $\mathbf{A}$ (which has up to now been along the x axis) to point in the direction of $\mathbf{r}_0$; then

$$\mathbf{A} = \bar{B}(-k_xk_yx - k_y^2y, k_x^2x + k_xk_yy, 0)/(k_x^2+k_y^2) \ . \tag{12}$$

(There is a phase factor change to ψ associated with this gauge change.) The displacement $\mathbf{u}$ is once again perpendicular to $\mathbf{k}$ and the strain fields can be read off from Eq. (12) and the previous result for the special case remembering that k_y^2 of that is replaced by $k_x^2 + k_y^2$:

$$\begin{aligned}&\partial_x u_x = -\partial_y u_y = -Pk_xk_y\theta \ , \\ &\partial_y u_x = -Pk_y^2\theta, \quad \partial_x u_y = Pk_x^2\theta \ .\end{aligned} \tag{13}$$

The strains in Eq. (13) imply that the relationship between displacement $\mathbf{u}$ and a phase θ which also varies in the plane perpendicular to the applied field [as $\theta=\theta_0\cos(qz+k_x x+k_y y)$] must be

$$u_x=P\partial_y\theta,\quad u_y=-P\partial_x\theta\ . \tag{14}$$

Since Eq. (14) is valid for any value of q and $\mathbf{k}$, it must hold generally for any slowly varying spatial dependence of θ. Observe that it also guarantees that the flux motion generated is as if the lattice were incompressible since $(\partial_x u_x+\partial_y u_y)=0$.

An effective Hamiltonian associated with a slowly varying phase θ and consistent with Eq. (5) for the energy of the soft mode is

$$H=\tfrac{1}{2}\int d^3r\{\rho_s(\partial_z\theta)^2+c_{66}[(\partial_x u_x-\partial_y u_y)^2+(\partial_x u_y+\partial_y u_x)^2]\}\ , \tag{15}$$

with (u_x,u_y) related to θ via Eq. (14). The elastic energy term is that appropriate to a pure shear;[13] on integrating by parts and dropping surface terms the elastic energy density reduces to $\frac{1}{2}c_{66}P^2(\nabla_\perp^2\theta)^2$ where $\nabla_\perp^2=\partial^2/\partial_x^2+\partial^2/\partial_y^2$. In what follows ρ_s and c_{66} will be assumed to be nonvanishing despite the absence of ODLRO, just as in the KT theory.

Hamiltonians similar to Eq. (15) have been studied in connection with smectic liquid crystals[14] and the same renormalization-group (RG) techniques can be employed. Dividing the Hamiltonian by T, the temperature, one finds that under the length rescaling appropriate to Eq. (15) ($x'=bx$, $y'=by$, $z'=b^2z$) that at the new length scale the temperature $T'=T$. Thus temperature is a marginal operator and hence a line of fixed points should exist for all temperatures less than $T_c(\overline{B})$. For $T>T_c(\overline{B})$ the temperature would flow towards the infinite temperature sink. Addition to the Hamiltonian density of a term like $[(\partial_z u_x)^2+(\partial_z u_y)^2]$, which corresponds to a bending motion of the flux lines, is irrelevant in the RG sense, as also are all anharmonic terms (in contrast with the smectic liquid crystal case[14]). Hence one can conclude that these are no (logarithmic) corrections to the correlation functions which can be derived from Eq. (15).

To examine the stability of the flux lattice itself against thermal fluctuations we shall calculate $d^2(T)=\langle(u_x^2+u_y^2)\rangle=P^2\langle(\partial_x\theta)^2+(\partial_y\theta)^2\rangle$:

$$d^2(T)=P^2\int\frac{dq}{2\pi}\int_{\mathrm{BZ}}\frac{d^2k}{(2\pi)^2}\frac{k_BTk^2}{\rho_s q^2+c_{66}P^2k^4}=\frac{1}{4\pi}\frac{k_BT}{(\rho_s c_{66})^{1/2}}\ . \tag{16}$$

This is finite in the thermodynamic limit, indicating the stability of the flux lattice. However, according to the Lindemann criterion[6] one expects a crystal to melt for $d(T)=cl$, where c is a small constant (typically ~ 0.1) which from Eq. (16) gives the melting criterion

$$0.36\epsilon^{1/2}(\overline{B}/H_{c2}^0)(T/T_c)\times[(1-\overline{B}/H_{c2})(1-T/T_c)]^{-3/2}\approx c^2 \tag{17}$$

or when $(1-T/T_c)\sim 1.2\times 10^{-5}c^{-4/3}H^{2/3}$ for $T\to T_c$, using our previous values for the parameters ϵ and H_{c2}^0. Setting $c=0.06$ brings this expression into coincidence with the experimental results of Ref. 7 for the equation of the line which marks the onset of irreversible behavior. Whether genuine melting takes place at this line is beyond the scope of this paper, but it is perhaps significant that Bishop (quoted in Ref. 15) reports a melting of the flux lattice near H_{c2}.

Pinning of the flux lines by crystal defects, etc., will always be present. It has been modeled by adding to Eq. (1) a term $\tau(\mathbf{r})|\psi|^2$ and using the replica method to average over the disorder associated with the random spatially varying $\tau(\mathbf{r})$. The fluctuation calculations reported here remain essentially unchanged.[16]

If ODLRO is absent in the mixed state why is it that the high-T_c materials behave as superconductors in a magnetic field? One answer is that for the sizes of system experimentally accessible the fluctuation effects are usually small except for various special regions close to the phase transition boundaries. Even in these regions ODLRO would appear present on short time scales. This is because ODLRO is destroyed by the thermal fluctuations associated with the motion of flux lines and such processes are intrinsically slow and pinning effects will make them even slower. Superconducting phenomena where intrinsic time scales are much shorter than the time scales of flux lattice motion will be hardly affected by the fluctuations. Thus a full treatment of the effects discussed here inevitably requires a dynamical approach but I believe that they may have important consequences for the magnitude of critical currents and the decay of persistent currents.

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